The Mathematical Association of Victoria

Trial Examination 2016

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calcualtors or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Question 1 (4 marks)

a. If $f(x) = \log_e\left(\cos(4x)\right)$ find $f'(x)$. Express your answer in the form $A\tan\left(Bx\right)$ where A and B are real numbers. 2 marks

b. **i.** Factorise $x^3 - 3x^2 + 3x - 1$. 1 mark

ii. Hence, antidifferentiate $\dfrac{1}{(1-x)(x^3 - 3x^2 + 3x - 1)}$. 1 mark

TURN OVER

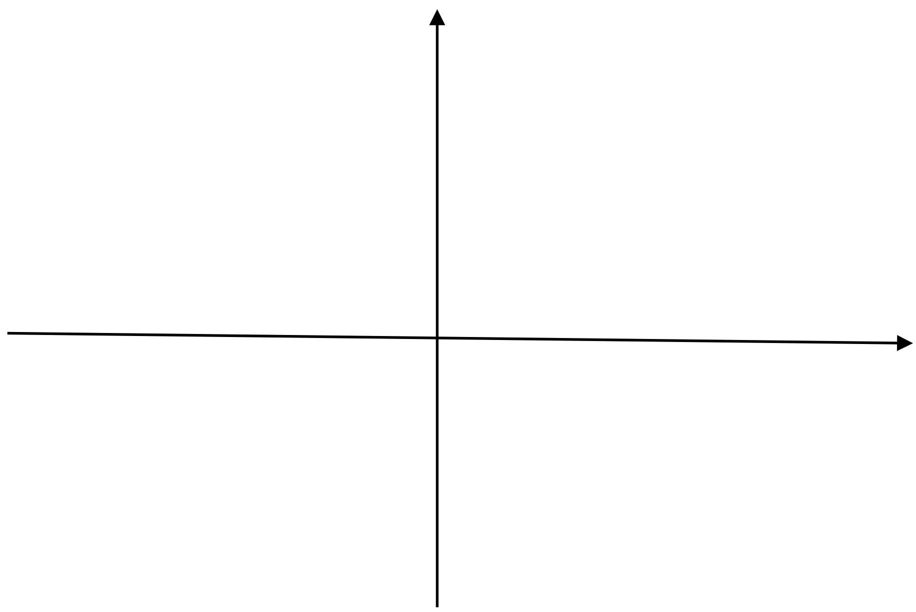
Question 2 (3 marks)
The depth, $d(t)$ m, of water at a pier t hours after midnight on a particular 24 hour day is given by $d(t) = 2\sin\left(\frac{\pi t}{6}\right) + 5$. Find the values of t for which the depth is greater than 6 m.

Question 3 (5 marks)

Consider the function $g:[0,2]\to R, g(x)=2x^5-10x^4+20x^3-20x^2+10x+2$.

a. Find A, B and C given $g(x)=A(x+B)^5+C$, where A, B and C are real constants. 3 marks

b. Sketch the graph of g on the set of axes below. Label the endpoints and any stationary points with their coordinates. 2 marks



TURN OVER

Question 4 (3 marks)

a. Show that $x = \log_e(3)$ is a solution of the equation $\frac{9}{e^{2x}} - \frac{6}{e^x} + 2e^x = 5$. 1 mark

b. Hence, or otherwise, solve the equation $\frac{9}{e^{2x}} - \frac{6}{e^x} + 2e^x = 5$ for x, given that there are only two real solutions. 2 marks

Question 5 (4 marks)

Let $f(x) = xe^{2x}$.

a. Find $f'(x)$. 1 mark

b. **Hence**, find the average value of *f* over the interval $\left[0, \frac{1}{2}\right]$. 3 marks

Question 6 (3 marks)

Find the values of A and B, where A and B are real constants, if the graph of $y = A\log_e\left(x - B\right)$ passes through the points (2, 10) and (8, 20).

Question 7 (4 marks)
Two linear equations can be written in the form of

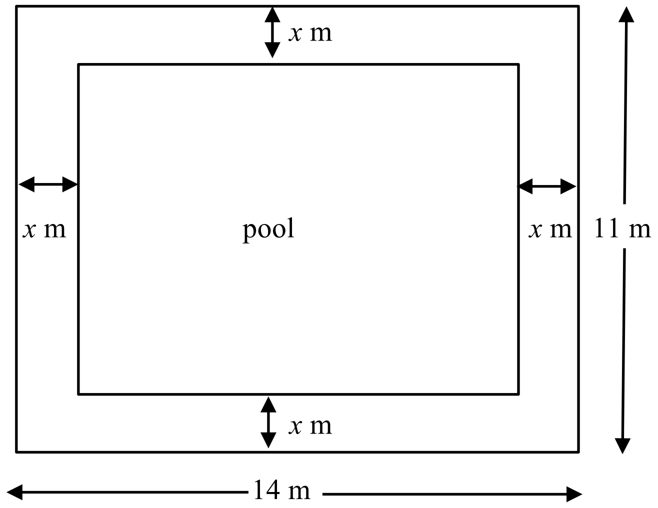
$$\begin{aligned} -2(m-1)x + my &= -m+4 \\ mx - 3y &= 2m+1 \end{aligned}$$ where m is a real constant.

a. Find the value(s) of m such that the graphs of the two lines have a unique solution. 2 marks

b. If $m = -1$, show that the line with equation $-2(m-1)x + my = -m+4$ is a tangent to the parabola with the equation $y = mx^2 + 2x - 6$. 2 marks

TURN OVER

Question 8 (6 marks)
Taren and Yao have a rectangular garden. It is 14 metres long and 11 metres wide. They want to put a rectangular swimming pool in the middle of the garden and a path of width x metres around the edge, as shown below.



a. Show that an expression for the length of the diagonal of the pool in terms of x is $\sqrt{8x^2 - 100x + 317}$. 2 marks

Taren's swimming instructor insists that the length of the diagonal of the pool is at least 15 metres for their pool dancing lessons.

b. For what value(s) of x will the diagonal be at least 15 metres in length? 3 marks

c. Yao wants the surface area of the floor of the pool to be at least 155 square metres. Show that this is not possible. 1 mark

Question 9 (8 marks)

a. Suppose that 80% of all 16 year olds play basketball. If a sample of size 4 is taken find the probability that the sample proportion lies within and including one standard deviation of the population proportion. 3 marks

Sam plays baskeball. The probability that Sam scores a goal every time she has a shot is 0.2.

b. Given that she scores no more than one goal in four shots, what is the probability the first two shots were **not** goals? 2 marks

c. What is the least number of shots she needs to make to ensure the probability that she gets at least one goal is more than 0.9, given $\log_{10}(8) \approx 0.903$? 3 marks

END OF QUESTION AND ANSWER BOOKLET