Year 2016

VCE

Mathematical Methods

Trial Examination 1

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9018 5376** **FAX: (03) 9817 4334** **kilbaha@gmail.com** **http://kilbaha.com.au**

IMPORTANT COPYRIGHT NOTICE

Victorian Certificate of Education
2016

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Differentiate $\sqrt{16-x^2}$ with respect to x. 1 mark

b. If $f(x)=\log_e\left(\tan(2x)\right)$, find $f'\left(\frac{\pi}{6}\right)$. Give your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b, c are integers. 2 marks

Question 2 (3 marks)

Consider the linear simultaneous equations

$(7-k)x-6y=10$

$4x+3k\,y=k+6$

where k is a real constant.

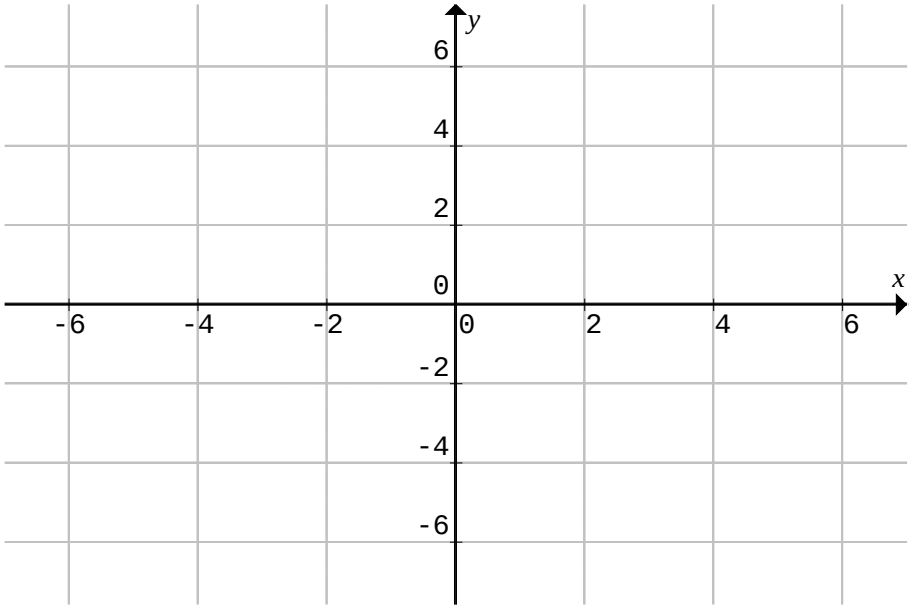
i. Find the value(s) of k, for which there is a unique solution.

ii. Find the value(s) of k, for which there is an infinite number of solutions.

Question 3 (4 marks)

a. Sketch the graph of $y = \frac{8}{(x-2)^2} - 2$ on the axes below, clearly indicating all axial intercepts and the equation of any asymptotes, stating the maximal domain and range.

2 marks



b. Find the area bounded by the graph of $y = \frac{8}{(x-2)^2} - 2$ the x-axis and the lines $x = 3$ and $x = 5$. 2 marks

Question 4 (4 marks)

The function f has the rule $f(x) = \log_4(x)$ and the function g has the rule $g(x) = \sqrt{4x^2 - 1}$. State the maximal domain for which

a. $f(g(x))$ is defined. 2 marks

b. $g(f(x))$ is defined. 2 marks

Question 5 (4 marks)

a. Find the general solution of the equation $2\cos^2(x)-\cos(x)-1=0$

2 marks

b. Solve $2\cos^2(x)-\cos(x)-1=0$ for x, where $0 \le x \le 2\pi$

2 marks

Question 6 (4 marks)

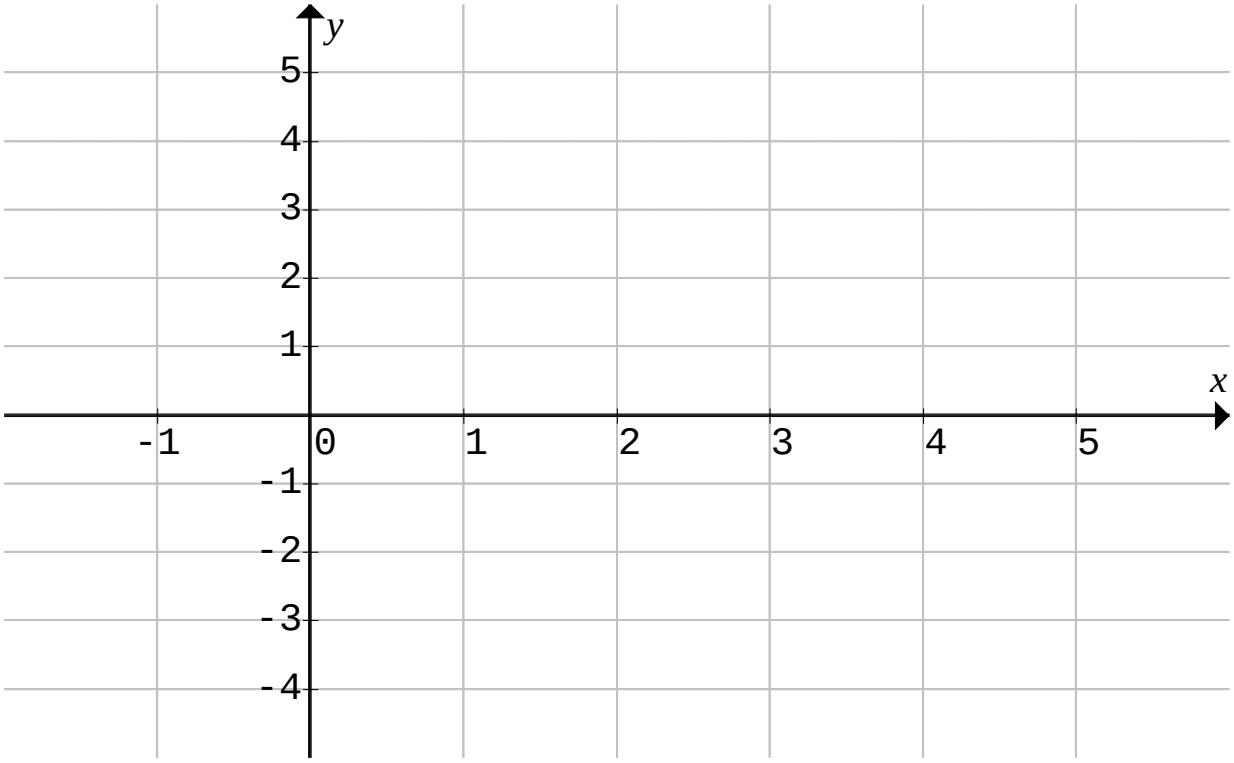
a. On the axes below, sketch the graphs of the functions

$f:[0,4] \to R$, $f(x) = 3\sin\left(\frac{\pi x}{2}\right)$ and $g:[0,4] \to R$, $g(x) = x$

Hence sketch the graph of $h:[0,4] \to R$, $h(x) = 3\sin\left(\frac{\pi x}{2}\right) + x$,

stating the co-ordinates of the endpoints.

3 marks



b. Define completely the function $h'(x)$. 1 mark

Question 7 (3 marks)

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$\frac{1}{2e^k}$	$\frac{e^k}{4}$

Find the value(s) of k.

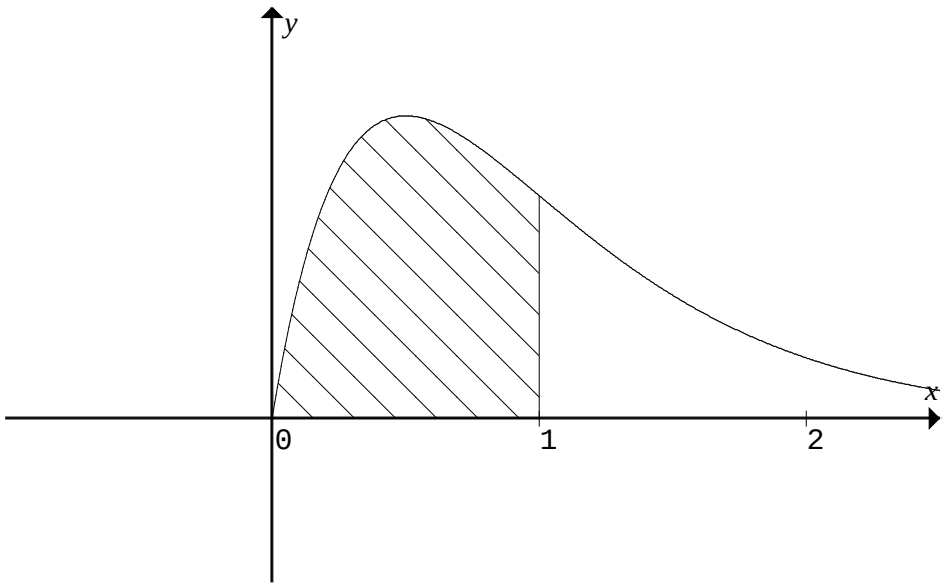
Question 8 (3 marks)

A random sample of size n, is drawn from a population. The proportion of the population with interest is equal to p. Exactly two standard deviations for p give a confidence interval as $(0.7, 0.9)$. Determine the values of p and n.

Question 9 (5 marks)

a. Differentiate xe^{-2x} with respect to x. 1 mark

The diagram shows part of the graph of $f:[0,\infty)\to R$, $f(x)=xe^{-2x}$.



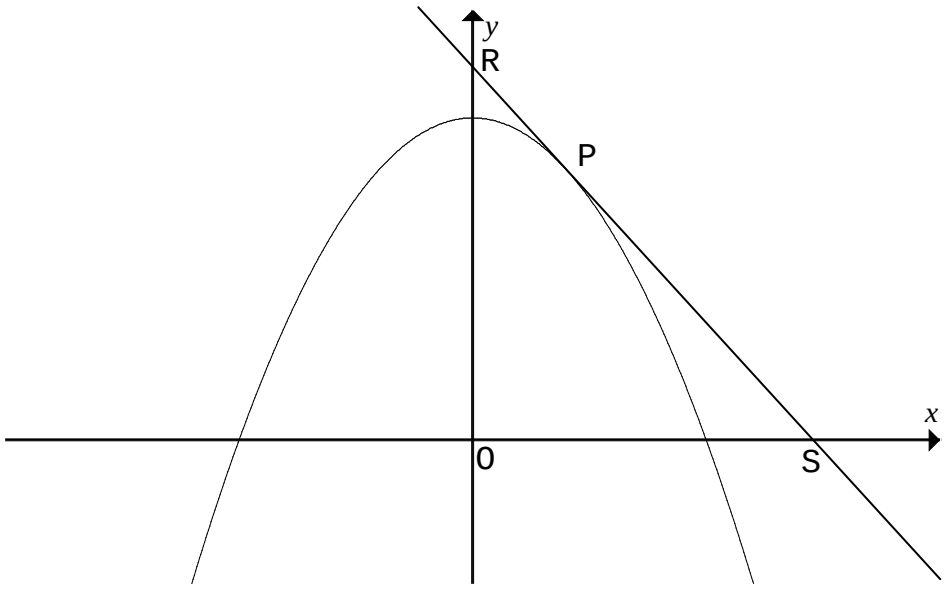
b. Find the co-ordinates of the turning point on the graph. 1 mark

c. Find the area of the shaded region.

3 marks

Question 10 (7 marks)

The diagram shows the graph of the function $f(x) = 9 - x^2$.



The graph of the tangent line to the curve at the point $\mathrm{P}\left(p, f\left(p\right)\right)$, **where** $1 \leq p \leq 3$ is also shown.

a. Determine the equation of this tangent line in terms of p.

1 mark

b. If this tangent line crosses the x-axis at the point S, and crosses the y-axis at the point R, find the co-ordinates of the points S and R in terms of p.

2 marks

If O is the origin, then the area A of the triangle OSR is given by $A = \dfrac{\left(9 + p^2\right)^2}{4p}$.

c. Find the **minimum** area of the triangle OSR and the value of p for which the area is a minimum.

3 marks

d. Find the **maximum** area of the triangle OSR and the value of p for which the area is a maximum.

1 mark

END OF QUESTION AND ANSWER BOOKLET
END OF EXAMINATION

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET