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Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2016

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where a numerical answer is required, an exact value must be given unless otherwise directed.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on pages 12 and 13 of this exam.

Question 1 (5 marks)

a. If $y = \sqrt{x^2 + 1}$, find $\frac{dy}{dx}$. 2 marks

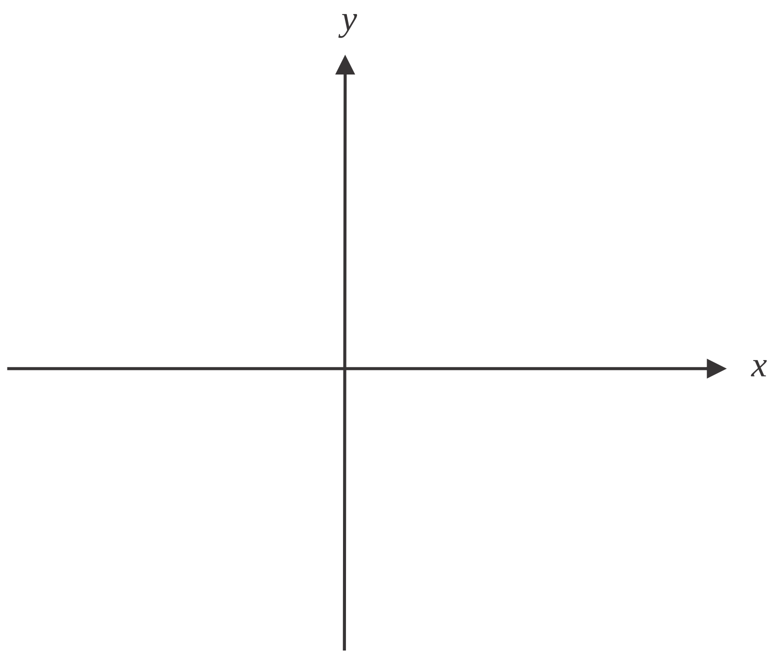
b. Let $f(x) = \frac{\log_e(3x)}{x}$.

i. Find $f'(x)$. 2 marks

ii. Evaluate $f'\left(\frac{1}{3}\right)$. 1 mark

Question 2 (3 marks)

Sketch the graph of $f: R \setminus \{2\} \to R,\ f(x) = -1 + \dfrac{3}{x-2}$ on the set of axes below. Label axes intercepts with their coordinates. Label asymptotes with their equations.



Question 3 (2 marks)

Find $\int_1^3 \left(\frac{2}{x}+1\right)dx$.

Question 4 (3 marks)

Let $f(x)=\frac{1}{\sqrt{3}}\cos(x)$ and $g(x)=\sin(x)$.

a. Solve the equation $f(x)=g(x)$ for $x \in [0, 2\pi]$. 2 marks

b. Evaluate $f(g(0))$. 1 mark

Question 5 (4 marks)

The number of deliveries made to a business on any given work day can be represented by the random variable X.
The probability distribution of X is shown in the table below.

x	0	1	2	3
$\Pr(X = x)$	0.3	0.4	0.2	0.1

The mean number of deliveries is 1.1.

a. Find the variance of X. 2 marks

b. Find the probability that on a work day when there is a delivery, the number of deliveries is two or more. 2 marks

Question 6 (3 marks)

Solve the equation $\log_e(x) + \log_e(3x+2) = 2\log_e(x+1)$ for x, where $x > 0$.

Question 7 (4 marks)

A continuous random variable *X*, has a probability density function given by

$$f(x) = \begin{cases} 2\sin(4x), & x \in \left[0, \frac{\pi}{4}\right] \\ 0, & \text{otherwise} \end{cases}.$$

a. Find $\Pr\left(X < \frac{\pi}{6}\right)$. 2 marks

b. Find $\Pr\left(X < \frac{\pi}{8} \middle| X < \frac{\pi}{6}\right)$. 2 marks

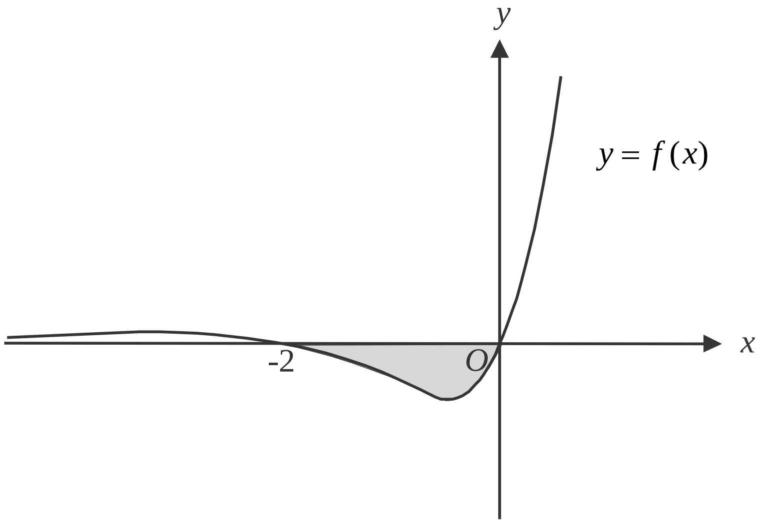
Question 8 (5 marks)

a. A car dealership has eight new BX3 cars in stock. It is found that three of these cars have defective airbags.
Two of the eight BX3 cars in stock are randomly selected.
Find the probability that less than two of the BX3 cars selected have defective airbags. 3 marks

b. The car manufacturer that produces the BX3 cars, finds that worldwide, the proportion of cars with defective airbags is 0.2.
A random sample of four BX3 cars is taken from the many car dealerships around the world.
Find the probability that for this sample, exactly two BX3 cars have defective airbags. 2 marks

Question 9 (4 marks)

The graph of $f : R \rightarrow R, f(x) = (x^2 + 2x)e^x$ is shown below.



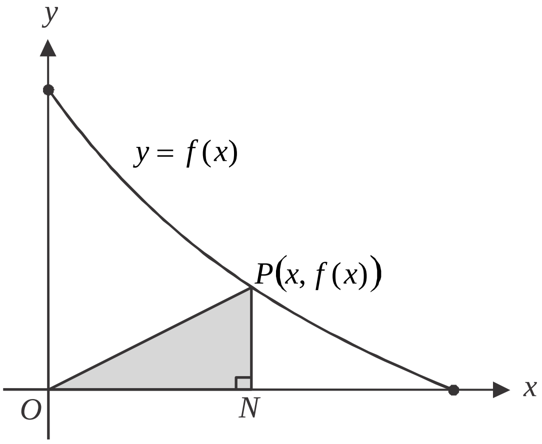
The region enclosed by the graph of *f* and the *x*-axis is shaded.

a. Find the derivative of $(3 - x^2)e^x$. Give your answer in the form $ae^x - f(x)$, where a is a positive constant. 1 mark

b. Use your answer to part **a.** to find the area of the shaded region. 3 marks

Question 10 (7 marks)

Let $f:[0,1]\to R,\ f(x)=1-x^{\frac{2}{3}}$. The graph of f is shown below.



The right-angled triangle *NOP* has vertex *N* on the *x*-axis, and vertex *O* at the origin. The vertex *P* lies on the graph of *f* and has coordinates $(x, f(x))$ as shown.

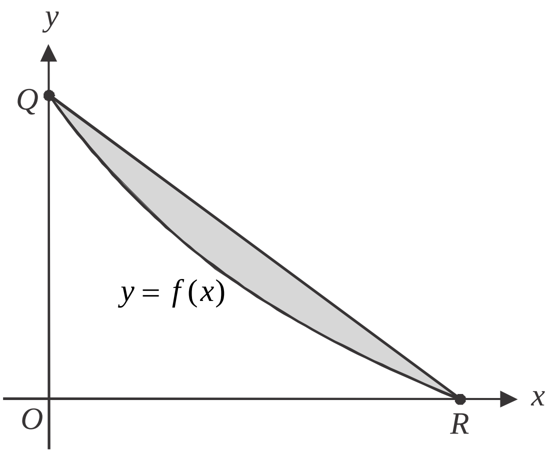
a. Find the area *A*, of triangle *NOP* in terms of *x*. 1 mark

b. Find

i. the value of *x* for which *A* is a maximum. 2 marks

ii. the maximum area of triangle *NOP*. Give your answer in the form $\dfrac{a\sqrt{b}}{c}$ where *a*, *b* and *c* are positive integers. 1 mark

c. The point *Q* lies on the graph of *f* and on the *y*-axis. The point *R* lies on the graph of *f* and on the *x*-axis.



Find the area enclosed by the line segment *QR* and the graph of *f*. 3 marks

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2\, f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$