The Mathematical Association of Victoria

Trial Exam 2015

MATHEMATICAL METHODS (CAS)

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 8 pages, with a detachable sheet of miscellaneous formulas in the centerfold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.
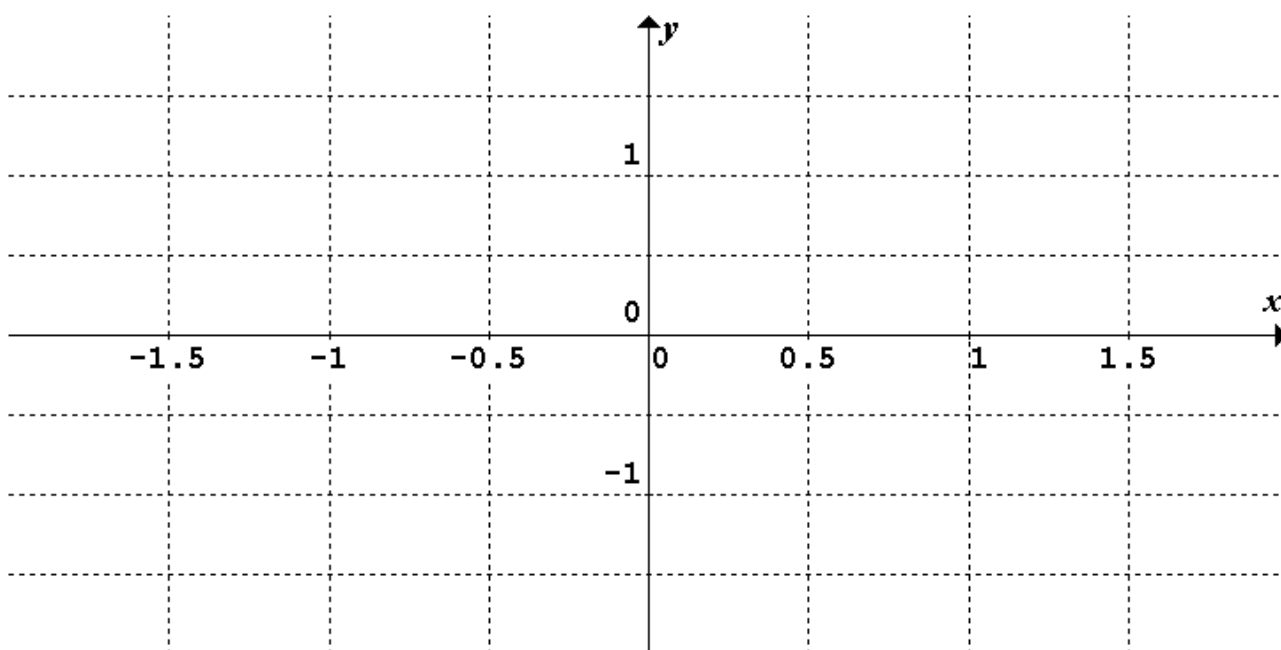
Question 1 (4 marks)

a. If $y = xe^{2x}$ find $\dfrac{dy}{dx}$.   2 marks

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b. Hence find an expression for $\int \left(xe^{2x} \right) dx$.   2 marks

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Question 2 (2 marks)

For what value(s) of k is there a unique solution to the following system of equations?

$$(k-1)x + 2y = 1$$
$$x + (k-1)y = -k$$

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Question 3 (5 marks)

Consider $f: \left[-\dfrac{1}{3}, \infty\right) \to R, f(x) = \log_e(3x+2)$ and $g: [0, \infty) \to R, g(x) = |x-1|$.

a. Write down the rule and the domain for $g(f(x))$. 2 marks

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b. Hence sketch the graph of $y = g(f(x))$, labelling the endpoint and axial intercepts with their coordinates. 3 marks

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Question 4 (4 marks)

Consider the function $h:[0,14] \to R, h(t) = 2\sin\left(\dfrac{\pi}{30}(t+1)\right) + 2$ where h is the height of water, in cm, in a container at time t seconds. The water is being poured into the container at a rate of 2 cm³/s.

a. Find $h'(t)$.   1 mark

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b. Find the rate of change of the volume with respect to the height of the water in the container when $h = 3$.   3 marks

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Question 5 (3 marks)

Let $f(x) = \dfrac{1}{2}\log_e\big(x(x+1)\big)\log_e(2x-1)$.

a. Find $f'(x)$. 1 mark

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b. Hence evaluate $f'(2)$, expressing your answer in the form of $\log_e\big(a^m b^n\big)$ where a and b are prime numbers and m and n are real numbers. 2 marks

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Question 6 (3 marks)

Find the general solution of $2\sqrt{3}\cos(2x) = -3$.

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Question 7 (5 marks)

a. Given $f(x) = 2(x-1)^{\frac{1}{3}}$, find an approximate value for $f(27.99)$ using
$f(x+h) \approx f(x) + hf'(x)$.   3 marks

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b. Will the answer to **part a.** be an under or over estimate of the actual value of $f(27.99)$?
Explain.   2 marks

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Question 8 (5 marks)

A random variable X has a probability density function

$$f(x) = \begin{cases} -(x-1)(x-2) & 1 \le x \le 2 \\ \dfrac{1}{2}x - 1 & 2 < x \le a \\ 0 & \text{elsewhere} \end{cases}.$$

Find the value of a, where a is a real constant.

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Question 9 (9 marks)

An examination consists of 22 multiple choice questions. Each question has five possible answers. Max has not done any work and he has to guess his answers.

a. What is the probability he will get the first three answers correct.　　　　　　　　1 mark

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b. What is the probability he will get two of the first five questions correct?　　　　2 marks

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c. Calculate $\mu \pm 2\sigma$ for the number of questions Max will get right. Interpret the result.　　3 marks

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d. What is the least number of multiple choice questions Max must attempt to ensure that the

probability of getting at least one correct is more than $\dfrac{369}{625}$?　　　　　　3 marks

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END OF QUESTION AND ANSWER BOOKLET