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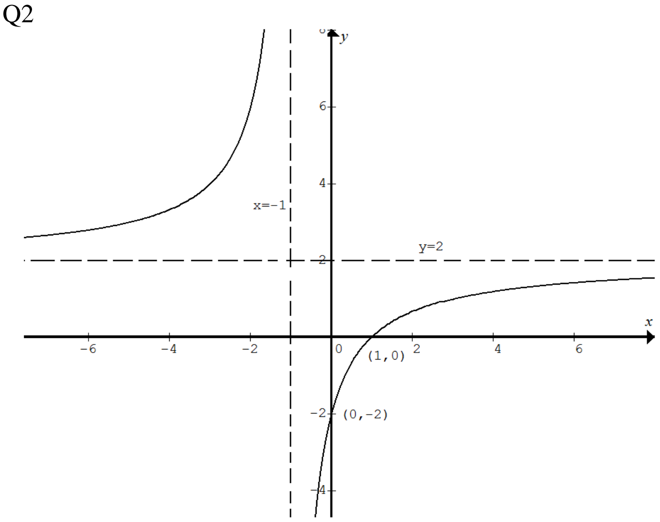
2016 VCAA Math Methods Sample (v2 April) Exam 1 Solutions

Q1a $\frac{d}{dx}\sqrt{4-x} = -\frac{1}{2\sqrt{4-x}}$

Q1b $f(x) = \frac{x}{\sin x}$, $f'(x) = \frac{(\sin x)(1) - x(\cos x)}{\sin^2 x}$

$f'\left(\frac{\pi}{2}\right) = \frac{\left(\sin\frac{\pi}{2}\right)(1) - \frac{\pi}{2}\left(\cos\frac{\pi}{2}\right)}{\sin^2\frac{\pi}{2}} = 1$

Q2



Q3a $\int \frac{1}{(2x-1)^3}dx = \frac{1}{2(-2)(2x-1)^2} = -\frac{1}{4(2x-1)^2}$

Q3b $g'(x) = \sin(2\pi x)$, $g(x) = \int \sin(2\pi x)dx = -\frac{\cos(2\pi x)}{2\pi} + c$

Given $g(1) = \frac{1}{\pi}$, $g(1) = -\frac{\cos(2\pi)}{2\pi} + c = \frac{1}{\pi}$, .: $c = \frac{3}{2\pi}$

.: $g(x) = \frac{3 - \cos(2\pi x)}{2\pi}$

Q4a $\overline{X} = 0\times0.2 + 1\times0.2 + 2\times0.5 + 3\times0.1 = 1.5$

Q4b $0.2^3 = 0.008$

Q5a At the point of intersection, $\cos\frac{\pi}{3} = a\sin\frac{\pi}{3}$

.: $\frac{1}{a} = \tan\frac{\pi}{3} = \sqrt{3}$, .: $a = \frac{1}{\sqrt{3}}$

Q5b $\tan x = \sqrt{3}$, $x = \frac{\pi}{3}, \frac{4\pi}{3}$, .: the other point of intersection is at $x = \frac{4\pi}{3}$.

Q6a $2\log_3 5 - \log_3 2 + \log_3 x = 2$, $\log_3\frac{5^2 x}{2} = 2$

$\frac{5^2 x}{2} = 3^2$, $x = \frac{18}{25}$

Q6b $3e^t = 5 + 8e^{-t}$, equation$\times e^t$, $3(e^t)^2 - 5e^t - 8 = 0$

$(3e^t - 8)(e^t + 1) = 0$.

Since $e^t + 1 \neq 0$, .: $3e^t - 8 = 0$, $t = \log_e\frac{8}{3}$

Q7a $sd(\hat{P}) \le \frac{1}{100}$, $\sqrt{\frac{p(1-p)}{n}} \le \frac{1}{100}$ where $p = \frac{1}{5}$

.: $\frac{2}{5\sqrt{n}} \le \frac{1}{100}$, $\sqrt{n} \ge 40$, $n \ge 1600$, .: smallest $n = 1600$

Q7b $\Pr(\text{exactly one success}) = \Pr(SF) + \Pr(FS)$

$= \Pr(S)\Pr(F \mid S) + \Pr(F)\Pr(S \mid F) = \frac{1}{23}\times\frac{22}{22} + \frac{22}{23}\times\frac{1}{22} = \frac{2}{23}$

Q8a $\int_0^m \frac{1}{5}e^{-\frac{x}{5}}dx = 0.5$, $\left[-e^{-\frac{x}{5}}\right]_0^m = 0.5$, $-e^{-\frac{m}{5}} + e^0 = 0.5$

$e^{-\frac{m}{5}} = 0.5$, $m = 5\log_e 2$

Q8b $\Pr(X < 1 \mid X \le 5\log_e 2) = \frac{\Pr(X < 1)}{\Pr(X \le 5\log_e 2)}$

$= \frac{\int_0^1 \frac{1}{5}e^{-\frac{x}{5}}dx}{0.5} = \frac{\left[-e^{-\frac{x}{5}}\right]_0^1}{0.5} = \frac{-e^{-\frac{1}{5}} + e^0}{0.5} = 2\left(1 - e^{-\frac{1}{5}}\right)$

Q9a $\frac{d}{dx}x^2\log_e x = x^2\left(\frac{1}{x}\right) + 2x\log_e x = x + 2x\log_e x$

Q9b $\frac{d}{dx}x^2\log_e x = x + 2x\log_e x$

.: $x\log_e x = \frac{1}{2}\left(\frac{d}{dx}x^2\log_e x - x\right)$

Area of the shaded region

$= \int_1^3 x\log_e x\,dx = \frac{1}{2}\left(\int_1^3 \frac{d}{dx}x^2\log_e x\,dx - \int_1^3 x\,dx\right)$

$= \frac{1}{2}\left(\left[x^2\log_e x\right]_1^3 - \left[\frac{x^2}{2}\right]_1^3\right) = \frac{1}{2}\left(9\log_e 3 - \frac{9}{2} + \frac{1}{2}\right) = \frac{9}{2}\log_e 3 - 4$

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Q10a $y = ax^2 + bx$, $\frac{dy}{dx} = 2ax + b$

At $(2, 4)$, $4 = a(2^2) + b(2)$, .: $4a + 2b = 4$, .: $2a + b = 2$ ... (1)

also gradient of the tangent $= 2a(2) + b = 4a + b = \frac{0-4}{6-2}$,

.: $4a + b = -1$ … (2)

Solve (1) and (2) simultaneously, $a = -\frac{3}{2}$ and $b = 5$

Q10bi Gradient of VQ = gradient of QU

.: $\frac{v-4}{0-2} = \frac{4-0}{2-u}$, $v - 4 = \frac{8}{u-2}$, $v = 4 + \frac{8}{u-2}$, $v = \frac{4u}{u-2}$

Q10bii Shaded area $A(u) = \frac{1}{2}uv - 8 = \frac{2u^2}{u-2} - 8$

Let $\frac{dA}{du} = 0$ to find the turning point(s).

$\frac{(u-2)(4u) - (2u^2)(1)}{(u-2)^2} = 0$.: $2u^2 - 8u = 0$, $2u(u-4) = 0$.

Since $u > 0$, .: $u = 4$ and $A = \frac{2(4^2)}{4-2} - 8 = 8$

First end point at $u = \frac{5}{2}$, $A = \frac{2\left(\frac{5}{2}\right)^2}{\frac{5}{2} - 2} - 8 = 17$

Second end point at $u = 6$, $A = \frac{2(6^2)}{6-2} - 8 = 10$

.: $A_{min} = 8$ square units

Q10biii From part ii, $A_{max} = 17$ square units at $u = \frac{5}{2}$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors