Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 11 pages and a formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

a. If $y = x\ln(x)$, find $\frac{dy}{dx}$.

1 mark

b. If $y = (x^3 - 2x)^2$, find $\frac{dy}{dx}$ at $x = 2$.

2 marks

Total: 3 marks

Question 2

Find an antiderivative of $\cos(-2x+3)$.

2 marks

Question 3

Given $g(x) = 2e^{3x} + 1$:

a. Find the inverse function of g.

2 marks

b. Find $g(g^{-1}(x))$, including its domain.

2 marks

Total: 4 marks

Question 4

The number of phone calls, X, that Bob receives at home in a day is a random variable with probability distribution given by:

x	0	1	2	3
$\Pr(X = x)$	0.25	0.1	0.25	0.4

a. Find the mean of X.

2 marks

b. What is the probability that Bob receives two calls three days in a row?

1 mark

c. What is the probability that Bob receives three calls over a two day span?

3 marks

Total: 6 marks

Question 5

The transformation $T: R^2 \rightarrow R^2$ is equivalent to a dilation in the x-direction by a factor of three, followed by a reflection in the y-axis, followed by a translation of 2 units in the positive y-direction.

a. Find the coordinates of the image of the point $(0, -3)$ under T.

1 mark

b. Express T in the form $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$, where a, b, c and d are real numbers.

2 marks

c. Find the **pre-image**, g, of $f: [3,5) \rightarrow R, f(x) = \ln(x)$ obtained under T.

3 marks

Total: 6 marks

Question 6

Find the solutions of $2\sin(2x - \frac{\pi}{2}) + \sqrt{2} = 0$ for $x \in [0, 2\pi]$.

3 marks

Question 7

Solve $\ln(x-3) - 2\ln(x+1) + \ln(2x-1) = 0$.

3 marks

Question 8

a. The random variable X is normally distributed with mean 150 and standard deviation 11. If $\Pr(X < 139) = q$, find $\Pr(X > 161 \mid X > 150)$ in terms of q.

2 marks

b. The probability density function f of the random variable X is given by $f(x) = \frac{x-2}{8}, 2 \le x \le 6$. Find k such that $\Pr(X \le k) = \frac{1}{2}$.

3 marks

Total: 5 marks

Question 9

a. $f(x) = x^2 \ln x$. Find $f'(x)$.

1 mark

b. Use your answer from part a to find $\int_1^e x \ln(x)\ dx$.

3 marks

Total: 4 marks

Question 10

Given $f(x) = x^3 - kx$, $g(x) = kx$, show that the area bounded by f and g is four times the size of the area bounded by f and the x-axis. Consider only the regions satisfying $x > 0$, and assume $k > 0$.

4 marks

Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Booklet

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