Neap:

Trial Examination 2016

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 25 pages.

Formula sheet.

Answer sheet for multiple-choice questions.

Instructions

Write your **name** and **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in the booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2016 VCE Mathematical Methods Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The horizontal translation undergone by the graph of $g(x) = 5\cos\left(\frac{1}{3}x - \pi\right) - 3$ from the graph of $f(x) = 5\cos(x)$ is

A. -3

B. 3

C. $\frac{\pi}{3}$

D. π

E. 3π

Question 2

If the equation $f(3x) - 3f(x) = 0$ is true for all $x \in R$, then the rule for f could be

A. $\sqrt{3x}$

B. $\frac{x^2}{3}$

C. $3x$

D. $3e^x$

E. $x + 3$

Question 3

In a complete survey of the population of a particular rural township, it is found that 73% of families have at least two children.

The '73%' represents the value of a

A. sample.

B. sample parameter.

C. sample statistic.

D. population.

E. population parameter.

Question 4

Let $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x+1}$.

If $h(x) = f(x)g(x)$, then the maximal domain of the derivative of h is

A. $[-1, 3]$
B. $(-1, 3)$
C. $[-1, \infty)$
D. $[3, \infty)$
E. $(\infty, -1) \cup (3, \infty)$

Question 5

The random variable X has the probability distribution shown, where $0 < p < \frac{1}{3}$.

x	-10	0	10
$\Pr(X = x)$	$2p - p^2$	$(p-1)(3p-1)$	$2p - 2p^2$

The mean of this distribution is

A. 0
B. 1
C. 10
D. $-10p^2$
E. $-40p$

Question 6

To halve the margin of error in an approximate 95% confidence interval, one could

A. use a 47.5% confidence interval.
B. halve the sample size.
C. double the sample size.
D. decrease the sample size by a factor of 4.
E. increase the sample size by a factor of 4.

Question 7

Using the left rectangle approximation with rectangles of width 1 unit, the area of the region bounded by the x-axis, the lines $x = 2$ and $x = 5$ and the curve with equation $y = \log_e(x-1)$ is approximately

A. $\log_e(2) + \log_e(3) + \log_e(4) + \log_e(5)$
B. $\log_e(1) + \log_e(2) + \log_e(3) + \log_e(4)$
C. $\log_e(2) + \log_e(3) + \log_e(4)$
D. $\log_e(3) + \log_e(4) + \log_e(5)$
E. $\log_e(2) + \log_e(3)$

Question 8

The average rate at which the volume of a sphere changes with respect to the radius as the radius increases from $r = a$ to $r = b$ is

A. $\frac{4}{3}\pi(a^3 - b^3)$

B. $-\frac{4}{3}\pi(a^3 - b^3)$

C. $\frac{4}{3}\pi(a^2 + ab + b^2)$

D. $4\pi(a^2 + ab + b^2)$

E. $4\pi(b + a)$

Question 9

The graph of a normal distribution whose mean ($\mu > 0$) has doubled and standard deviation halved, when compared to the original, would appear to be

A. wider and have moved left.
B. wider and have moved right.
C. narrower and have moved left.
D. narrower and have moved right.
E. narrower and taller only.

Question 10

The function $f(x) = a\sqrt{3}\cos(x) + b\sin(x)$, where $a, b > 0$, will have a maximum turning point at $x = \frac{\pi}{6}$ if

A. $b = 3a$

B. $b = -3a$

C. $a = \frac{b}{3}$

D. $a = b$

E. $b = -a$

Question 11

X has a probability function given by the following:

$$p(x) = \frac{1}{5} \text{ for } x = 0, 1, 2, 3, 4$$

The median of X is

A. 0

B. $\frac{1}{5}$

C. 2

D. 2.5

E. 3

Question 12

The function $f: R \to R$ satisfies the functional equation $f(u - \pi) = f(u)$ for all $u \in R$.

The rule could be

A. $\sin(x)$

B. $\cos(x)$

C. $\tan\left(\frac{x}{2}\right)$

D. $\sin\left(\frac{x}{2}\right)$

E. $\cos(2x)$

Question 13

14 of a random sample of 88 people say they prefer to rent a movie to watch at home rather than go to the cinema.

An approximate 95% confidence interval for the proportion of people in the population who prefer to rent a movie is given by

A. (0.083, 0.236)
B. (0.085, 0.233)
C. (0.120, 0.198)
D. (0.157, 0.161)
E. (0.259, 0.359)

Question 14

The equation of the tangent to the curve $y = \log_e\left(\frac{a}{x}\right)$ at the point $x = e$ is

A. $y = -\frac{1}{e}x + \log_e(a)$

B. $y = -\frac{1}{e}x + \log_e(a) + 2$

C. $y = \frac{1}{e}x + \log_e(a) - 2$

D. $y = -\frac{e}{x} + \log_e(a) - 2$

E. $y = -\frac{e}{x} - \log_e(a) + 2$

Question 15

The following simultaneous equations

$$(m-1)x + 7y = 3m$$
$$6x + (3m+2)y = -24n$$

will have infinite solutions for

A. $m \in \{-\frac{11}{3}, 4\}, n = -1$

B. $m \in \{-\frac{11}{3}, 4\}, n \in R\backslash\{-1\}$

C. $m \in R\backslash\left\{\frac{-11}{3}, 4\right\}, n = -1$

D. $m \in R\backslash\left\{\frac{-11}{3}, 4\right\}, n \in R\backslash\{-1\}$

E. $m = 4, n = -1$

Question 16

A researcher wants to calculate some statistics regarding the health of the tens of thousands of bats populating the trees near a local river.

Consider the following statistics:

I the population size

II an exact sampling distribution of sample proportions using a table

III a point estimate for the proportion of healthy bats

IV a confidence interval estimate for the proportion of healthy bats

After sampling 100 of the bats, which of the above statistics would be appropriate to calculate?

A. I only
B. II only
C. III only
D. IV only
E. III and IV

Question 17

For $y = \log_e(\sqrt{f(x)})$, $\frac{dy}{dx}$ is equal to

A. $\frac{1}{\sqrt{f(x)}}$

B. $\frac{f'(x)}{\sqrt{f(x)}}$

C. $\frac{f'(x)}{2\sqrt{f(x)}}$

D. $\frac{f'(x)}{2f(x)}$

E. $\frac{1}{2}\log_e(\sqrt{f(x)})$

Question 18

The equation $ax^4 + 4x^3 + 2x^2 = 0$, where $a > 0$, has exactly two distinct solutions.

Hence the graph of $y = ax^4 + 4x^3 + 2x^2$ has

A. local maximum turning points at $x = 0$ and $x = -1$.

B. local minimum turning points at $x = 0$ and $x = -1$.

C. local minimum turning points at $x = 0$ and $x = 1$.

D. a local maximum turning point at $x = -1$ and a local minimum turning point at $x = 0$.

E. a local maximum turning point at $x = 0$ and a local minimum turning point at $x = -1$.

Question 19

If $\int_0^2 f(x)dx = 3$, then $\int_0^8 f\left(\frac{1}{4}x\right) + 2dx =$

A. $\int_0^8 f\left(\frac{1}{4}x\right)dx + 4$

B. $\int_0^2 f\left(\frac{1}{4}x\right)dx + 2x$

C. $\frac{1}{4}\int_0^8 f(x)dx + 16$

D. 14

E. 28

Question 20

It is known that the probability of a person becoming ill after returning on a particular flight from a remote country is 0.3. A random sample of 10 people who were on that flight was taken and the people were monitored.

The probability that at the most 2 of them will **not** become ill, correct to four decimal places, is

A. 0.0015
B. 0.0016
C. 0.3545
D. 0.3827
E. 0.3828

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

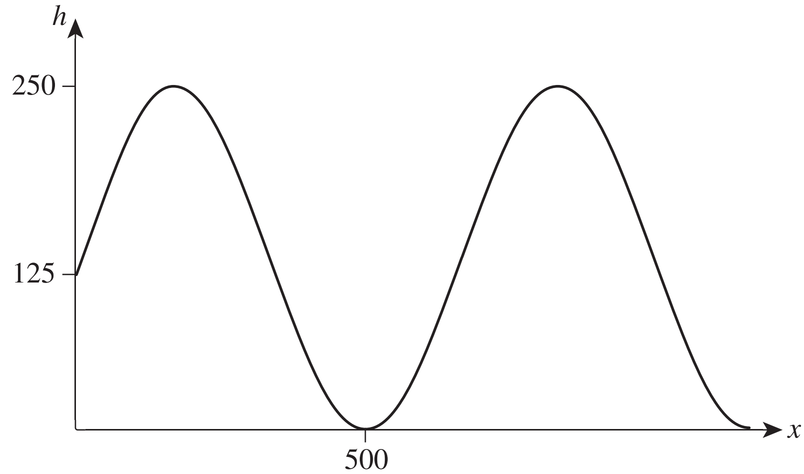
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (10 marks)

The side view of a roller coaster in a hamster's cage is shown below, where h is the vertical height, in mm, of the roller coaster above the bottom of the cage and x is the horizontal distance, in mm, from the start of the roller coaster. The start of the roller coaster is at (0, 125), where it is attached to the side of the cage. The maximum height reached by the roller coaster is 250 mm and the roller coaster first reaches the bottom of the cage when $x = 500$. The end of the roller coaster is where it touches the bottom of the cage for the second time.

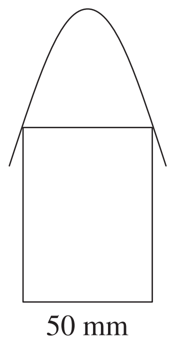


The highest point of the roller coaster can be modelled by the equation $h = A\sin(nx) + B$, where A, n and B are real constants.

a. Find the values of A, n and B. 2 marks

b. Find the average rate of change of the height of the roller coaster from the start to the first time it touches the ground. 2 marks

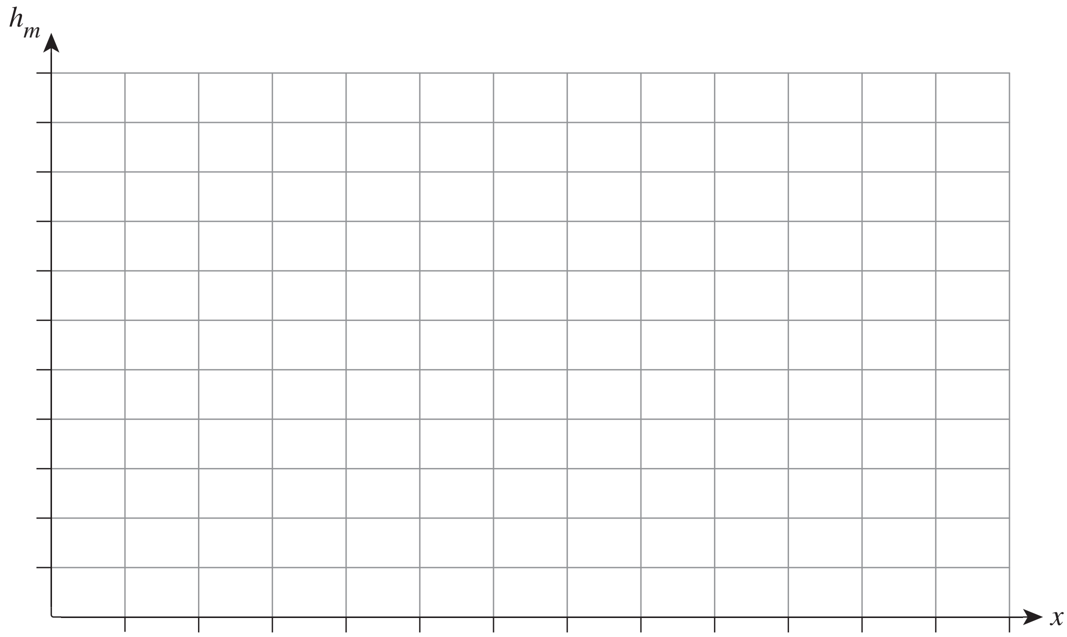
It is found that the roller coaster is not strong enough to support large hamsters. It is decided that supports are required under each of the 'high points' of the roller coaster. The support is a cuboid of width 50 mm. The side view is shown below.



c. What is the maximum height the cuboid structure can be if its edges touch the arch? Give your answer correct to the nearest mm. 3 marks

A smaller scale version of the hamster's cage, suitable for a mouse, is to be constructed. The roller coaster in the mouse's cage is modelled by the equation h_m, which is a dilation of the graph of h by a factor of $\frac{1}{5}$ from the x-axis.

d. Sketch the graph of h_m on the axes below, labelling intercepts and turning points with their coordinates. 2 marks



e. Write down the rule for the roller coaster in the mouse's cage. 1 mark

Question 2 (16 marks)

Xavier is a dental inspector. He has just opened a new dental practice and has been informed that in his local area, about 20% of young children have tooth decay. Xavier wants to conduct some of his own research.

a. If Xavier randomly selected a sample of 100 children from his area,

i. how many would be expected to have tooth decay? 1 mark

ii. what would be the probability, correct to three decimal places, of finding exactly this number? 2 marks

b. Xavier would like to reduce the number of children with tooth decay.

Find the probability that the number of children in the sample with decay would be no more than 15. Give your answer correct to three decimal places. 1 mark

c. Xavier examines all the children in the sample and finds 29 of them have tooth decay.

i. Calculate a 2 standard deviation limit to determine whether this number is significantly higher than expected. Comment on whether Xavier could have cause to doubt the accuracy of information he was given. 2 marks

ii. What is the sample proportion of children with tooth decay? 1 mark

iii. Calculate an approximate 95% confidence interval for the proportion of children in the area who have tooth decay. Give your answer correct to three decimal places. 1 mark

d. Xavier takes a further 200 samples of 100 children, calculating the sample proportion and 95% confidence intervals for each.

i. How many of these confidence intervals could Xavier expect to contain the population proportion of children with tooth decay? 1 mark

ii. Give the appropriate type of distribution that you would expect would be a good approximation for these sample proportions. 1 mark

iii. What would the parameters (mean and standard deviation) of this distribution be, correct to two decimal places? 1 mark

iv. What is the approximate probability, correct to three decimal places, that the proportion of children with tooth decay in his next 200 samples is between 0.15 and 0.20, given the probability is less than 0.25? 3 marks

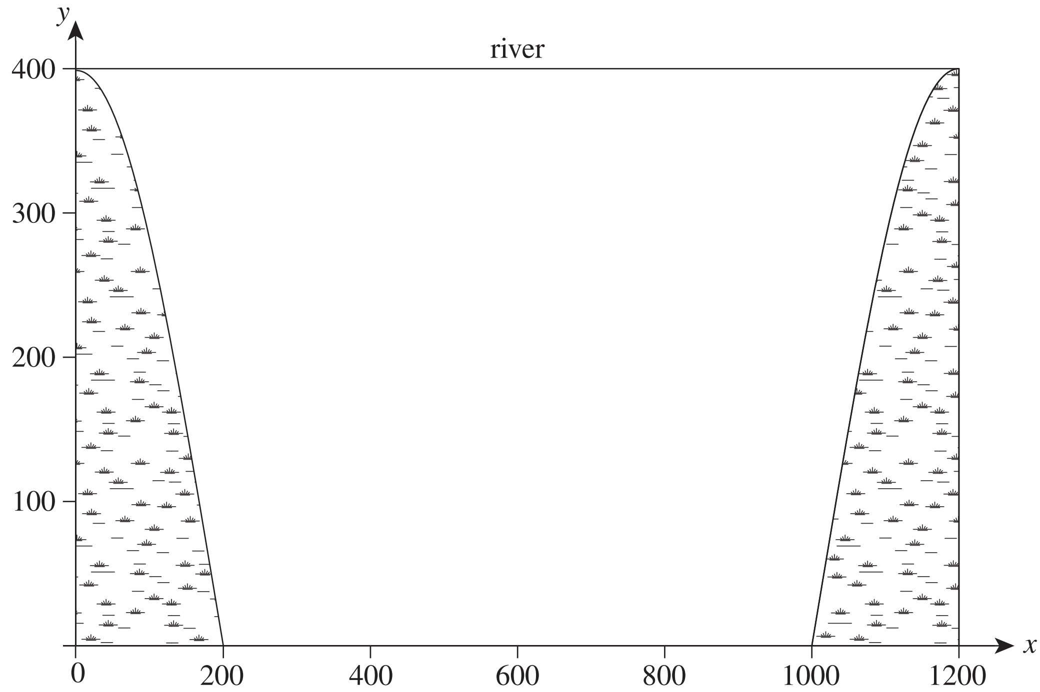
Xavier intends to embark on an education program regarding dental health for these children. His aim is to reduce the proportion of young children with tooth decay to half its current expectation. He would like to find out how many children he will need to sample to have a 95% confidence interval with a margin of error that is equal to 3%.

e. How many children will Xavier need to sample? 2 marks

Question 3 (8 marks)

The Melville Town Council is planning a new residential suburb. They have engaged a landscape designer, Penny Planner, to design the space, which will be a parkland in the centre of the town.

Penny has a large area at her disposal. The rectangular piece of land the council has allocated for the park measures 1200 m by 400 m. A river runs along the length of the land and the riverbank forms one of the boundaries of the park. A section of garden will be planted at each end, as shown in the graph below.



The border of the garden at the left end of the park is modelled by the function

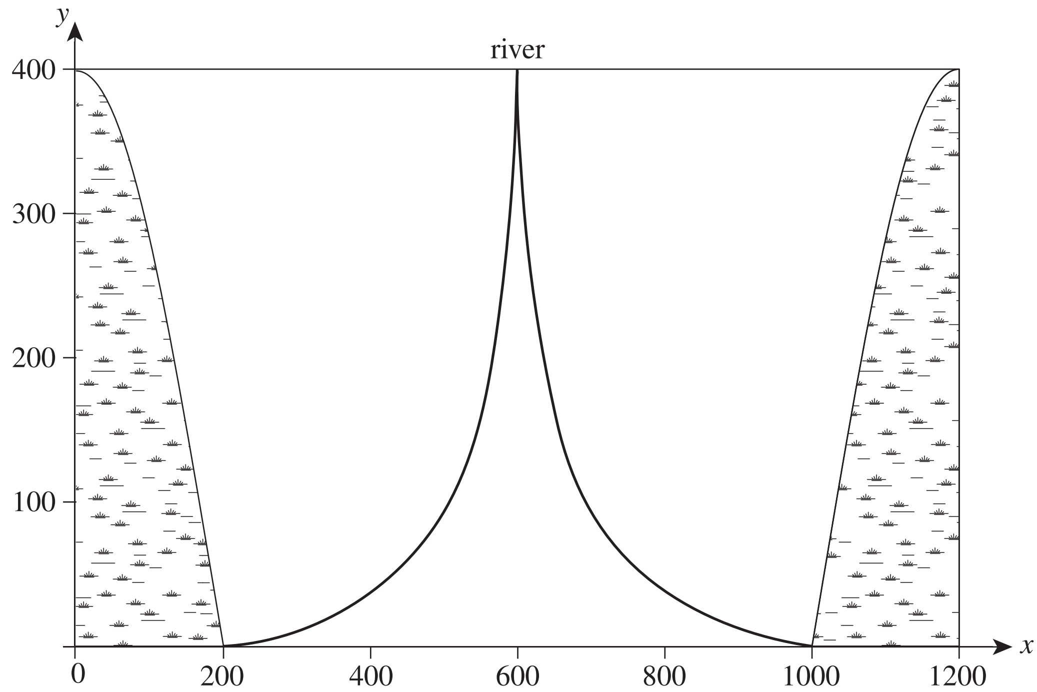
$$g : [0, 200] \to R,\ g(x) = 400 - \frac{x^2}{100},$$

where x is the horizontal distance, in metres, from the left end of the park.

The border of the garden at the other end of the park, which is modelled by the function f, is a reflection of the graph of g in the line $x = 600$.

a. Find the function f. 2 marks

Penny decides to design a walking path through the park that leads to the river, as shown in the graph below.



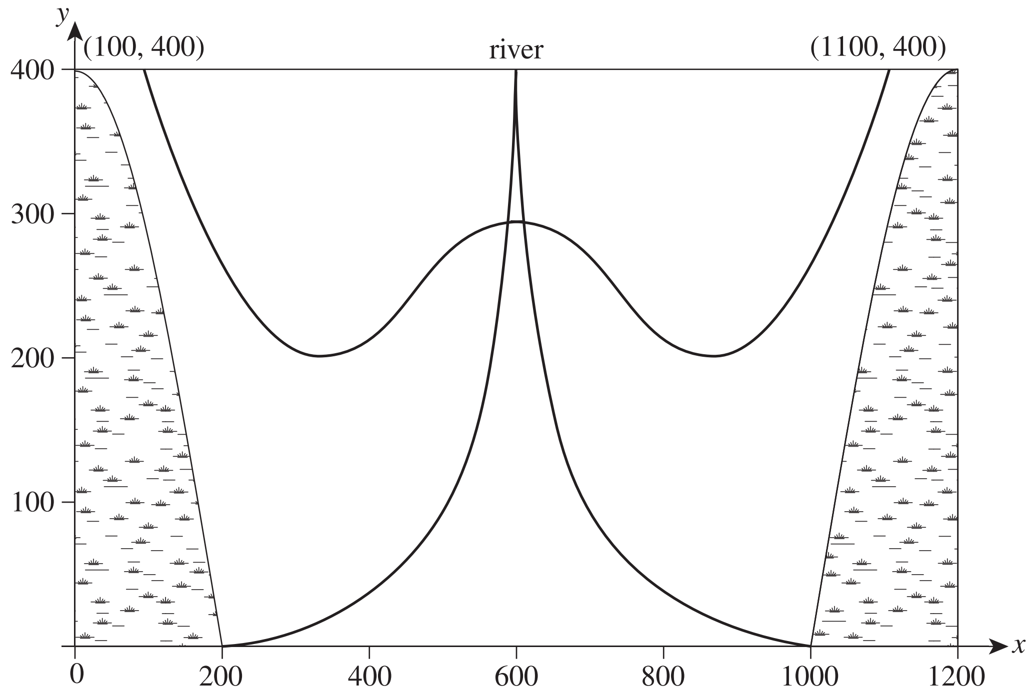
She models the path by the hybrid function

$$p(x) = \begin{cases} e^{-6}(x-200)e^{\frac{x}{100}} & 200 \le x \le 600 \\ e^{-6}(b-x)e^{\frac{c-x}{100}} & 600 < x \le 1000 \end{cases}$$

where b and c are real constants.

b. If the right-hand side of the graph of the hybrid function is a reflection in the line $x = 600$ of the left-hand side, find the values of b and c. 2 marks

Penny also decides a small, artificial creek would be nice for families to use for paper-boat races. She plans to feed the creek from the river, branching off the river at the point (100, 400) and returning to the river at the point (1100, 400), as shown in the diagram below.



The creek is modelled by the function

$$c : [100, 1100] \rightarrow R, \; c(x) = \frac{1}{128\ 000\ 000}(x - 300)^2(x - 900)^2 + 200$$

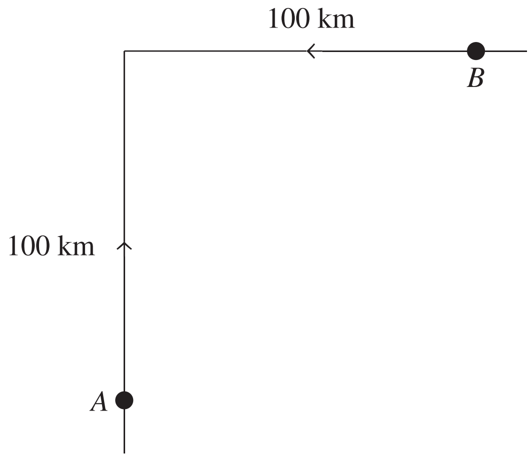
Two bridges will need to be constructed for the path to go over the creek.

c. Find the coordinates of the points where these two bridges will need to be constructed, correct to one decimal place. 2 marks

d. Find the average value of the function modelling the creek, over the interval between the two bridges. Give your answer correct to one decimal place. 2 marks

Question 4 (18 marks)

Two straight roads intersect at right angles. Two truck drivers, A and B, are 100 kilometres from the intersection, one on each road. They drive towards the intersection at 30 km/hr and 40 km/hr respectively.



a. **i.** Find the distance of each driver from the intersection one hour after they have been driving. 1 mark

ii. Find the distance between the drivers at this time, correct to one decimal place. 1 mark

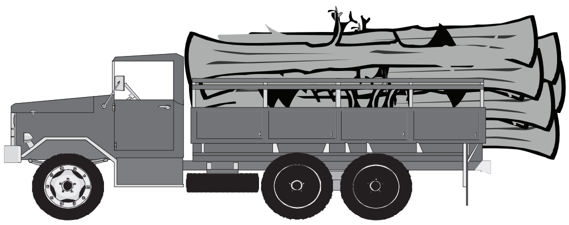
b. **i.** Express the distance of each driver from the intersection as a function of t, the time in hours for which they are driving, where d_A and d_B are the distances A and B have travelled respectively. 1 mark

ii. Hence, find their distance apart, $d(t)$, at any time. 2 marks

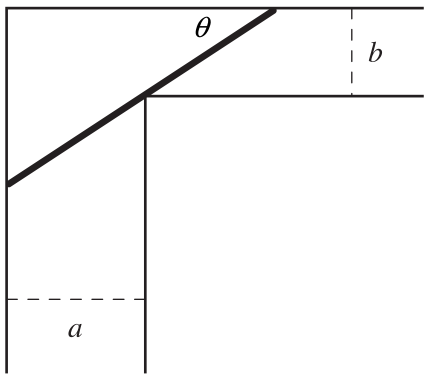
iii. For what value of t is their distance apart least? 2 marks

iv. What is the minimum distance? 1 mark

The two trucks are logging trucks which carry long, cut down, stripped pine trees to the lumber yard at the wharf. The drivers need to be sure their load will be able to be carried around corners as in the situation described previously.



The two roads, which meet to form a right-angled corner, are of widths a and b. The angle which driver B's truck makes with the road as it turns the corner is shown in the diagram below as θ.



c. If the two roads of width a and b are 8 m and 6 m respectively,

i. show that the equation for the maximum length, L, of the diagonal, in terms of θ, is $L = \dfrac{6}{\sin(\theta)} + \dfrac{8}{\cos(\theta)}$. 2 marks

ii. find the longest length of pine the truck can carry if it to make it around the corner, correct to two decimal places. 3 marks

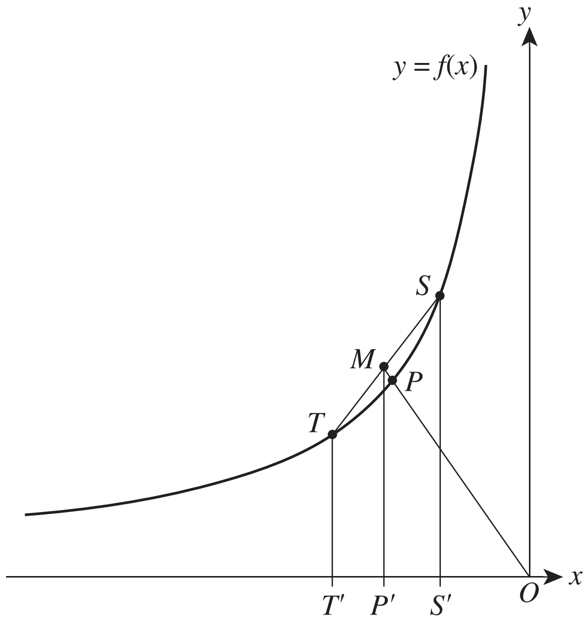
d. **i.** Show that for the roads of arbitrary widths a and b, the maximum length of pine which can be carried around the corner can be found by solving $a\sin^3(\theta) - b\cos^3(\theta) = 0$. 2 marks

ii. Hence, show that the maximum length of pine which can be carried around the corner of arbitrary roads of widths a and b is $\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)^{\frac{3}{2}}$. 3 marks

Question 5 (8 marks)

The diagram below shows part of the graph of the function $f:(-\infty, 0) \to R, f(x) = -\frac{1}{x}$.

The points $S(s, f(s))$ and $T(t, f(t))$, where $t < s < 0$, are two points on the graph $y = f(x)$. The points $S'(s, 0)$ and $T'(t, 0)$ lie on the x-axis, while M is the midpoint of the line segment joining S and T. The line through OM, where O is the origin, intersects the function at the point $P(p, f(p))$ and the point $P'(p, 0)$ also lies on the x-axis.



a. Find the equation of the tangent to the curve at the point P, expressing constants in terms of p. 1 mark

b. **i.** Find the coordinates of the point M in terms of s and t. 1 mark

ii. Show that $p^2 = st$. 2 marks

c. Let A be the area bounded by the curve $y = f(x)$, the x-axis and the lines with equations $x = s$ and $x = t$.

Express A in terms of s and t. 1 mark

d. The area B is bounded by the curve $y = f(x)$ and the line segments OT and OS.

If $B = \int_t^s g(x)dx$, write down the rules for $g(x)$. 3 marks

END OF QUESTION AND ANSWER BOOKLET

