MATHEMATICS
HIGHER LEVEL
PAPER 1

1.5 hours

NAME:_____

November 2015

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is not permitted for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the *Mathematics HL information booklet* is required for this paper.

SECTION A:	**55 MARKS**
SECTION B:	**35 MARKS**
TOTAL:	**90 MARKS**

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A
*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1.   (a) Find $f'(x)$ given $f(x) = x^2 \ln x$.

   (b) Evaluate $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cos(2x)\,dx$

(Total 5 marks)

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2. The polynomial $f(x) = x^3 + 3x^2 + ax + b$ leaves the same remainder when divided by $(x - 2)$ as when divided by $(x + 1)$. Find the value of a.

(Total 6 marks)

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3. The function f' is given by $f'(x) = 2\sin\left(5x - \dfrac{\pi}{2}\right)$.

(a) Write down $f''(x)$. (2)

(b) Given that $f\left(\dfrac{\pi}{2}\right) = 1$, find $f(x)$. (4)

(Total 6 marks)

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4. Consider the function $f(x) = \dfrac{1}{2}(x^3 + 3x^2 - 4)$, $-3 \leq x \leq 2$.

(a) Find the coordinates of the stationary points of the function.

The rule for f can also be expressed as $f(x) = \dfrac{1}{2}(x-1)(x+2)^2$.

(b) Sketch the graph of f, clearly indicating axis intercepts and turning points. Label the end points with their coordinates.

(Total 6 marks)

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5. The three terms a, 1, b are in arithmetic progression. The three terms 1, a, b are in geometric progression. Find the value of a and of b given that $a \neq b$.

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6. *A, B* and *C* are the angles in a triangle such that $\cos A = \dfrac{3}{5}$ and $\cos B = \dfrac{5}{13}$.

Find the value of:

(a) $\tan 2A$,

(b) $\cos(A+B)$

(c) $\cos C$

(Total 9 marks)

7.   (a) Solve $\log_2(6-x) - \log_2(4-x) = 2$ for x, where $x < 4$.

   (b) Solve $3e^t = 5 + 8e^{-t}$.

                                                                                (Total 6 marks)

8. The function f is defined by $f : x \mapsto x^3$.

Find an expression for $g(x)$ in terms of x in each of the following cases

(i) $(f \circ g)(x) = x + 1$;

(ii) $(g \circ f)(x) = x + 1$.

(Total 4 marks)

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9. In the game of darts, a dart is thrown at a board with concentric circles (circles with a common centre) painted on it. Points are awarded according to how close the centre your dart lands.

The smallest circle on this dartboard (the bullseye) has a radius of 2cm and the radii of the circles increase by 2cm each time. The probability of getting a bullseye is based on the area of the landing space.

(a) Show that the probability of getting a bullseye is $\dfrac{1}{25}$.

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(b) What would be the probability of getting 2 bullseyes in a row?

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(c) If you score 50 points for a bullseye, and 10 points less for each circle as you move away from the centre:

(i) complete a distribution table for your score

Score (x)	50	40	30	20	10
P($X = x$)					

(ii) what would be your expected score for one throw of a dart?

(Total 7 marks)

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Do **not** write solutions on this page.

Section B

Answer **all** questions on lined paper provided. Please start each question on a new page.

10. *[Maximum 10 marks]*

The diagram shows the graph of the function *f* given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \le x \le 5$, where *A* and *B* are constants, and *x* is measured in radians.
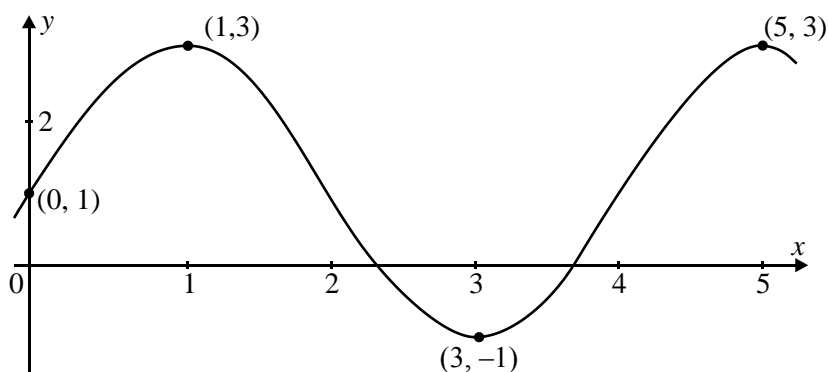
The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a)   Write down the values of *A* and *B*.

(2)

(b)   Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(2)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \le x \le 5$.

(c)   Find

    (i)   the point where this tangent meets the curve;

    (ii)   the value of *k*.

(6)
(Total 10 marks)

11. *[Maximum 11 marks]*

Bag A contains 2 red and 3 green balls.

(a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen.

(2)

Bag B contains 4 red and *n* green balls.

(b) Two balls are chosen without replacement from this bag. If the probability that two red balls are chosen is $\dfrac{2}{15}$, show that *n* = 6.

(3)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(4)

(d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die.

(2)
(Total 11 marks)

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12. *[Maximum 14 marks]*

Consider the functions $f(x), g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	−14	−18
$f'(x)$	1	1
$g'(x)$	−5	−3
$h''(x)$	−6	0

(a) Write down the value of $g(3), f'(3)$ and $h''(2)$

The following diagram shows parts of the graphs of h and h''.
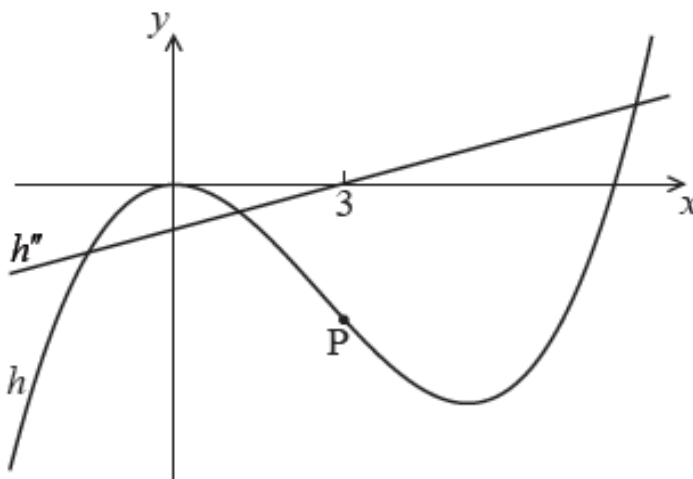
There is a point of inflexion on the graph of **h** at P, when $x = 3$.

(b) Explain why P is a point of inflexion.

Given that $h(x) = f(x) \times g(x)$,

(c) find the *y*-coordinate of P.

(d) find the equation of the normal to the graph of $h(x)$ at the point P.