Neap:

Trial Examination 2015

VCE Mathematical Methods (CAS) Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 22 pages and a sheet of miscellaneous formulas.

Answer sheet for multiple-choice questions.

Instructions

Write your **name** and **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Mathematical Methods (CAS) Units 3&4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The maximal domain, D, of the function $f: D \to R$ with rule $f(x) = \frac{\sqrt{x+2}}{\sqrt{x+1}} + \frac{1}{\sqrt{x-1}}$ is

A. $R \backslash \{-1, 1\}$

B. $(1, \infty)$

C. $[-1, \infty) \backslash \{1\}$

D. $(-1, \infty)$

E. $(-1, \infty) \backslash \{1\}$

Question 2

The equation of the normal to the graph of the function whose rule is $y = x^3 + 1$ at the point (1, 2) is

A. $y = 3(x - 1) + 2$

B. $y = -3(x - 1) + 2$

C. $y - 2 = \frac{1}{3}(x - 1)$

D. $y - 2 = \frac{1}{3}(1 - x)$

E. $y + 2 = -\frac{1}{3}(1 + x)$

Question 3

The graph of the function $y = x^2(x - a) + b$, where a and b are constants, has two distinct turning points: a local minimum at the point (2, 10) and a local maximum.

The slope of the straight line joining the turning points is

A. 2

B. –2

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

E. 0

Question 4

Which one of the following is **not** true of the function $f: R \to R,\ f(x) = -|3x - 1| - 1$?

A. The graph of f is continuous everywhere.

B. The graph of f' is not continuous everywhere.

C. $f(x) \le 0$ for all values of $x \le \frac{1}{3}$

D. $f'(x) = 3$ for all values of $x \le \frac{1}{3}$

E. $f'(x) = -3$ for all values of $x > \frac{1}{3}$

Question 5

The inverse of the function $f: R^+ \to R,\ f(x) = 2 - \frac{1}{\sqrt{x}}$ is

A. $f^{-1}: R^+ \to R,\ f^{-1}(x) = \frac{1}{(2-x)^2}$

B. $f^{-1}: R^+ \to R,\ f^{-1}(x) = \frac{1}{x^2} + 2$

C. $f^{-1}: (-\infty, 2) \to R,\ f^{-1}(x) = \frac{1}{(x-2)^2}$

D. $f^{-1}: (-\infty, 2] \to R,\ f^{-1}(x) = \frac{1}{(2-x)^2}$

E. $f^{-1}: (2, \infty) \to R,\ f^{-1}(x) = \frac{1}{(2-x)^2}$

Question 6

The solutions to the equation $4\sin(x)\cos(x) - \sqrt{2} = 0$ for $0 < x < 2\pi$ are

A. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$ and $\frac{11\pi}{4}$

B. $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

C. $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}$ and $\frac{11\pi}{8}$

D. $\frac{\pi}{8}$ and $\frac{3\pi}{8}$

E. $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}$ and $\frac{15\pi}{8}$

Question 7

Given that $f(x) = \cos(x)$, $g(x) = -e^{-x}$ and $h(x) = g(f(x))$, then $h'(x)$ is

A. $\cos(x)e^{-x}$

B. $\sin(x)e^{-x}$

C. $\dfrac{\sin(x)e^{-x-1}}{-x-1}$

D. $-\sin(x)e^{-x}$

E. $-\sin(x)e^{-\cos(x)}$

Question 8

The average value of the function $y = \sin(3x)$ over the interval $\left[0, \dfrac{\pi}{4}\right]$ is

A. $\dfrac{2(\sqrt{2}-2)}{3\pi}$

B. $\dfrac{2(\sqrt{2}+2)}{3\pi}$

C. $\dfrac{-2(\sqrt{2}-2)}{3\pi}$

D. $\dfrac{(2+\sqrt{2})\pi}{24}$

E. $\dfrac{2+\sqrt{2}}{6}$

Question 9

Using a linear approximation with $f(x) = e^{-2x}$, $\dfrac{1}{e^{1.96}}$ is equal to

A. $f(2) - 0.04f'(2)$

B. $f(2) + 0.04f'(2)$

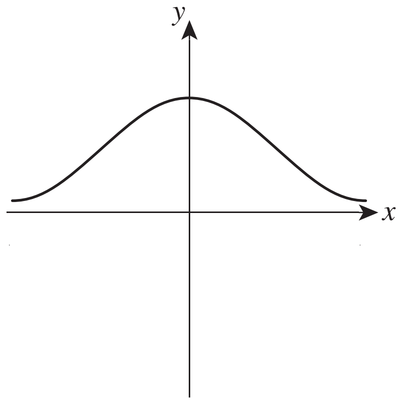
C. $f(1) + 0.02f'(1)$

D. $f(1) - 0.02f'(1)$

E. 0.1409

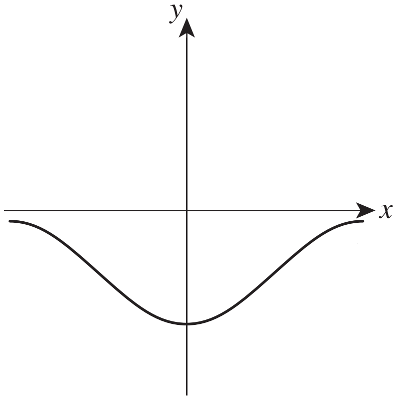
Question 10

The graph of the function *h* is shown below.

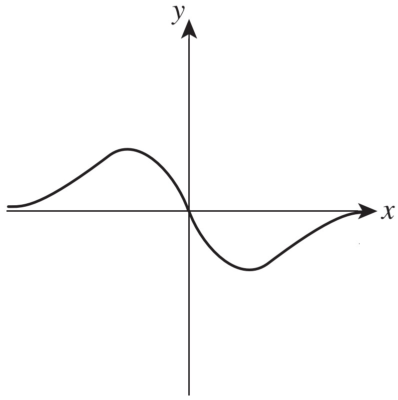


The graph of an antiderivative of *h* could be

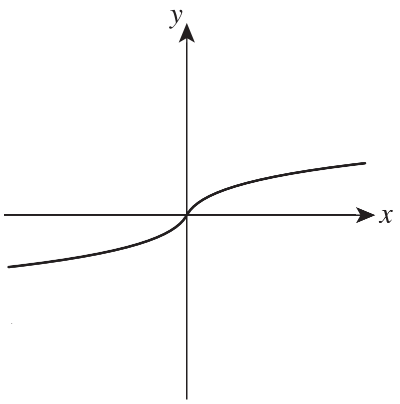
A.



B.



C.



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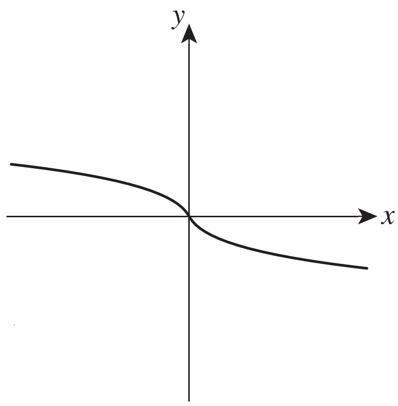

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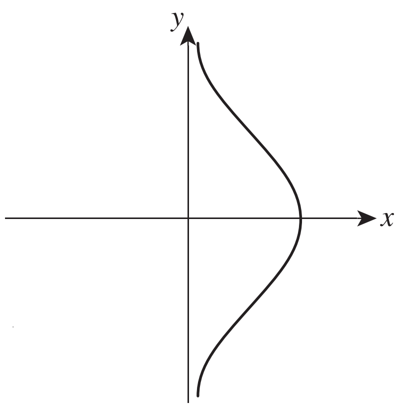


Question 11

The random variable X has a normal distribution with mean 12 and standard deviation 0.75.

If the random variable Z has the standard normal distribution, then the probability that X is greater than 13.5 is equal to

A. $\Pr(Z < -2)$

B. $\Pr(Z > 1.5)$

C. $\Pr(Z \leq -1.5)$

D. $\Pr(Z < 2)$

E. $1 - \Pr(Z < -2)$

Question 12

Given that $f(x) = -x^3$ and $g(x) = -x^2$, which of the following is **incorrect**?

A. $f(x)f(y)f(z) = f(xyz)$

B. $g(f(x)) = -f(g(x))$

C. $f(g(x+y) - g(x-y)) = g(8)f(x)f(y)$

D. $g(g(g(x))) = f(f(x))$

E. $g(y)g(z) = -g(yz)$

Question 13

Given that $f'(x) = 2^{3x+1}$, $f(x)$ could be

A. $f(x) = \dfrac{2 \times 8^x}{\log_e(8)} - \pi$

B. $f(x) = \log_2(x) - 1$

C. $f(x) = \frac{1}{3}(2^{3x})$

D. $f(x) = \log_e(8)2^{3x+1}$

E. $f(x) = -\log_e(8)2^{3x+1}$

Question 14

Suppose $g(x)$, $h(x)$ and $f(x)$ are all differentiable functions with domain and range R.

If $h(x) = f(g(x))$, which one of the following is true?

A. If $h(x)$ has a stationary point at $x = a$, then $f(x)$ will have a stationary point at $x = a$.

B. If $g(x)$ does not have a stationary point at $x = a$, then $h(x)$ will not have a turning point at $x = a$.

C. If $g(x)$ has a stationary point at $x = a$, then $h(x)$ has a stationary point at $x = a$.

D. If $f(x)$ has a stationary point at $x = a$, then $h(x)$ has a stationary point at $x = a$.

E. If $f(a) = b$ and $g(b) = c$, then $h(a) = c$.

Question 15

A water tank is constructed in the shape of a cone whose diameter is equal to its height. The cone is inverted and supported on a suitable stand. The tank is filled and water leaks out from the lowest point. It is reasonable to assume that the water will leak out at a rate proportional to the depth (h) of water in the tank. If V is the volume of water remaining in the tank, then the rate of decrease of volume is $\frac{dv}{dt} = kh$, where k is a constant.

Therefore $h'(t)$, the rate that the depth decreases with time, is

A. $\pi k h^3$

B. $4\pi k h^2$

C. $\frac{k}{\pi h}$

D. $\frac{4k}{\pi h}$

E. $\frac{\pi k h^3}{4}$

Question 16

Let $f\colon \left[0, \frac{\pi}{3}\right] \to R,\ f(x) = 1 + \cos(3x)$.

The graph of f is transformed by a dilation of factor 3 from the y-axis, followed by a reflection in the x-axis. The resulting graph is defined by

A. $g\colon \left[0, \frac{\pi}{3}\right] \to R,\ f(x) = -1 - \cos(x)$

B. $g\colon [0, \pi] \to R,\ f(x) = -1 - \cos(3x)$

C. $g\colon \left[0, \frac{\pi}{3}\right] \to R,\ f(x) = -1 - 3\cos(3x)$

D. $g\colon [0, \pi] \to R,\ f(x) = -1 - \cos(x)$

E. $g\colon [0, \pi] \to R,\ f(x) = 1 - \cos(9x)$

Question 17

If $\int_1^4 f(x)dx = 6$, then $\int_1^4 3(f(x) - 1)dx$ is equal to

A. 3
B. 9
C. 13
D. 15
E. 17

Question 18

Let X be a discrete random variable with a binomial distribution. The mean of X is 3.2 and the standard deviation is 0.8.

The values of n (the number of independent trials) and p (the probability of success in each trial) are

A. $n = 8, p = 0.4$

B. $n = 4, p = 0.8$

C. $n = 16, p = 0.2$

D. $n = 2, p = 1.6$

E. $n = 4, p = 0.75$

Question 19

The average rate of change of the function with rule $f(x) = x^3 - 2\sqrt{x+1}$ over the interval [0, 3] is

A. 0

B. 5

C. $\frac{23}{3}$

D. $\frac{25}{3}$

E. $\frac{31}{3}$

Question 20

If A and B are independent events of a sample space with $\Pr(A) = a$ and $\Pr(B) = b$, where $a \in (0, 1)$ and $b \in (0, 1)$, then $\Pr(A' \cup B')$ is equal to

A. $1 - 2a - 2b = ab$

B. $2 - a - b$

C. $ab - (1 - a)(1 - b)$

D. $ab - 1$

E. $1 - ab$

Question 21

The system of simultaneous equations

$$y - 4x = x^2$$

$$2y - 2ax = -b$$

where a and b are positive real constants, has a unique solution consisting of a single point (x, y), provided that the constants a and b satisfy

A. $a = 4$

B. $a = 4 \pm \sqrt{2b}$

C. $a \neq 4$

D. $a \neq 4 \pm \sqrt{2b}$

E. $a = 4 \pm \sqrt{b}$

Question 22

Consider the equation $||x-4|-4| = k$, where k is a real constant. Let the number of solutions, N, to this equation for a given value of k be given by the function $N(k)$.

The range of the function N is

A. $\{2\}$

B. $\{0, 2\}$

C. $\{2, 4\}$

D. $\{0, 2, 3, 4\}$

E. $\{0, 1, 2, 3, 4\}$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (12 marks)

Consider the region *A* bounded by the curve $y = 4x - x^2$ and the *x*-axis for $x \in [0, 4]$.

a. Find the area of region *A*. 1 mark

b. Region *A* can be subdivided into two regions, *B* and *C*, by the line $y = x$. The points in *B* are above the line $y = x$ and the points in region *C* are below it.

Find the coordinates of the points of intersection of the line $y = x$ with the curve $y = 4x - x^2$. 1 mark

c. Find the area of region *B*. 2 marks

d. Consider the equation $y = mx$, where m is a real constant such that $0 < m < 4$. This equation defines a whole family of lines. For each value of m, the line will intersect the curve $y = 4x - x^2$ at the origin and also at one other point, P_m. The line $y = mx$ will divide the region A into two regions, B_m and C_m (with points in region B_m above the line $y = mx$).

Find the coordinates of the point P_m as a function of m. 2 marks

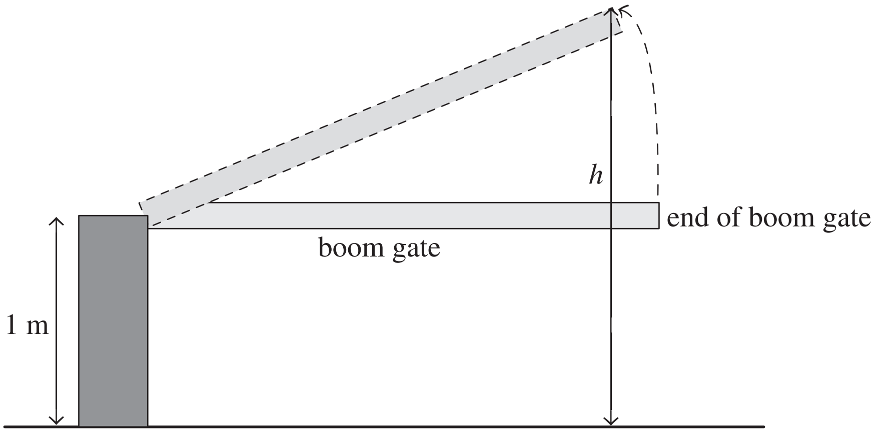
e. Find the area of region B_m as a function of m. 2 marks

f. Hence, find an expression for the area of region C_m in terms of m. 2 marks

g. Find the particular value of m for which the two regions B_m and C_m are equal in area. 2 marks

Question 2 (14 marks)

The boom gate at the entrance to an industrial estate is malfunctioning. It is not responding to manual operation, but periodically opening and closing in a pattern that repeats every hour.



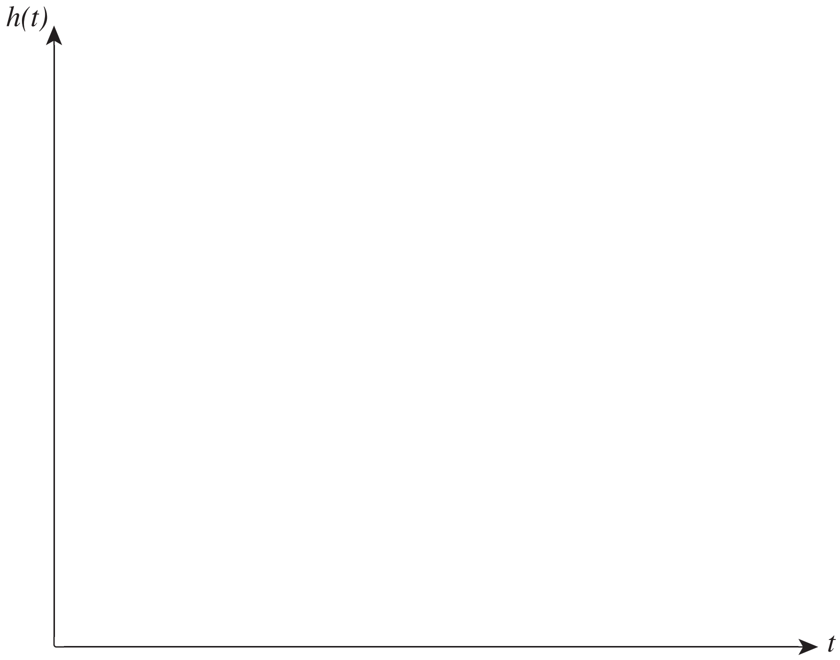
The height above the ground, h metres, of the end of the boom gate at time t minutes is given by the continuous function

$$h(t) = \begin{cases} \left|3\sin\left(\frac{\pi}{20}(t-10)\right)\right| + 1 & t \in [0, 50] \\ k & t \in [0, 10) \cup (50, 60] \end{cases}$$

where $t = 0$ corresponds to 6:00 am on Saturday.

a. Find the value of k. 1 mark

b. Sketch the graph of $h(t)$ for $t \in [0, 60]$, labelling the endpoints with coordinates. 3 marks



c. Find at what time(s) during the first hour the boom gate reaches its maximum height above the ground. 1 mark

A van has come to make a delivery to one of the warehouses in the estate. It can only gain access when the boom gate is at least 3 metres above the ground. The driver of the van arrives at the boom gate at 9:10 am.

d. How long does the driver have to wait to gain access through the boom gate and into the estate? Express your answer in minutes, correct to two decimal places. 2 marks

e. How many minutes of each hour would the driver be able to drive the van through the boom gate? Express your answer in minutes, correct to two decimal places. 2 marks

The time, T minutes, that the driver takes to enter through the boom gate, unload his van and return to the boom gate is dependent on the mass, m kg, of his load, where

$$T = e^{0.005m} + 5 \qquad 0 \le m \le 1000$$

Assume that the driver enters through the boom gate at the first possible opportunity after his arrival at 9:10 am.

f. Find the total time, in minutes, that elapsed between the driver arriving at the boom gate and exiting the boom gate, given that the load he is delivering is 200 kg. Give your answer correct to two decimal places. 2 marks

A second delivery van arrives at 9:28 am, carrying a load of mass 500 kg.

g. What is the earliest time this van will be able to leave the estate after having delivered its load to the warehouse? Give your answer correct to the nearest minute. 3 marks

Question 3 (16 marks)

The 'Tinman' is an event with two different disciplines: a swim and kayak. Timothy is training for this event. Each morning he will either swim or paddle his kayak.

If Timothy swims one morning, the probability he swims the next morning is 0.3. If he paddles one morning, the probability he swims the next morning is 0.6. On Monday morning Timothy goes for a paddle.

a. **i.** What is the probability that he will paddle the next two mornings? 1 mark

ii. What is the probability he will paddle just once over the next three mornings? 2 marks

iii. What is the probability, correct to three decimal places, that Timothy will swim on Friday morning? 2 marks

b. Over the long term of his training, what percentage of mornings does Timothy train for the paddle? Express your answer correct to two decimal places. 1 mark

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When Timothy goes paddling, the time taken to complete his paddle, X (minutes), is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{1000}(x-20) & 20 \le x \le 40 \\ \frac{1}{4000}(120-x) & 40 \le x \le 120 \\ 0 & \text{elsewhere} \end{cases}$$

c. **i.** Find the probability that Timothy will paddle for less than 30 minutes on a given morning. 2 marks

ii. Hence, find the probability, correct to three decimal places, that Timothy paddles less than 30 minutes on three of the next ten mornings that he paddles. 2 marks

d. Due to work commitments, Timothy finds he will now only be able to paddle for more than 30 minutes on the weekends.

What is the probability that Timothy paddles more than 60 minutes on a given day on the weekend? 3 marks

e. Timothy paddles less than n minutes 80% of the time.

Find the values of n. 3 marks

Question 4 (8 marks)

A new solar system has recently been found with a sun called Mathica. This solar system has two known planets, P1 and P2, which orbit Mathica within the same plane and in circular orbits. In this solar system, the period of planet P1 is found to be one year and the radius of its orbit is taken to be a length of 1 unit.

Planet	Orbit radius (years)	Orbit period (units)
P1	$R_1 = 1$	$T_1 = 1$
P2	R_2	T_2

With Mathica at the origin, the centre of planet P1 has coordinates $(\cos(2\pi t), \sin(2\pi t))$ at time t, and the centre of the planet P2 has coordinates $\left(4\cos\left(\frac{\pi t}{4}\right), 4\sin\left(\frac{\pi t}{4}\right)\right)$.

a. Deduce the values of R_2 and T_2 from the expressions given for the coordinates of P2. 2 marks

b. The radius, R, and period, T, of a planetary orbit around any star are known to be related by a general law of the form $R = kT^q$, where k and q are constants whose values can be found by using observed values of R and T.

i. By substituting the values of R_2 and T_2 into this law, derive an equation involving the values of k and q. Then derive another equation by substituting the values of R_1 and T_1. 1 mark

ii. Deduce the values of the constants k and q, and hence write down the law relating the radius and the period for planets in the Mathica system. 1 mark

Another general law for planetary orbits describes at what orbital radii planets are found. Suppose R_n is the orbital radius of the nth planet from the central star (counting outward from the innermost). Such a law is given by $R_n = a + b \times 2^n$, where a and b are real constants that depend on the particular star and the units of measurement chosen.

c. Find the values of a and b for the Mathica system, assuming P1 is the innermost planet and P2 is the second planet. 2 marks

d. Assume that a third planet, P3, of Mathica exists.

Assuming that no other planets of Mathica exist, and using the laws discussed above, predict the orbital radius and deduce the orbital period of this third planet from Mathica. 2 marks

Question 5 (8 marks)

Consider the function $f: [-1, 2] \rightarrow R, f(x) = x^5 - 5x + 2$.

a. Find the coordinates of the endpoints and any stationary points of the function f. 3 marks

b. Find the x- and y-intercepts of the graph of the function f, correct to four decimal places. 2 marks

c. Sketch a graph of the function f over its domain. 2 marks

d. Find the range of the function *f*. 1 mark

END OF QUESTION AND ANSWER BOOKLET

