2016 Teacher-exclusive Exam
For 2016-18 VCE Study design

Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	20	20	20
B	4	4	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers and a bound reference.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- A calculator is allowed in this examination.

Materials supplied:

- This question and answer booklet of 19 pages including a formula sheet.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Questions

Question 1

The maximal domain of the function $f(x) = 2\,log_e\left(\frac{x}{2} - 1\right)$ is:

A. $(2, \infty)$
B. $(1, \infty)$
C. $\left(\frac{1}{2}, \infty\right)$
D. $(-2, \infty)$
E. R

Question 2

Let $f(x) = -3x^2 - 4x + 1$. The average rate of change with respect to x, between $x = 1$ and $x = 3$ is:

A. -32
B. -16
C. $\frac{1}{16}$
D. -22
E. -11

Question 3

The equation of the inverse of $\boldsymbol{f(x) = e^{x-3} - 2}$ is:

A. $f^{-1}(x) = -log_e(x+2) - 3$
B. $f^{-1}(x) = -log_e(x+2) + 3$
C. $f^{-1}(x) = log_e(x+2) + 3$
D. $f^{-1}(x) = log_e(x-2) + 3$
E. $f^{-1}(x) = -log_e(2-x) - 3$

Question 4

Let $f: (-1,2] \to R, f(x) = -2x^2 + 7$. The range of $f(x)$ is:

A. $[-1,7]$
B. $[-1,5]$
C. $[-1,5)$
D. $(1,7)$
E. $[5,7]$

Question 5

The following simultaneous equations have an infinite number of solutions when:

$$(m+2)x + y = -n$$

$$4x - (1-m)y = 8$$

A. $\mathrm{m} = -3$ and $\mathrm{n} = 2$
B. $\mathrm{m} = -3$ and $\mathrm{n} \neq 2$
C. $\mathrm{m} = 2$ and $\mathrm{n} \neq -8$
D. $\mathrm{m} = -3 \text{ or } 2$
E. $\mathrm{m} \in \mathrm{R}\backslash\{-3, 2\}$

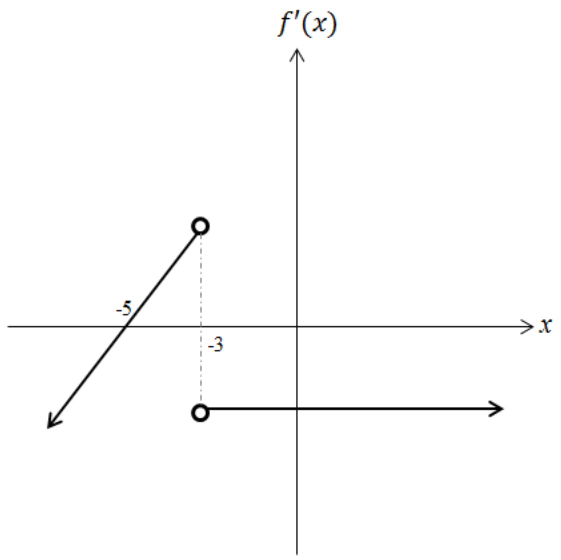
Question 6

Given that $\int_{-1}^{3} f(x)dx = 2$, the value of $\int_{3}^{-1} 3f(x) - 2\, dx$ is:

A. -14
B. 14
C. 10
D. 2
E. 6

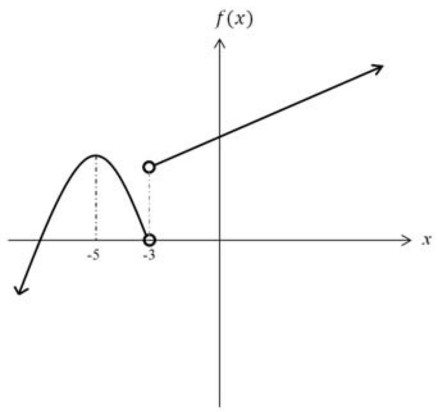
Question 7

The gradient function $f'(x)$ is shown below.

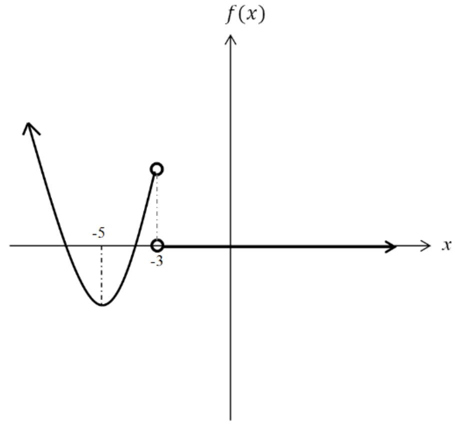


The graph of $f(x)$ could be:

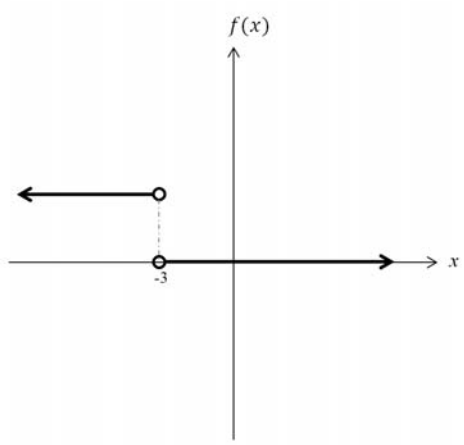
A.



B.



C.



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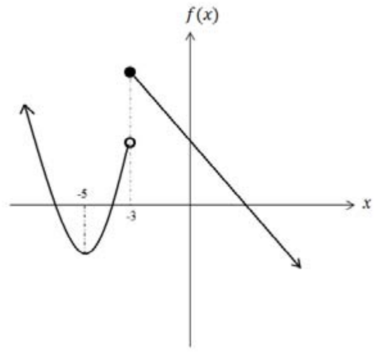
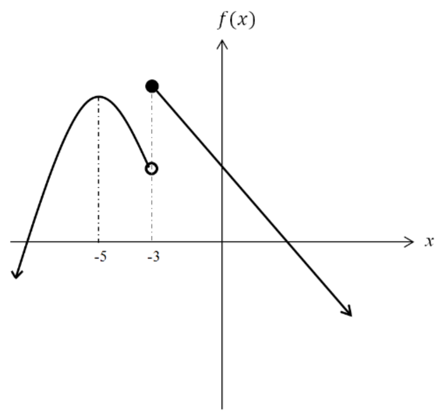


E.



Question 8

The function $f(x) = x^3 - 3x + 2$ is strictly increasing over the domain:

A. $(-\infty, 1]$
B. $[-1,1]$
C. $(-\infty, -1]$
D. R
E. $[1, \infty)$

Question 9

The image of the equation $y = -x^2$ after applying the following series of transformations is:

- Reflection in the y-axis
- Translation by 3 units in negative y-direction
- Dilation by factor $\frac{1}{2}$ from the y-axis

A. $\mathrm{y} = -\frac{1}{2}x^2 - 3$
B. $y = -4x^2 - 3$
C. $\mathrm{y} = 4x^2 - 3$
D. $\mathrm{y} = -\frac{1}{2}x^2 + 3$
E. $\mathrm{y} = \frac{1}{2}x^2 - 3$

Question 10

Let $\mathrm{g}'(\mathrm{x}) = -3\mathrm{x}^2 + \mathrm{x} - 1$. If $\mathrm{g}(1) = \frac{9}{2}, \mathrm{g}(-2)$ is:

A. 12
B. 18
C. 14
D. 2
E. 6

Question 11

The average value of the function $h(x) = -\frac{\sin(x)}{x}$ for $\frac{1}{2} \leq x \leq 3$ is approximately:

A. -0.0175

B. -1.8487

C. 0.0733

D. -1.3555

E. 0.6489

Question 12

The signed area enclosed by the function $f(x) = 10xe^{2x}$ and the x axis is:

A. A. $\frac{4}{3}$
B. B. $-\frac{5}{2}$
C. C. ∞
D. D. $\frac{5}{2}$
E. E. $\frac{3}{2}$

Question 13

The tangent to the curve $y = \sin(2x)$ at $x = \frac{\pi}{6}$.

A. $y = x - \frac{\pi}{6} + \frac{\sqrt{3}}{2}$
B. $y = x + \frac{\pi}{6} - \frac{\sqrt{3}}{2}$
C. $y = \sqrt{3}x - \frac{\sqrt{3}}{6}\pi + \frac{\sqrt{3}}{2}$
D. $y = x - \frac{\pi}{6} + \frac{1}{2}$
E. $y = \sqrt{3}x - \frac{\sqrt{3}}{6}\pi + \frac{1}{2}$

Question 14

A random variable, X, is binomially distributed with standard deviation of 9. Find the mean and p when the number of trials is 900 and p<0.5.

A. mean=90, p=0.9
B. mean=81, p=0.1
C. mean=810, p=0.9
D. mean=900, p=0.1
E. mean=90, p=0.1

Question 15

For events A and B, $\Pr(A) = 0.55$, $\Pr(A \cap B) = 0.30$ and $\Pr(A' \cap B') = 0.10$. $\Pr(A|B)$ is closest to:

A. 0.8564
B. 0.4615
C. 0.4013
D. 0.3575
E. 0.2755

Question 16

A factory produces blue ball point pens. The number of faulty pens produced per month, X, is normally distributed with mean 21 and variance 16. The probability that there are less than 14 faulty pens produced in a month is:

A. 0.4010
B. 0.0058
C. 0.0401
D. 0.9599
E. 0.0528

Question 17

Random variable X is normally distributed with mean of 20. If the 70th percentile is 25.6, the standard deviation of X is approximately:

A. 11.5
B. 4.6
C. 9.2
D. 10.8
E. 10.7

Question 18

The probability density function for the continuous random variable is given by:

$$\mathrm{f(x)} = \begin{cases} e^{-2x} + 3, & 0 \leq x \leq a \\ 0, & \text{elsewhere} \end{cases}$$

The value of a is closest to:

A. 1.55
B. 1.41
C. 0.98
D. 0.26
E. -1.28

Question 19

In Engage High School, 50% of 496 Year 11 students are female and 55% of 380 Year 12 students are male. If a sample size of 100 is required for a survey, how many Year 11 and Year 12 boys respectively should be selected?

A. 28, 24
B. 28, 20
C. 24, 20
D. 24, 24
E. 28, 20

Question 20

In a class of 20, 17% of students are planning on going overseas during the term break. The 99% confidence interval for the proportion of school population that will travel overseas is:

A. $(0.17 - 1.96\sqrt{\frac{0.17*0.83}{20}}, 0.17 + 1.96\sqrt{\frac{0.17*0.83}{20}})$
B. $(0.17 - 2.58\sqrt{\frac{0.17*0.83}{20}}, 0.17 + 2.58\sqrt{\frac{0.17*0.83}{20}})$
C. $(0.17 - 2.58\sqrt{0.17 * 0.83}, 0.17 + 2.58\sqrt{0.17 * 0.83})$
D. $(0.17 - 0.99\sqrt{\frac{0.17*0.83}{20}}, 0.17 + 0.99\sqrt{\frac{0.17*0.83}{20}})$
E. $(0.17 - 1.96\sqrt{\frac{0.17*0.17}{20}}, 0.17 + 1.96\sqrt{\frac{0.17*0.17}{20}})$

Section B – Short-answer questions

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

The depth, of the ocean varies throughout the day according to the equation,

$$h: [0,24] \rightarrow R, h(t) = 2\,sin\left(\frac{\pi t}{6} - \frac{3\pi}{4}\right) + 4$$

where t is time in hours and $h(t)$ is depth in meters. Midnight, 12:00 am, is denoted as $t = 0$.

a. Find:

i) the amplitude of $h(t)$

1 mark

ii) the period of $\mathrm{h(t)}$

1 mark

b. Sketch the graph of $h(t)$, labelling the end points, maximum and minimum.

4 marks

c. What is the rate of change of the depth of the ocean at 2:30 am?

2 marks

d. Determine the time after 12:00 pm when the ocean first reaches

i. Its lowest depth

1 mark

ii. Its highest point.

1 mark

e. A ferry that transports people from the mainland to the Lemon Island can only operate when the depth of the water is greater than 2.6 m. If the boat operates everyday between 6 am to 8pm and takes 45 minutes for a round trip, find the maximum number of round trips that the ferry can do on a given day.

4 marks

Total: 14 marks

Question 2

Consider the following equation:

$$f(x) = -\frac{x^3}{3} + 4x - \frac{16}{3}$$

a. Factorise $f(x)$, given that $(x-2)$ is one of the factors.

3 marks

b. Find the coordinates of all turning points and determine their nature.

3 marks

c. Sketch $f(x)$.

3 marks

d. If $f^{-1}(x)$ exists for $x \in [a, \infty)$, find the minimum value of a.

1 mark

e. Find the equation of tangent to $f(x)$ at $x = -3$.

3 marks

f. A point $Q(a, b)$is on the tangent found in (e.). If the distance, L, from $Q(a, b)$ to the origin $O(0,0)$ is minimum, find a, b and L to 3 decimal places.

4 marks

g. Find the area enclosed by the tangent from (e.) and $f(x)$.

3 marks

Total: 20 marks

Question 3

Jen, Amy, Dennis and Calvin are all State swimmers.

a. On Thursday, Jen and Amy did a total of seven 100m freestyle races. The probability that Jen wins the 100m race is 0.48.

i. Find the probability that Jen won more than Amy.

3 marks

ii. Jen and Amy did 50 races this week. Find the probability that Amy won 29 times given that she won more than half of the total number of races.

2 marks

b. The time taken, T, for Dennis to complete 100m freestyle is a continuous random variable with probability density function given by the following:

$$f(t) = f(x) = \begin{cases} -\frac{1}{800}(x-60)^2 + \frac{1}{10}, & M-a \le x \le M+a \\ 0, & elsewhere \end{cases}$$

i) Find M.

1 mark

ii) Find a to 3 decimal places.

3 marks

iii) Find the average time taken for Dennis to complete 100m freestyle to 3 decimal places.

2 marks

iv) Find the probability that Dennis finishes within 56 seconds.

2 marks

Total: 13 marks

Question 4

The increase in population of bacteria *E.coli* incubated at 21°C follows the exponential function, where t is time in hours and N is the number of *E.coli*:

$N(t) = 15 * e^{t} + 200$

a. Find the initial *E.coli* population.

1 mark

b. Approximate the number of *E.coli* after 20 minutes of incubation to the nearest whole number.

1 mark

c. Due to the limited availability of the nutrient in the Petri dish where *E.coli* is incubated, the number of E.coli will reach its maximum value when $\mathrm{N(t)}$ reaches 100 times the initial value. Once this maximum value has reached, the *E.coli* population undergoes an exponential decay modelled by the equation $D(T) = ae^{-\frac{1}{100}T} + 500$, where T is the time in hours. Find the value of a to the nearest whole number.

3 marks

A microbiologist realised that the initial *E.coli* population is contaminated with antibiotic resistant strain of *E.coli*. Assume that in *E.coli* population of interest, 2% is resistant to antibiotics.

d. If the *E.coli* population of interest is 120 million, find the smallest sample size of *E.coli* that would be considered large.

1 mark

e. The microbiologist collects multiple samples of 5000 *E.coli* and finds out the percentage of antibiotic resistant bacteria each sample contains. Find the mean and standard deviation for the distribution of the sample proportion $\hat{p}$.

3 marks

f. In a sample of 100 000 *E.coli*, the microbiologist found that 3.5% of them belong to pathogenic strains that cause severe abdominal pain. Find the 90% confidence interval for the proportion of the *E.coli* population of interest that is pathogenic and give a brief interpretation of this interval.

4 marks

Total: 13 marks

Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Booklet