PERA AD ASTRA

Girton

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NAME: Solutions. YEAR/HOUSE: ____________

TEACHER'S NAME: Students circle Mr Jones Mr James Mrs Itter

SEMESTER 2 EXAMINATIONS NOVEMBER 2016

Year Eleven Mathematical Methods

Reading time: 15 Minutes

Writing time: 120 Minutes

Marks Allocated:

Section	Number of Questions	Number of Marks
Section B: Multiple Choice	20 Questions	20 Marks
Section C: Extended Answer	5 Questions	60 Marks

Specific Instructions

Use of calculators and summary book are permitted

SECTION 1

Answer all questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is correct for the question.

A correct answer scores 1, an incorrect answer scores 0. Marks will not be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question

SECTION 2

Answer all questions in the spaces provided. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Supplies and Equipment

Supplies: Please ensure you have the correct supplies/instruments for taking the examination before you enter the examination venue (e.g. pencils, pens, calculator, ruler, etc). There will be no sharing allowed. No other paper, etc. will be allowed to come in with you unless instructed as Specific Instructions. A clear bottle containing only water is permissible.

At the Conclusion: Please wait quietly for specific instruction as to how you will be dismissed. Leave your examination paper on your table. Pick up unwanted papers around you, push your chair under the table, and put your rubbish in the bin on your way out of the examination room.

Multiple Choice Answer Sheet

Year 11 Mathematical Methods : Exam 2 Semester 2 2016

Section B Multiple Choice **NAME:** Solutions

Teacher (please circle): Mr James Mr Jones Mrs Itter

Clearly circle the best answer for each question. If you wish to alter your choice, rewrite the question number and the new answer next to it.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E

Section A	**/40**
Section B	**/20**
Section C	**/60**
Total	**/120**
Percentage	____%

SECTION B: Multiple Choice. **20 questions** **20 marks**

Calculator and summary book allowed.

Place answers on provided multiple choice answer sheet.

Question 1

The period and the amplitude of the graph of $y = 2\cos(3x) - 1$ are given respectively by

A. $\frac{2\pi}{3}$ and 2

B. $\frac{3\pi}{2}$ and 2

C. 6π and 1

D. 6π and 2

E. 2π and 3

Question 2

When converted from degrees to radians the angle $135°$ is equal to

A. $\frac{3\pi}{4}$

B. 0.75

C. $\frac{4\pi}{3}$

D. 4.2

E. $\frac{3\pi}{2}$

Question 3

An approximation for $\sin\left(\frac{\pi}{20}\right)$ is

A. 0.00274

B. $\frac{\pi}{10}$

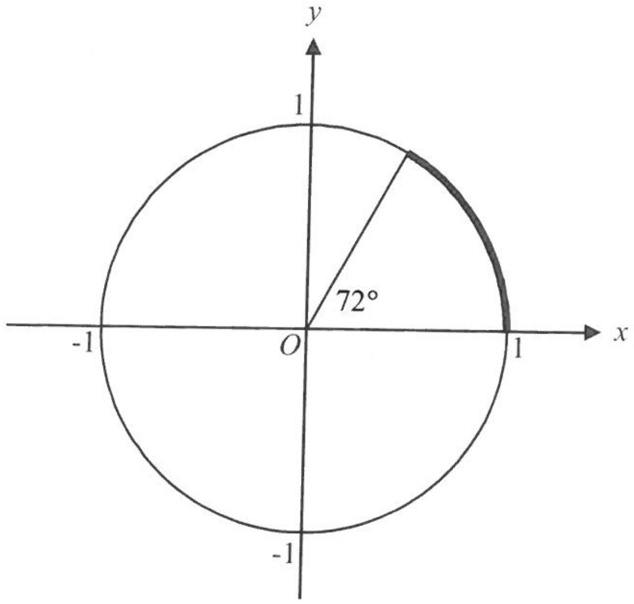
C. 0.01571

D. $\frac{\pi}{20}$

E. 0.03141

Question 4

The diagram below shows a unit circle. The bold section of the unit circle represents the arc subtended by the angle of 72°



The length of the arc subtended by the angle 72° is

A. $\frac{1}{5}$ units

B. $\frac{\pi}{5}$ units

C. 72 units

D. $\frac{2}{5}$ units

E. $\frac{2\pi}{5}$ units

Question 5

The height of a machine part changes according to the rule $h = 4 - 2\cos\left(\frac{\pi t}{3}\right)$, where h is the height of the part, in metres above the ground, and t is the number of hours after 3pm. At 5pm on the same day, the height of this machine part above the ground is

A. 2 metres
B. 3 metres
C. 4 metres
D. 5 metres
E. 6 metres

$t = 2$

$h = 4 - 2\cos\frac{2\pi}{3}$

$= 4 - 2 \times -1/2$

$= 4 + 1$

$= 5$

Question 6

Water-flows in a canal are being monitored. It is found that the depth of water, d, in metres, in the canal after t hours of monitoring is given by

$$d = 5 + 2\sin\left(\frac{\pi t}{6}\right), \ t \geq 0.$$

The time that elapses before the canal first returns to its original depth is

A. 3 hours
B. 5 hours
C. 6 hours
D. 7 hours
E. 12 hours

Question 7

The maximal domain of the function with rule $y = 3\log_e(2 - x)$ is

A. $x \in (-\infty, 2)$
B. $x \in (-\infty, 2]$
C. $x \in (-\infty, 6)$
D. $x \in (2, \infty)$
E. $x \in [2, \infty)$

Question 8

Let $f:[0,\infty) \to R, f(x) = 5^{2x} + 3$.
The range of *f* is

A. $(-\infty, 3]$
B. $(3, \infty)$
C. $[3, \infty)$
D. $(4, \infty)$
E. $[4, \infty)$

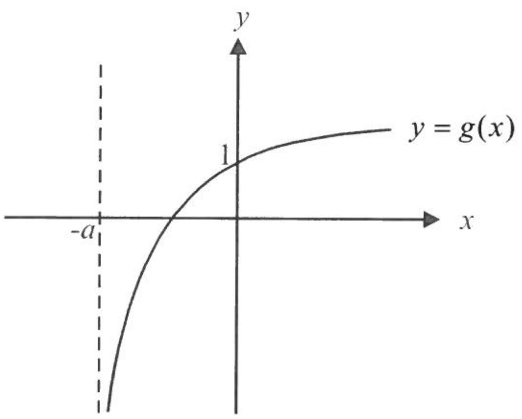
Question 9

A circle has its centre located at the point $(2,-1)$ and has a radius of 4 units.
The equation of the circle is

A. $(x-2)^2 + (y+1)^2 = 4$
B. $(x-2)^2 + (y+1)^2 = 16$
C. $(x+2)^2 + (y-1)^2 = 16$
D. $(x-1)^2 + (y+2)^2 = 16$
E. $(x+1)^2 + (y-2)^2 = 4$

Question 10

The graph of *g* is shown below.



The rule for *g* could be

A. $y = \log_a(x-a)$
B. $y = 2\log_a(x-a)$
C. $y = \log_a(x+a)$
D. $y = 2\log_a(x+a)$
E. $y = a\log_a(x+1)$

Question 11

If **A** = $\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$ then **A**$^{-1}$ =

A $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ **B** $\begin{bmatrix} -1 & -1 \\ -3 & -2 \end{bmatrix}$ **C** $\begin{bmatrix} -2 & -3 \\ -1 & -1 \end{bmatrix}$ **D** $\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$ **E** cannot be calculated

Question 12

If **A** = $\begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix}$ and **AX** = $\begin{bmatrix} 14 & 20 \\ 38 & 48 \end{bmatrix}$ then **X** is equal to

$$\begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix} X = \begin{bmatrix} 14 & 20 \\ 38 & 48 \end{bmatrix}$$

A $\begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix}$ **B** $\begin{bmatrix} 6 & -2 \\ -8 & 4 \end{bmatrix}$ **C** $\begin{bmatrix} 8 & 24 \\ 40 & 32 \end{bmatrix}$ **D** $\begin{bmatrix} -4 & 8 \\ 2 & -6 \end{bmatrix}$ **E** $\begin{bmatrix} -4 & 2 \\ 8 & -6 \end{bmatrix}$

Question 13

If $M = 50 \times 2^{5t}$, then when *M* = 300, *t* equals:

A. 0.095
B. 0.6
C. 1.593
D. 2.585
E. 0.517

$$300 = 50 \times 2^{5t}$$
$$6 = 2^{5t}$$
$$\log_{10} 6 = \log_{10} 2^{5t}$$
$$\log_{10} 6 = 5t \log_{10} 2$$
$$5t = \frac{\log_{10} 6}{\log_{10} 2}$$

Question 14

Matt has four hard boiled eggs which got mixed up in his fridge with five raw eggs. He randomly selects three eggs from the fridge to put in a cake.
The probability that two of the eggs he selects are hard-boiled is closest to

A. $\frac{3}{8}$
B. $\frac{5}{14}$
C. $\frac{17}{56}$
D. $\frac{20}{81}$
E. $\frac{28}{243}$

$$\frac{{}^4C_2 \times {}^5C_1}{{}^9C_3}$$
$$= \frac{6 \times 5}{84}$$
$$= \frac{30}{84}$$
$$= 5/14$$

Question 15

Jane is late for work 5% of the time. Her colleague Mardi is late 20% of the time. The probability of Jane being late is independent of the probability of Mardi being late.
The probability that one or more of the women are late for work on a particular morning is

A. 0.01
B. 0.23
C. 0.24
D. 0.25
E. 0.76

Question 16

For the two independent events *A* and *B*, $\Pr(A) = 0.5$ and $\Pr(A \cap B) = 0.2$.
$\Pr(A \cup B)$ is equal to

A. 0.3
B. 0.4
C. 0.5
D. 0.6
E. 0.7

Question 17

If $h(x) = 6x^{\frac{1}{3}} - 1$, then $h'(-1)$ is equal to

A. -7
B. -2
C. 2
D. 3
E. 5

Question 18

The rate of change of the function $y = (x-1)^2$ at the point $(2,1)$ is

A. -1
B. 0
C. 1
D. $\frac{1}{3}$
E. 2

Question 19

The gradient of the curve $y = x^3 + 3x^2 + 4x$ at the point where $x = 1$ is

A 0
B 4
C 13
D 8
E 16

$\frac{dy}{dx} = 3x^2 + 6x + 4$

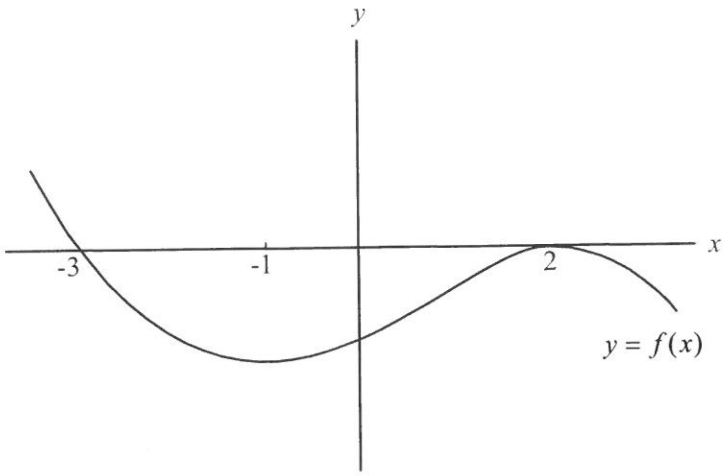
when $x = 1$

$dy/dx = 3 + 6 + 4$

$= 13.$

Question 20

The graph of the function *f* is shown below.



Which one of the following statements is **not** true?

A. $f'(-1) = 0$ ✓
B. $f(2) = 0$ ✓
C. $f'(2) = 0$ ✓
D. $f(-3) = 0$ ✓
E. $f'(-3) = 0$

END OF SECTION B

SECTION C: Extended answer. **5 questions** **60 marks**

Calculator and summary book allowed.
Working out required where appropriate
Place answers in the space provided on the exam paper.

Question 1 (15 marks)

A dinner is attended by 250 guests, 180 of whom are male. Of these 250 guests, 110 are employees of the company organising the dinner. There are 60 female guests present at the dinner who are employees of the company.

a.

i. Complete a Karnaugh map that represents this information 2 marks

	E	E'	
M	50	130	180
M'	60	10	70
	110	140	250

Find the probability that a randomly selected guest at the dinner will be

ii. a female not employed by the company. 1 mark

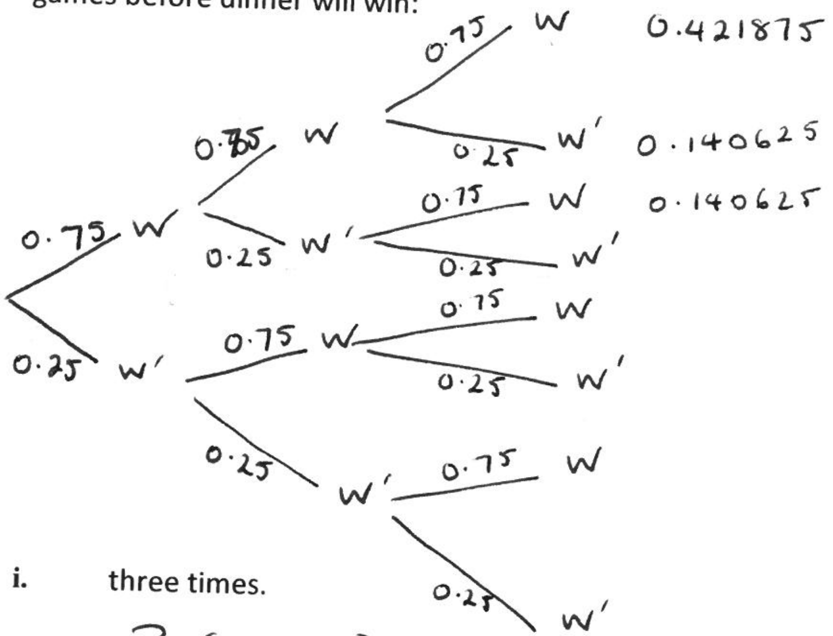
$Pr(M' \cap E') = \frac{10}{250} = \frac{1}{25}$

iii. employed by the company given that they are male. 1 mark

$Pr(E/M) = \frac{50}{180} = \frac{5}{18}$

At the dinner, guests are invited to play a game. Anyone playing the game has a 75% chance of winning. The game is played three times before main course is served.

b. Using a tree diagram or otherwise, find the probability that a guest playing all three games before dinner will win:



i. three times. 1 mark

$Pr(WWW) = 0.421875$ or $\frac{3^3}{4^3} = \frac{27}{64}$

ii. at least twice. Pr (2 wins or 3 wins) 1 mark

$= 3 \times 0.140625 + 0.421875$

$= 0.84375$

On Table 7 at the dinner, the 4 female and 6 male guests pose in a line for a photo.

c. How many different arrangements of these guests are possible if

i. there are no restrictions? 1 mark

$10! = 3\,628\,800$ arrangements

ii. all the female guests are together? 1 mark

$7! \times 4! = 120\,960$ arrangements

iii. there is a male at each end of the line? 1 mark

$6 \times 8! \times 5 = 1\,209\,600$ arrangements

During the evening when guests are seated, three guests at each table are randomly selected to win a prize.

d. Find the probability that on Table 7

i. all three winners are male. 2 marks

$$Pr(3M) = \frac{{}^{6}C_3 \times {}^{4}C_0}{{}^{10}C_3}$$

$$= \frac{1}{6}$$

ii. there is 1 male and 2 female winners. 2 marks

$$Pr(1M \,\&\, 2F) = \frac{{}^{6}C_1 \times {}^{4}C_2}{{}^{10}C_3}$$

$$= \frac{3}{10}$$

iii. all three winners are female given that at least one of the winners is female. 2 marks

$$Pr(3F / \text{at least } 1F) = \frac{Pr(3F)}{1 - Pr(0F)}$$

$$= \frac{{}^{6}C_0 \times {}^{4}C_3}{{}^{10}C_3} \div \left(1 - \frac{1}{6}\right)$$

$$= \frac{1}{25}$$

Question 2 (13 marks)

A former quarry site is to be developed into a housing estate. The quarry site is to be filled with material which must compact down before building can begin.

The height of the proposed soil material, *h*, in metres above the base of the quarry *t* years after it is placed there is modelled by

$$h(t) = 17.5 \times 2^{-0.1t} + 20, \qquad t \geq 0.$$

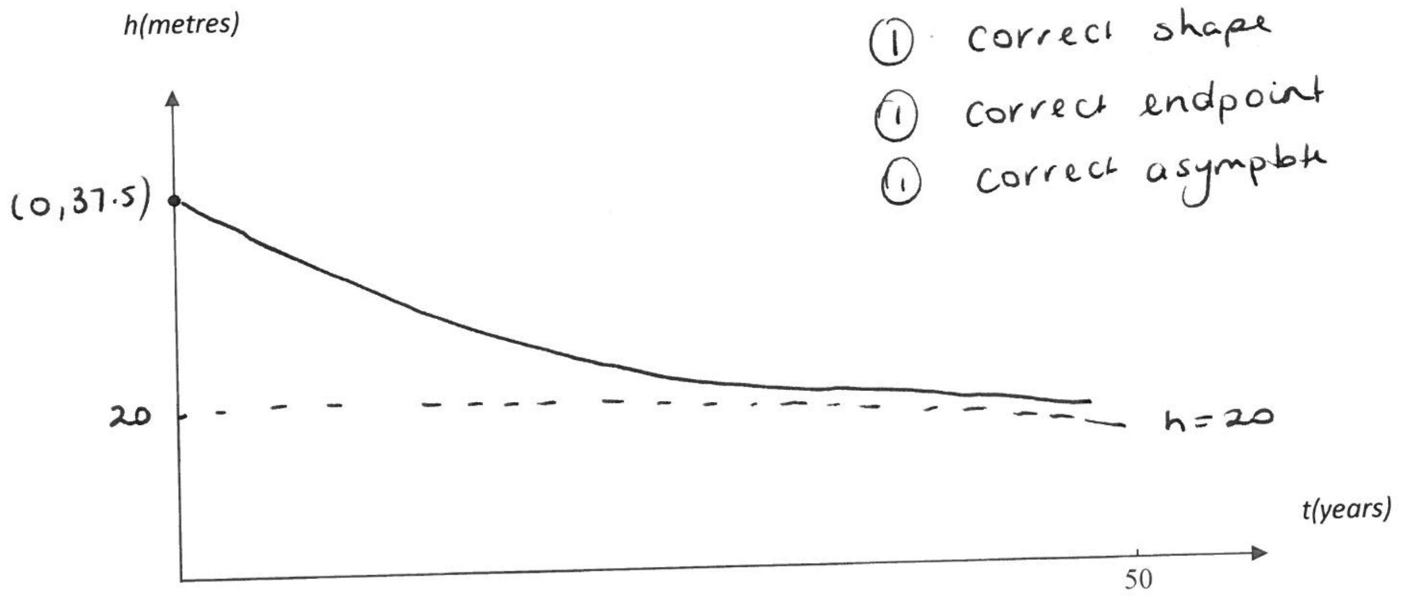
a. Find the height of the proposed soil material immediately after it is placed in the quarry according to this model. 1 mark

$h(0) = 17.5 \times 2^{-0.1 \times 0} + 20$

$= 17.5 \times 1 + 20$

$= 37.5$ m

b. On the set of axes below, sketch the graph of the function *s*. Indicate any asymptotes or endpoints clearly on the graph. 3 marks



(1) correct shape

(1) correct endpoint

(1) correct asymptote

c. Over the long term, what height does the proposed soil material approach? 1 mark

20m

In theory, the height h, in metres, of material that is used as fill above a base t years after it is placed there is modelled by the function

$$h(t) = 17.5 \times 2^{-kt} + 20, \qquad t \geq 0,$$

where k is a positive constant and depends on the type of material that is used as fill.

In an effort to have the fill in the quarry compact more quickly, a crushed rock material is being considered. It is known that crushed rock will have a height above the base of the quarry of 24.375m after 5 years.

d. Show that for this crushed rock material, $k = 0.4$. 2 marks

Using CAS - $k = 0.4$

By hand:

$24.375 = 17.5 \times 2^{-5k} + 20$

$4.375 = 17.5 \times 2^{-5k}$

$\frac{4.375}{17.5} = 2^{-5k}$

$\frac{1}{4} = 2^{-5k}$

$2^{-2} = 2^{-5k}$

$-2 = -5k$

$k = 2/5$ or 0.4

Another type of material, that contains recycled tyres, is known to compact down more quickly than the soil material described in part **a.** but more slowly than the crushed rock material described in part **d.**

e. Find the possible values of k that this type of recycled material would have. 1 mark

from part a $\Rightarrow$ $k = 0.1$

part d $\Rightarrow$ $k = 0.4$

Since $h(t) = 17.5 \times 2^{-kt} + 20$,

we require $0.1 < k < 0.4$

f. Use the respective models for the height of the materials to find how much higher the height of the soil material described in part **a.** will be compared to the height of the crushed rock material described in part **d.** when $t = 10$.
Express your answer in metres correct to one decimal place. 2 marks

$h_1(10) = 28.75$

$h_2(10) = 21.09375$

$28.75 - 21.09375 = 7.65625$

Proposed material will be $\underline{\underline{7.7}}$ metres higher compared to crushed rock after 10 years

When the height of the fill material reaches 21m above the base of the quarry it is considered safe to be built on.

g. How much time will be saved in preparing the site to be built on, by using the crushed rock material described in part **d.** compared to using the soil material described in part **a.**

Express your answer in years correct to one decimal place. 3 marks

$h_1(t) = 21$ for t using CAS

$t = 41.2928$ 1

& $h_2(t) = 21$ using CAS

$t = 10.3232$. 1

$41.2928 - 10.3232 = 30.9696$

so 31.0 years. 1

Question 3 (12 marks)

In a city building, the internal temperature is centrally controlled. This temperature, T, in degrees Celsius, at time t hours is given by the function

$$T = 2.5\sin\left(\frac{\pi}{12}(t-6)\right) + 20, \quad t \in [0, a]$$

where $t = 0$ corresponds to midnight on Sunday and $t = a$ corresponds to midnight on Friday.

a. What is the amplitude of this function? 1 mark

2.5

b. What is the temperature inside the building

i. at midnight on Sunday? $T = 2.5\sin\left(\frac{\pi}{12}(t-6)\right) + 20$ 1 mark

$t = 0$

$T = 2.5\sin\left(\frac{\pi}{12}x - 6\right) + 20$ $T = 17.5\,°C$

$T = 2.5\sin\left(-\frac{\pi}{2}\right) + 20$

$T = 2.5 \times -1 + 20$

ii. at 10am on Tuesday?
Express your answer correct to two decimal places. 1 mark

$t = 34$

$T = 2.5\sin\left(\frac{\pi}{12} \times (34-6)\right) + 20$ ~~$T = 21.25\,°C$~~

$T = 2.5\sin\frac{28\pi}{12} + 20$ $T = 22.17\,°C$

$T = 2.5\sin\frac{7\pi}{3} + 20$

$T = 2.5 \times \sqrt{3}/2 + 20$

c. What is the period of the temperature function? 1 mark

$\text{Period} = \frac{2\pi}{\pi/12}$

$= 2\cancel{\pi} \times 12/\cancel{\pi}$

$= 24$

d. Find the

i. maximum temperature inside the building during the week. 1 mark

$20 + 2.5 = 22.5\,°C$

ii. minimum temperature inside the building during the week. 1 mark

$20 - 2.5 = 17.5\,°C$

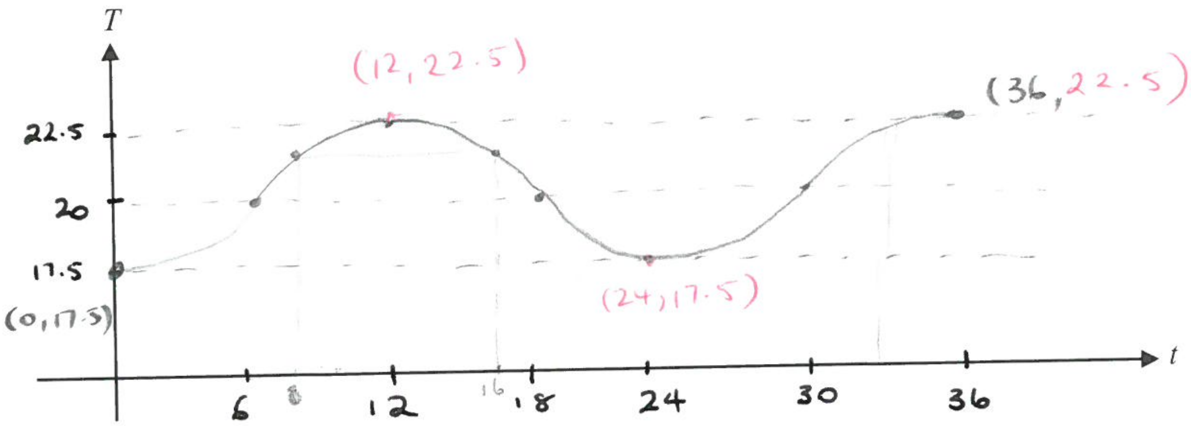
e. Find the value of a. 1 mark

$a = 120$

f. Sketch the function $T = 2.5\sin\left(\frac{\pi}{12}(t-6)\right) + 20$ for $t \in [0,36]$ on the set of axes below.

Indicate clearly on your graph the coordinates of the endpoints and turning points.

3 marks



g. When Gayle walked into the building on Thursday afternoon the temperature was 21.25°C. At what time did Gayle walk into the building?

2 marks

$21.25 = 2.5\sin\left(\frac{\pi}{12}(t-6)\right) + 20$

$\frac{1.25}{2.5} = \sin\left(\frac{\pi}{12}(t-6)\right)$

$\frac{1}{2} = \sin\left(\frac{\pi}{12}(t-6)\right)$

$\frac{\pi}{6} = \frac{\pi}{12}(t-6)$

$\frac{\pi}{6} \times \frac{12}{\pi} = t - 6$ 8 am & 4pm

$t = 8$ 4pm

Question 4 (10 marks)

A circular cylinder, **open at the top ~~and bottom~~**, is constructed of thin sheet metal. The area of the sheet is 432π cm2. The cylinder has a radius of r cm and a height of h cm.

(The curved surface area of a cylinder = $2\pi rh$ and the volume of a cylinder = $\pi r^2 h$.)

a **i** Write down a formula for A cm^2, the outer surface area of the open cylinder.

$$A = \pi r^2 + 2\pi r h$$

1 mark

ii Use the fact that $A = 432\pi$ to find a formula for h in terms of r.

$$432\pi = \pi r^2 + 2\pi r h$$

$$432 = r^2 + 2rh$$

$$\frac{432 - r^2}{2r} = h$$

1 mark

b Show that the volume, V cm^3, of the cylinder is given in terms of r by

$V = \frac{\pi}{2}(432r - r^3)$.

2 marks

$$\cancel{432\pi = \pi r^2 + 2\pi r h}$$

$$\cancel{432 = r^2 + 2rh}$$

$$\cancel{43}$$

$$V = \pi r^2 h$$

$$V = \pi r^2 \left(\frac{432 - r^2}{r}\right)$$

$$V = \pi r \left(\frac{432 - r^2}{2}\right)$$

$$V = \frac{\pi}{2} r (432 - r^2)$$

$$V = \frac{\pi}{2}(432r - r^3)$$

c Find $\frac{dV}{dr}$. 1 mark

$dV/dr = \pi/2 \ (432 - 3r^2)$

d Solve the equation $\frac{dV}{dr} = 0$ for r. 1 mark

$\pi/2 \ (432 - 3r^2) = 0$

$3r^2 = 432$

$r^2 = 144$

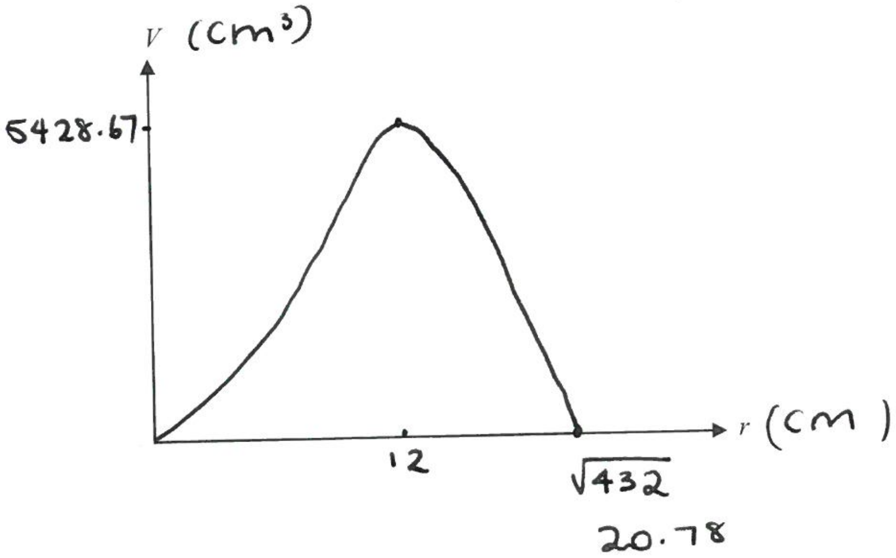
$r = 12$ cm

e Sketch the graph of $V = \frac{\pi}{2}(432r - r^3)$ over a suitable domain. 3 marks

$V = \frac{\pi r}{2}(432 - r^2)$

intercepts: $x = 0$, $r^2 = 432$

$r = \sqrt{432}$



f State the maximum volume of the cylinder, correct to the nearest cm^3. 1 mark

$$V_{max} = \frac{\pi}{2}(432 \times 12 - 12^3)$$

$$\approx 5428.672...$$

$$\approx 5429 \text{ cm}^3$$

Question 5 (10 marks)

The matrix equation $X' = AX + B$ defines a transformation where $A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

The curve with equation $y = x^3$ undergoes this transformation.

a. Show that the image produced is given by $y = -\frac{3}{8}(x-1)^3 - 1$. 4 marks

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2x \\ -3y \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2x+1 \\ -3y+1 \end{bmatrix}$$

$x' = 2x + 1$ $\quad$ $y' = -3y - 1$

$2x = x' - 1$ $\quad$ $3y = -y' - 1$

$x = \frac{x'-1}{2}$ $\quad$ $y = \frac{-y'-1}{3}$

$$y = x^3$$

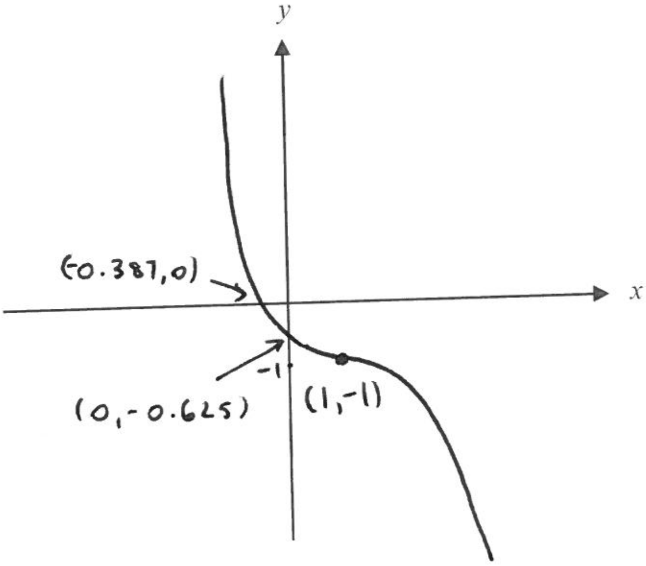
$$\frac{-y'-1}{3} = \left[\frac{x'-1}{2}\right]^3$$

$$-y' - 1 = \frac{3}{8}(x'-1)^3$$

$$-y' = \frac{3}{8}(x'-1)^3 + 1$$

$$y = -\frac{3}{8}(x-1)^3 + 1$$

b. On the set of axes below, sketch the graph of the image with equation $y = -\frac{3}{8}(x-1)^3 - 1$, showing axes intercepts expressed correct to three decimal places. 3 marks



A straight line intersects the image graph given by $y = -\frac{3}{8}(x-1)^3 - 1$ at three points.

Two of these points are located at $(-1,2)$ and $(1,-1)$.

c. Find the equation of this straight line and hence find the coordinates of the third point of intersection. 3 marks

Straight line equation: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - -1}{-1 - 1}$

$= -\frac{3}{2}$

eq for intersection

$-\frac{3}{8}(x-1)^3 - 1 = \frac{-3x+1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{3}{2}(x - -1)$

Solve for x using CAS

$2y - 4 = -3x - 3$

$x = -1, 1$ or 3

$y = \frac{-3x+1}{2}$

Sub $x = 3 \Rightarrow y = \frac{-3 \times 3 + 1}{2}$

$y = -4$

$(3, -4)$ third point of intersection.

END OF SECTION C