# Year 2016 VCE Mathematical Methods



**Trial Examination 1** 

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# Victorian Certificate of Education 2016

### STUDENT NUMBER

						Letter
Figures						
Words						

# **MATHEMATICAL METHODS**

# **Trial Written Examination 1**

Reading time: 15 minutes Total writing time: 1 hour

# **QUESTION AND ANSWER BOOK**

## Structure of book

	2	
Number of	Number of questions	Number of
questions	to be answered	marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

#### **Materials supplied**

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

#### **Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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## **Instructions**

Answer all questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (3 marks)

**a.** Differentiate  $\sqrt{16-x^2}$  with respect to x.

**b.** If  $f(x) = \log_e(\tan(2x))$ , find  $f'(\frac{\pi}{6})$ . Give your answer in the form  $\frac{a\sqrt{b}}{c}$  where a, b, c are integers.

**Question 2** (3 marks)

Consider the linear simultaneous equations

$$(7-k)x-6y=10$$

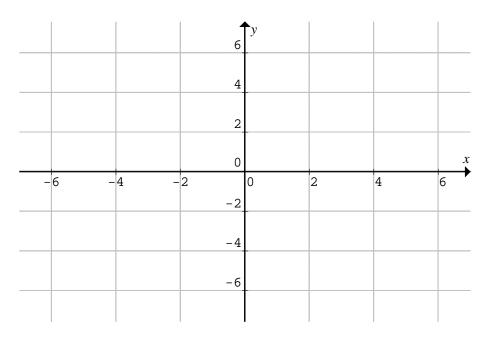
$$4x + 3k \ y = k + 6$$

where k is a real constant.

- i. Find the value(s) of k, for which there is a unique solution.
- **ii.** Find the value(s) of *k*, for which there is an infinite number of solutions.


**Question 3** (4 marks)

Sketch the graph of  $y = \frac{8}{(x-2)^2} - 2$  on the axes below, clearly indicating all axial a. intercepts and the equation of any asymptotes, stating the maximal domain and range. 2 marks



Find the area bounded by the graph of  $y = \frac{8}{(x-2)^2} - 2$  the x-axis and the lines b. x = 3 and x = 5. 2 marks

<b>Question 4</b>	(4 marks)
Question 4	(4 IIIai KS)

The function f has the rule  $f(x) = \log_4(x)$  and the function g has the rule

 $g(x) = \sqrt{4x^2 - 1}$ . State the maximal domain for which

**a.** f(g(x)) is defined.

2 marks

<b>b.</b> $g(f(x))$ is defined	b.	g(f)	(x)	is defined
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2 marks

<b>Ouestion</b>	5	(4	marks`
Oueshon	3	(+	marks

a.	Find the general solution of the equation 2 c	$\cos^2(x) - \cos(x) - 1 = 0$	
			2 marks


b.	Solve $2\cos^2(x) - \cos(x) - 1 = 0$ for x, where $0 \le x \le 2\pi$	
		2 marks

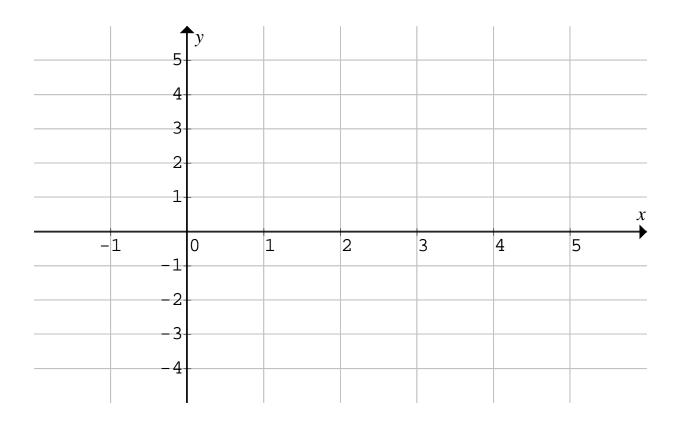

**Question 6** (4 marks)

**a.** On the axes below, sketch the graphs of the functions

$$f:[0,4] \to R$$
,  $f(x) = 3\sin(\frac{\pi x}{2})$  and  $g:[0,4] \to R$ ,  $g(x) = x$ 

Hence sketch the graph of  $h:[0,4] \to R$ ,  $h(x) = 3\sin\left(\frac{\pi x}{2}\right) + x$ , stating the co-ordinates of the endpoints.

3 marks



**b.** Define completely the function h'(x). 1 mark

# **Question 7** (3 marks)

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X=x)$	$\frac{1}{2e^k}$	$\frac{e^k}{4}$

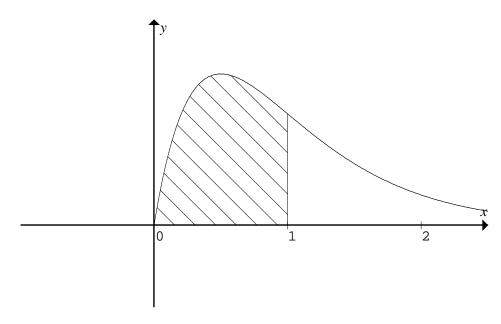
Find the value(s) of $k$ .				

Question 8	(3 marks)
A random sar	mple of size $n$ , is drawn from a population. The proportion of the population
with interest i	s equal to $p$ . Exactly two standard deviations for $p$ give a confidence interval
as (0.7, 0.9)	. Determine the values of $p$ and $n$ .

**Question 9** (5 marks)

**a.** Differentiate  $xe^{-2x}$  with respect to x. 1 mark

The diagram shows part of the graph of  $f:[0,\infty) \to R$ ,  $f(x) = xe^{-2x}$ .



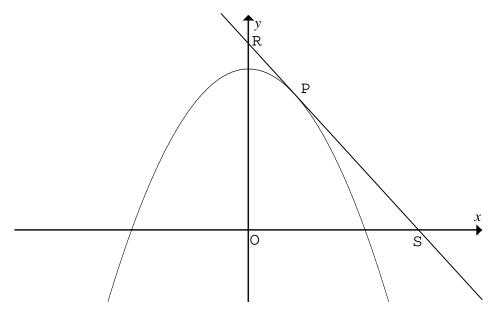
**b.** Find the co-ordinates of the turning point on the graph.

1 mark

c.	Find the area of the shaded region.	3 marks

# **Question 10** (7 marks)

The diagram shows the graph of the function  $f(x) = 9 - x^2$ .



The graph of the tangent line to the curve at the point P(p, f(p)), where  $1 \le p \le 3$  is also shown.

a.	Determine the equation of this tangent line in terms of $p$ .	
		1 mark
b.	If this tangent line crosses the <i>x</i> -axis at the point S, and crosses the <i>y</i> -axis at the	
D.	point R, find the co-ordinates of the points S and R in terms of $p$ .	
	point 14, 1ma the 60 ordinates of the points 5 and 14 m terms of p.	2 marks

If	O is the origin, then the area A of the triangle OSR is given by $A = \frac{(9+p^2)^2}{4p}$ .	
c.	Find the <b>minimum</b> area of the triangle OSR and the value of $p$ for which the	
	area is a minimum.	
		3 marks
d.	Find the <b>maximum</b> area of the triangle OSR and the value of $p$ for which the	
•	area is a maximum.	
		1 mark

# END OF QUESTION AND ANSWER BOOKLET END OF EXAMINATION

# **MATHEMATICAL METHODS**

# Written examination 1

# **FORMULA SHEET**

# **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

# **Mathematical Methods formulas**

# Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

# **Calculus**

$\frac{d}{dx}(x^n) = 1$	$nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}$	$x^{n+1} + c , n \neq -1$
$\frac{d}{dx}\Big(\Big(ax+b\Big)$	$(b)^n = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, \ n \neq -1$	
$\frac{d}{dx}(e^{ax}) =$	ae <sup>ax</sup>	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$	
$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$	
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$		$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$	
$\frac{d}{dx}(\tan(ax))$	$(x)$ = $\frac{a}{\cos^2(ax)}$ = $a \sec^2(ax)$		
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

# **Probability**

Pr(A) = 1 - Pr(A')		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$		
$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$				
mean	$\mu = E(X)$	variance	$\operatorname{var}(X) = \sigma^{2} = E((X - \mu)^{2}) = E(X^{2}) - \mu^{2}$	

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x  p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x  f\left(x\right) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

# **Sample proportions**

$\hat{P} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

# **END OF FORMULA SHEET**