NAME: ____________________________

UNITS 3 & 4 Practice Examination

VCE® Mathematical Methods

Written examination 2

Reading time: 15 minutes **Writing time: 2 hours**

QUESTION AND ANSWER BOOK
Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- A question and answer book of 22 pages.
- A double sided page of formulas.
- An answer sheet for multiple-choice questions.

Instructions
- Write your name in the space provided above on this page.
- Write your name on the multiple-choice answer sheet.
- Unless otherwise indicated the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At end of Examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The point $P(1,2)$ lies on the graph $y = f(x)$ of a function f. The graph is translated two units along the positive direction of the x −axis and then reflected in the x −axis.

The coordinates of the final image of P are

A. $(-1,4)$

B. $(-1,0)$

C. $(-1,-2)$

D. $(3,-2)$

E. $(3,2)$

Question 2

The linear function $f: D \to R, f(x) = 2 - x$ has range $(-2,4]$.

The domain D of the function is

A. $(-2,4]$

B. $[-2,4)$

C. $(4,-2]$

D. $[4,-2)$

E. $(-4,8]$

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Question 3

The function with rule $f(x) = -2\tan(\pi x)$ has period

A. -2

B. 1

C. 2

D. π^2

E. $2\pi^2$

Question 4

If $x + a$ is a factor of $5x^3 + 3x^2 - 7ax$, where $a \in R\backslash\{0\}$, then the value of a is

A. -2

B. $-\frac{4}{5}$

C. $\frac{4}{5}$

D. 1

E. 2

Question 5

The function $g: [-a, a] \to R, g(x) = \sin\left(2\left(x - \frac{\pi}{12}\right)\right)$ has an inverse function.

The maximum possible value of a is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{5\pi}{12}$

D. $\frac{7\pi}{12}$

E. $\frac{13\pi}{12}$

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Question 6

If $\int_2^5 f(x)dx = 4$, then $\int_2^5 (3f(x) - 2)dx$ is equal to

A. -2

B. 2

C. 6

D. 9

E. 10

Question 7

For events A and B, $\Pr(A \cap B) = p$, $\Pr\left(A' \cap B\right) = p - \frac{2}{3}$ and $\Pr\left(A \cap B'\right) = \frac{p}{2}$.

If A and B are independent, then the value of p is

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. 1

Question 8

Which one of the following functions satisfies the functional equation $f\left(f(x)\right) = x$ for every real number x?

A. $\sqrt{1 - x}$

B. $\sqrt{1 - x^2}$

C. $\sqrt[3]{1 - x}$

D. $\sqrt[3]{1 - x^2}$

E. $\sqrt[3]{1 - x^3}$

Question 9

A bag contains three red marbles, one black and three blue marbles.
Two marbles are drawn from the bag, **without replacement**, and the results are recorded.
The probability of only one red ball may be calculated using

A. $\frac{3}{7}\times\frac{4}{7}$

B. $\frac{3}{7}\times\frac{4}{6}$

C. $2\times\frac{3}{7}\times\frac{4}{7}$

D. $2\times\frac{3}{7}\times\frac{4}{6}$

E. $\frac{2!}{1!\,1!}\left(\frac{3}{7}\right)\left(\frac{4}{7}\right)$

Question 10

The transformation

$$T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

maps the line with equation $2x - y = 1$ onto the line with equation

A. $y = -2 + x$

B. $x + y = -2$

C. $-x + y = 2$

D. $x + y = 2$

E. $x - y = 2$

Question 11

If the tangent to the graph of $y = e^{\frac{x}{a}}, a \neq 0$, at $x = c$ passes through the origin, then c is equal to

A. 0

B. $\frac{1}{a}$

C. 1

D. a

E. $-\frac{1}{a}$

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Question 12

The simultaneous linear equations $ax - 2y = 4 - a$ and $2x - ay = a$ have **no solution** for

A. $a = -2$

B. $a = 2$

C. both $a = 2$ and $a = -2$

D. $a \in R\backslash\{-2\}$

E. $a \in R\backslash\{2\}$

Question 13

It is known that 13% of Australian adults prefer white chocolate to dark chocolate.
If a random sample of ten Australian adults is taken, the probability, correct to four decimal places, that more than half of them prefer **dark chocolate** is

A. 0.9947

B. 0.5000

C. 0.4336

D. 0.0260

E. 0.0006

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Question 14

In a random sample of 900 voters across the country, 52% indicated a preference to vote for the major opposition party at the next federal election.

An approximate 99.7% confidence interval for the proportion of the total voting population with a preference to vote for the major opposition party can be found by evaluating

A. $\left(52 - 0.15\sqrt{52 \times 48 \times 900}, 52 + 0.15\sqrt{52 \times 48 \times 900}\right)$

B. $\left(52 - 1.96\sqrt{\frac{52 \times 48}{900}}, 52 + 1.96\sqrt{\frac{52 \times 48}{900}}\right)$

C. $\left(0.52 - 1.96\sqrt{\frac{0.52 \times 0.48}{900}}, 0.52 + 1.96\sqrt{\frac{0.052 \times 0.48}{900}}\right)$

D. $\left(52 - 3\sqrt{\frac{52 \times 48}{900}}, 52 + 3\sqrt{\frac{52 \times 48}{900}}\right)$

E. $\left(0.52 - 3\sqrt{\frac{0.52 \times 0.48}{900}}, 0.52 + 3\sqrt{\frac{0.52 \times 0.48}{900}}\right)$

Question 15

The cubic function $f(x) = ax^3 + bx^2 + cx$, has **no** stationary points when

A. $b^2 - 3ac = 0$

B. $b^2 - 3ac \neq 0$

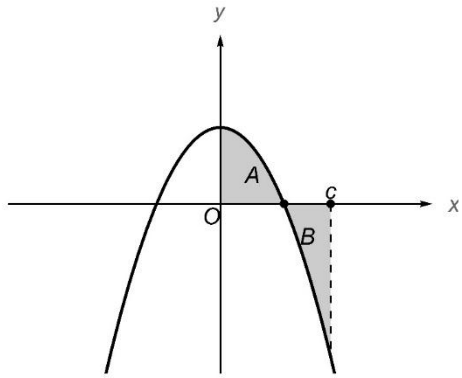
C. $c < \frac{b^2}{3a}$ and $a < 0$

D. $c < \frac{b^2}{3a}$ and $a > 0$

E. $b^2 - 3ac > 0$

Question 16

A part of the graph of $g: R \to R, g(x) = 9 - x^2$ is shown below.



The area of the region labelled *A* is the same as the area of the region labelled *B*.
The exact value of the positive real number c is

A. 3

B. $3\sqrt{3}$

C. 6

D. 9

E. 18

Question 17

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x −axis at $(-2,0)$ and $(3,0)$. The minimum value of the derivative function is -2.
The value of x for which the graph of $y = f(x)$ has a local maximum is

A. -2

B. 2

C. -3

D. 3

E. $-\frac{1}{2}$

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Question 18

The time T days required to process insurance claims at a large insurance company is a normally distributed random variable with a mean of 90 days and a standard deviation of σ days.
Of the last 1000 claims, 110 were processed within 70 days. The value of σ is closest to

A. 10

B. 12

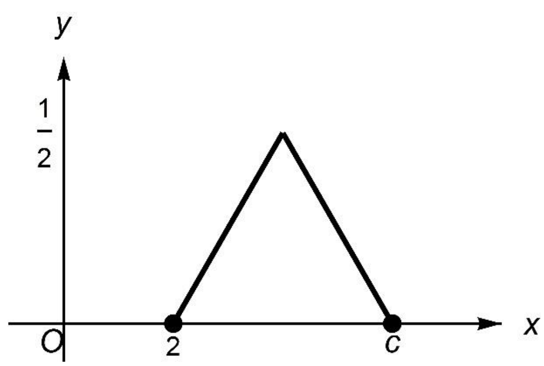
C. 14

D. 16

E. 18

Question 19

The graph of a probability density function of a continuous variable, X, is shown below.



If $c > 2$, and the mode occurs at the middle of the interval $[2, c]$, then $E(X)$ is equal to

A. 3

B. $\frac{7}{2}$

C. 4

D. $\frac{9}{2}$

E. 5

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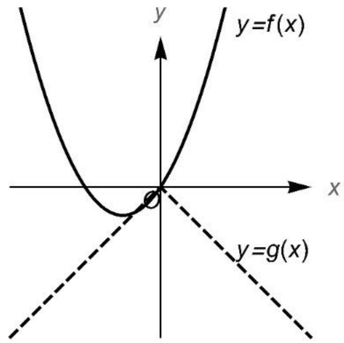
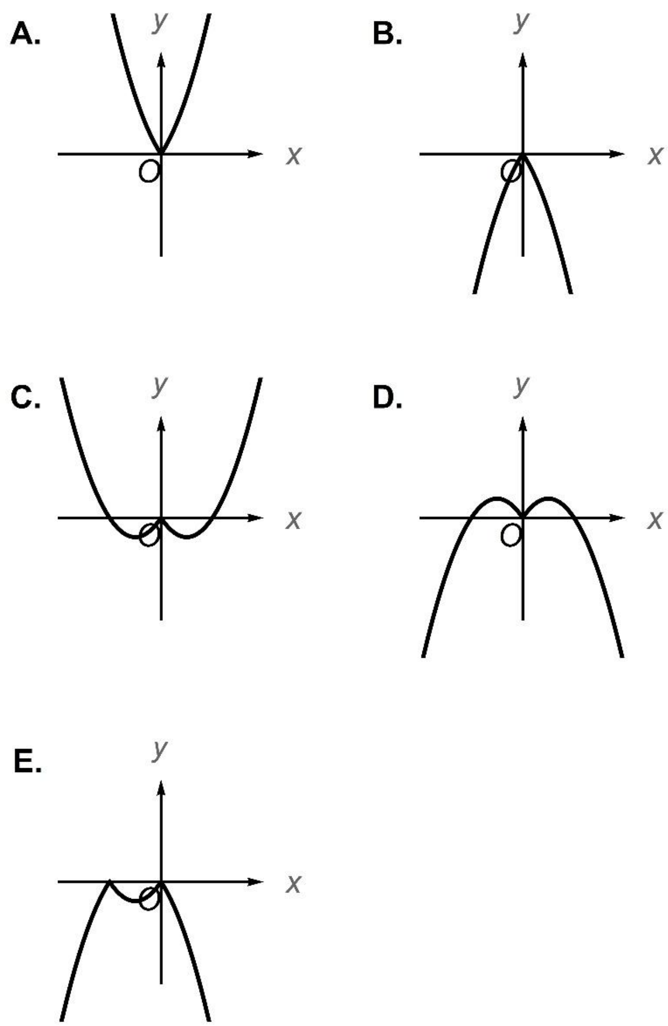
Question 20

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



The graph of $y = f(g(x))$ is best represented by



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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

The temperature, $T\ (°C)$, inside a climatically controlled greenhouse, t hours after midnight, is given by the rule

$$T(t) = 20 - 4\cos\left(\frac{\pi t}{12}\right).$$

a. Find the period and amplitude of the function T. 2 marks

b. Find the maximum and minimum temperature of the greenhouse. 2 marks

c. Find $T(8)$. 1 mark

d. Find the fraction of one period when the temperature of the greenhouse is at least as warm as $T(8)$. 2 marks

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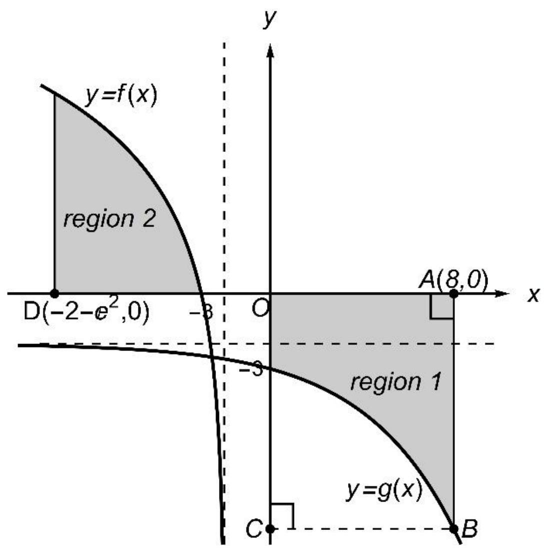
Question 2 (7 marks)

Let $f:(-\infty,-3)\to R, f(x)=4\log_e(-x-2)$ and $g:R\to R, g(x)=-e^{x/4}-2$.

The graphs $y=f(x)$ and $y=g(x)$ are shown in the diagram below.



a. Express the rule for each composite function in its simplest form:

i. $f(g(x))$ 1 mark

ii. $g(f(x))$ 1 mark

b. Find the area of the rectangle $OABC$ in the figure above. 1 mark

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c. Evaluate the integral $\int_0^8 g(x)\,dx$. 1 mark

d. Find the area of region 1 (shaded) in the figure above. 1 mark

e. **Hence**, find the area of region 2 (shaded) in the figure above. 2 marks

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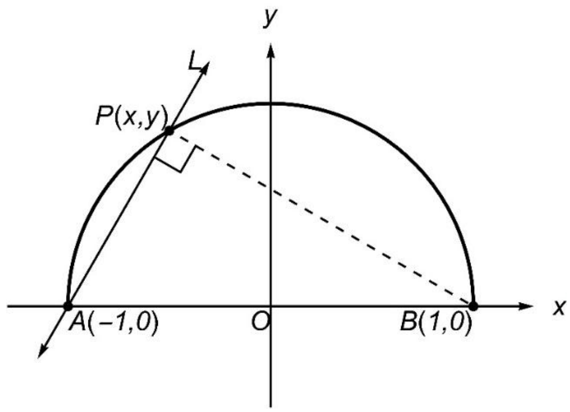
Question 3 (11 marks)

The points A, B and P are on the circumference of a semicircle of the unit radius, as shown in the figure below. The line L with gradient m, and equation $y = m(x+1)$ passes through point A and point P.



a. Solve the quadratic equation $x^2 + m^2(x+1)^2 = 1$ to express x in terms of m. 1 mark

b. Hence show that point P has coordinates $P\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$. 3 marks

c. Show that ΔAPB is a right angled triangle. 2 marks

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d. Express the area of ΔAPB in terms of m. 2 marks

e. Find $\frac{d}{dm}\left(\frac{2m}{1+m^2}\right)$ in simplest terms. 1 mark

f. Hence, find the value of m so that the area of ΔAPB is a maximum, and state the value of this maximum area. Justify your choice of m in the context of the current problem.

2 marks

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Question 4 (17 marks)

Let $f: R \rightarrow R, f(x) = (x-a)(x-b)(x-c)$, where a, b and c are real constants such that $a < b < c$

a. State the equation of the tangent line to the graph of $y = f(x)$ at the point with x −coordinate equal to $\frac{a+b}{2}$. Express your answer in a fully factored representation.

2 marks

b. Find x −intercept of the tangent line from part **a**. 1 mark

c. State the equation of the tangent line to the graph of $y = f(x)$ at the point with x −coordinate equal to $\frac{b+c}{2}$. Express your answer in a fully factored representation. 2 marks

d. Find the x −intercept of the tangent line from part **c**. 1 mark

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e. Use algebra to show that $b = \frac{a+c}{2}$ whenever the tangent line in part **b** is parallel to the tangent line in part **d**. 3 marks

In the remainder of this question, $b = \frac{a+c}{2}$.
Let $g: R \to R, g(x) = x + b$.

f. Find the **rule** for the composite function $h(x) = f\left(g(x)\right)$.
Express your answer in a factored form $h(x) = x(x-u)(x+u)$ where u is a suitably defined function of the parameters a and c. 2 marks

g. Verify that $h(x)$ is **a** solution of the functional equation $k(x) + k(-x) = 0$. 2 marks

h. Find the coordinates of the point of inflection of the graph of $y = h(x)$. 3 marks

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i. **Hence**, find the coordinates of the point of inflection of the graph of $y = f(x)$ when $b = \frac{a+c}{2}$. 1 mark

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Question 5 (18 marks)

Recent international medical research has revealed that individuals across the world with a particular gene have a $\frac{3}{5}$ probability of developing a specific disease.

a. A random sample of 24 gene carriers has been collected. Let X be the random variable that represents the number of these carriers who eventually develop the disease.

i. Find $\Pr(X \geq 12)$ correct to four decimal places. 2 marks

ii. Find $\Pr(X \geq 18 | X \geq 12)$ correct to three decimal places. 3 marks

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For samples of 24 gene carriers, $\hat{P}$ is the random variable of the distribution of sample proportions for the people who develop the disease.

iii. Find the expected value and variance of $\hat{P}$. 3 marks

iv. Find the probability that a sample proportion lies within two standard deviations of $\frac{3}{5}$.
Give your answer correct to three decimal places. Do not use a normal approximation.
3 marks

v. Find $\Pr\left(\hat{P} \geq \frac{3}{4} | \hat{P} \geq \frac{3}{5}\right)$. Give your answer correct to three decimal places.
Do not use a normal approximation. 2 marks

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b. The age (in years) when symptoms are first experienced by a sufferer of this disease is modelled by a random variable W, with a probability density function $g(w)$, given by the following hybrid function

$$g(w) = \begin{cases} 0 & w < 1 \\ -\dfrac{12(w-35)(w-1)}{99127} & 1 \le w \le 18 \\ -\dfrac{4(w-87)^3}{37559529} & 18 < w \le 87 \\ 0 & w > 87 \end{cases}$$

i. Find $E(W)$ correct to four decimal places. 2 marks

ii. In a random sample of 200 disease sufferers, how many would be expected to be older than 18 years of age? Give your answer to the nearest integer. 2 marks

c. From a random sample of 100 disease sufferers, it was found that 40 of these individuals developed symptoms of the disease before their 18th birthday.

State an approximate 95% confidence interval for the population proportion corresponding to this sample proportion. Express your values correct to three decimal places. 1 mark

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Multiple Choice Answer Sheet

Student Name:

Shade the letter that corresponds to each correct answer.

Question	A	B	C	D	E
Question 1	[ ]	[ ]	[ ]	[ ]	[ ]
Question 2	[ ]	[ ]	[ ]	[ ]	[ ]
Question 3	[ ]	[ ]	[ ]	[ ]	[ ]
Question 4	[ ]	[ ]	[ ]	[ ]	[ ]
Question 5	[ ]	[ ]	[ ]	[ ]	[ ]
Question 6	[ ]	[ ]	[ ]	[ ]	[ ]
Question 7	[ ]	[ ]	[ ]	[ ]	[ ]
Question 8	[ ]	[ ]	[ ]	[ ]	[ ]
Question 9	[ ]	[ ]	[ ]	[ ]	[ ]
Question 10	[ ]	[ ]	[ ]	[ ]	[ ]
Question 11	[ ]	[ ]	[ ]	[ ]	[ ]
Question 12	[ ]	[ ]	[ ]	[ ]	[ ]
Question 13	[ ]	[ ]	[ ]	[ ]	[ ]
Question 14	[ ]	[ ]	[ ]	[ ]	[ ]
Question 15	[ ]	[ ]	[ ]	[ ]	[ ]
Question 16	[ ]	[ ]	[ ]	[ ]	[ ]
Question 17	[ ]	[ ]	[ ]	[ ]	[ ]
Question 18	[ ]	[ ]	[ ]	[ ]	[ ]
Question 19	[ ]	[ ]	[ ]	[ ]	[ ]
Question 20	[ ]	[ ]	[ ]	[ ]	[ ]

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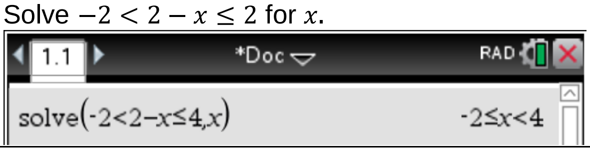
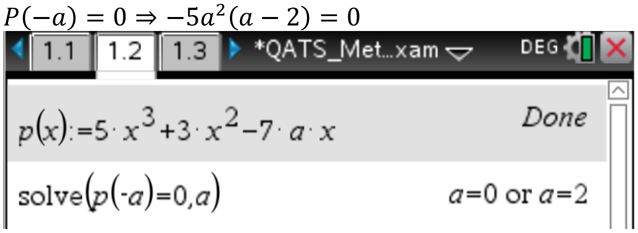
Solution Pathway

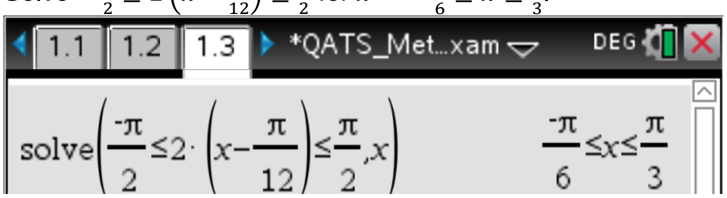
Below are sample answers. Please consider the merit of alternative responses.

Section A: Multiple-choice answers

1.	**D**	**6.**	**C**	**11.**	**D**	**16.**	**B**
2.	**B**	**7.**	**D**	**12.**	**A**	**17.**	**A**
3.	**B**	**8.**	**E**	**13.**	**A**	**18.**	**D**
4.	**E**	**9.**	**D**	**14.**	**E**	**19.**	**C**
5.	**A**	**10.**	**D**	**15.**	**C**	**20.**	**C**

Section A: Multiple-choice solutions

<table>
<tr><td>MC1</td><td>$(x,y) \to (x+2,-y) \Rightarrow (1,2) \to (3,-2)$</td><td>Answer D</td></tr>
<tr><td>MC2</td><td>Solve $-2 < 2-x \le 2$ for x.

</td><td>Answer B</td></tr>
<tr><td>MC3</td><td>$f(x) = a\tan(nx)$ has period $T = \frac{\pi}{n}$
So $f(x) = -2\tan(\pi x)$ has period $T = \frac{\pi}{\pi} = 1$</td><td>Answer B</td></tr>
<tr><td>MC4</td><td>If $(x+a)$ is a factor of $P(x)$ then by Factor Theorem
$P(-a) = 0 \Rightarrow -5a^2(a-2) = 0$


$\therefore a = 2$ since $a \ne 0$.</td><td>Answer E</td></tr>
<tr><td>MC5</td><td>$y = \sin(\theta)$ is one-one for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
Solve $-\frac{\pi}{2} \le 2\left(x - \frac{\pi}{12}\right) \le \frac{\pi}{2}$ for $x \Rightarrow -\frac{\pi}{6} \le x \le \frac{\pi}{3}$.


$\therefore a = \frac{\pi}{6}$</td><td>Answer A</td></tr>
<tr><td>MC6</td><td>$\int_2^5 (3f(x)-2)dx = 3\int_2^5 f(x)dx - 2\int_2^5 dx = 3\times 4 - 2\times(5-2) = 6$</td><td>Answer C</td></tr>
</table>

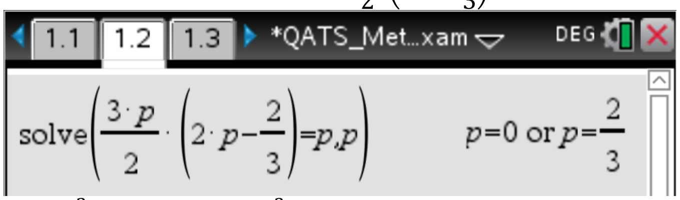
MC7

Using a 2-way table

	A	A'	
B	p	$p-\frac{2}{3}$	$2p-\frac{2}{3}$
B'	$\frac{p}{2}$		
	$\frac{3p}{2}$		1

For independence

$\Pr(A \cap B) = \Pr(A) \times \Pr(B) \Rightarrow \frac{3p}{2}\left(2p-\frac{2}{3}\right) = p$



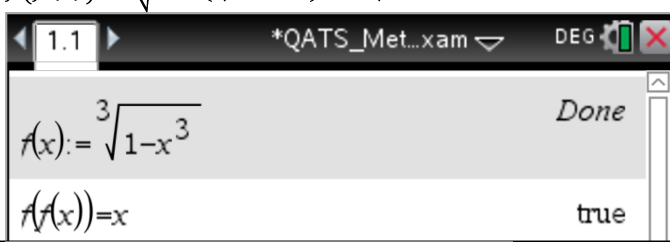
$\therefore p = \frac{2}{3}$ only since $p \geq \frac{2}{3}$.

Answer D

MC8

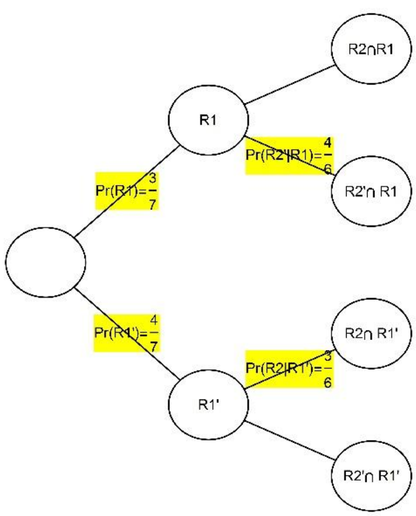
$f(x) = \sqrt[3]{1-x^3}$

$f(f(x)) = \sqrt[3]{1-\left(\sqrt[3]{1-x^3}\right)^3} = \sqrt[3]{x^3} = x$



Answer E

MC9



Answer D

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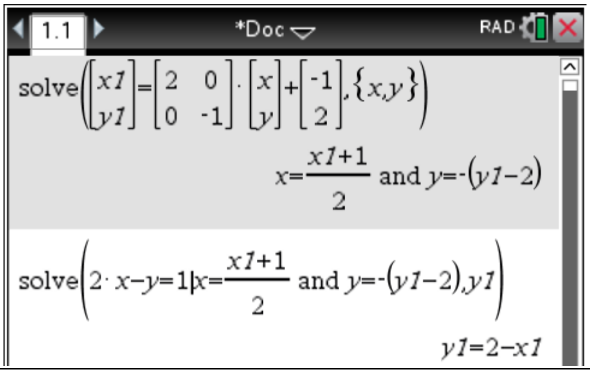
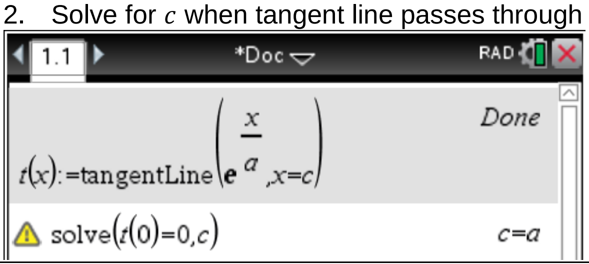
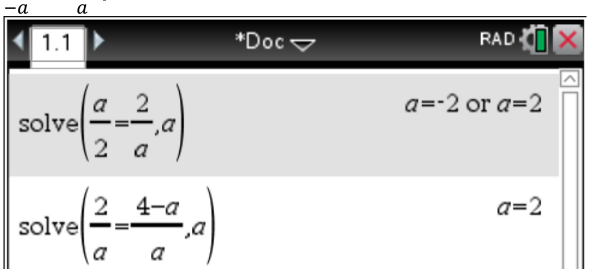
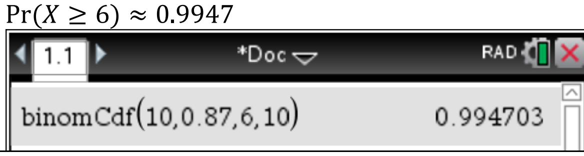
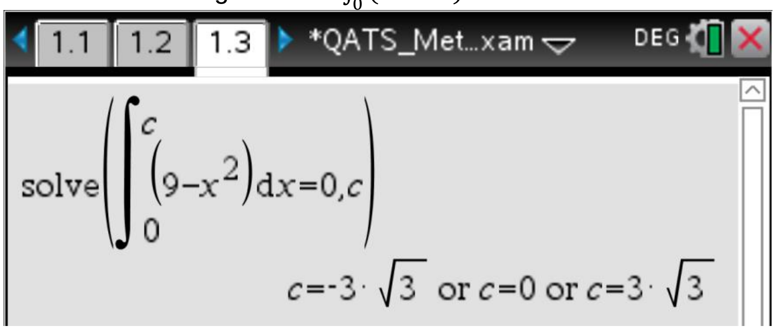
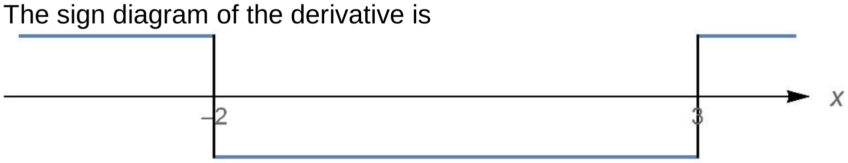
<table>
<tr><td></td><td>From tree diagram
$\Pr((R_1 \cap R_2') \cup (R_1' \cap R_2)) = \frac{3}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{3}{6} = 2 \times \frac{3}{7} \times \frac{4}{6}$</td><td></td></tr>
<tr><td>MC10</td><td>1. Find inverse transformation by solving the matrix equation for old x and old y in terms of new x and new y.
2. Then substitute into original equation $2x - y = 1$.
3. Finally, solve for new y in terms of new x to find the image of the original equation.
</td><td>Answer D</td></tr>
<tr><td>MC11</td><td>1. Find the equation of the tangent line at the general value $x = c$
2. Solve for c when tangent line passes through $(0,0)$.
</td><td>Answer D</td></tr>
<tr><td>MC12</td><td>For no solutions, the lines must be parallel, so they must have same slope.
Same slope $\Rightarrow \frac{a}{2} = \frac{-2}{-a}$.
For infinite solutions, system must be consistent with the same y –int⇒ $\frac{-2}{-a} = \frac{4-a}{a}$.

So, when $a = 2$ the system of equations is consistent, with all points on either line a solution.

$\therefore a = -2$ for an inconsistent system with no solutions.</td><td>Answer A</td></tr>
<tr><td>MC13</td><td>$X \sim Bi\left(n = 10, p = \frac{13}{100}\right)$
$\Pr(X \geq 6) \approx 0.9947$
</td><td>Answer A</td></tr>
</table>

MC14	If $1-\alpha = 0.997 = \Pr\left(-z_{1-\frac{\alpha}{2}} < Z < z_{1-\frac{\alpha}{2}}\right) \Rightarrow z_{1-\frac{\alpha}{2}} = 3$ Sub $\hat{P} = 0.52, z_{1-\alpha/2} = 3, n = 900$ into $(1-\alpha)\%\ \text{C.I.} \approx \left(\hat{P} - z_{1-\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}, \hat{P} + z_{1-\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}\right)$ Gives $99.7\%\ \text{C.I.} \approx \left(0.52 - 3\sqrt{\frac{0.52 \times 0.48}{900}}, 0.52 + 3\sqrt{\frac{0.52 \times 0.48}{900}}\right)$	**Answer E**
MC15	For stationary points $f'(x) = 3ax^2 + 2bx + c = 0$ If no stationary points, $\Delta \equiv (2b)^2 - 4(3a)c < 0$ $3ac > b^2$ $\therefore c < \frac{b^2}{3a}$ and $a < 0$	**Answer C**
MC16	When $x = c$ the signed area $\int_0^c (9 - x^2)\, dx = 0$  $c = 3\sqrt{3}$	**Answer B**
MC17	The derivative is a concave up parabola with zeros at $x = -2$ and $x = 3$. The sign diagram of the derivative is  By the first derivative test, $x = -2$ is a local maximum.	**Answer A**
MC18	$T \sim N(\mu = 90, \sigma)$ $\Pr(T \le 70) \approx \frac{110}{1000}$ $\Rightarrow \Pr\left(Z \le \frac{70-90}{\sigma}\right) \approx \frac{11}{100}$ $\Pr(Z \le -1.2265) \approx \frac{11}{100}$ Upon solving $\frac{70-90}{\sigma} = -1.2265$ for σ gives $\sigma \approx 16$	**Answer D**

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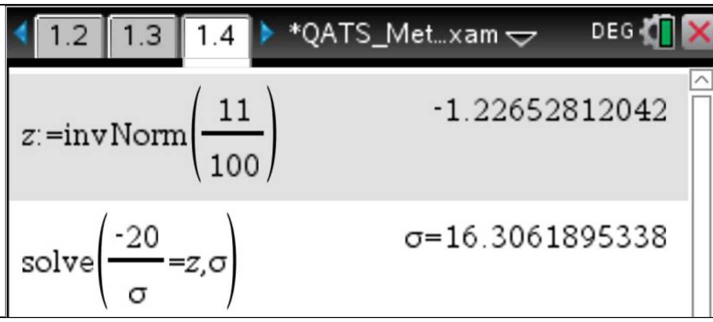
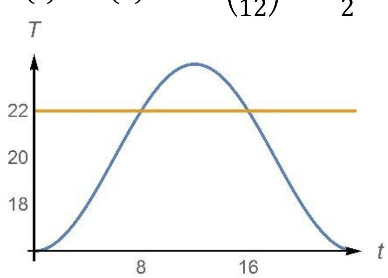
		
MC19	Mode in middle of interval $[2, c] \Rightarrow$ pdf symmetric about $x = \frac{c+2}{2}$ $\therefore mean = median = mode = \frac{c+2}{2}$ To find c, express total probability in terms of c. That is, $\frac{1}{2}(c-2) \times \frac{1}{2} = 1 \Rightarrow c = 6$. $\therefore E(X) = \frac{c+2}{2} = 4$	**Answer C**
MC20	$g(x) = \begin{cases} -x & x \geq 0 \\ x & x < 0 \end{cases}$ $\Rightarrow f(g(x)) = \begin{cases} f(-x) & x \geq 0 \\ f(x) & x < 0 \end{cases}$ $\therefore$ the left side of the graph is reflected about the $y -$ axis	**Answer C**

Section B: Extended Answer Solutions

Question 1 (7 marks)

(a)	$\text{Amplitude} = 4$ $Period = \frac{2\pi}{\left(\frac{\pi}{12}\right)} = 24$	(A1) (A1)
(b)	$\cos\left(\frac{\pi t}{12}\right) = -1 \Rightarrow T = 24 \text{ is max}$ $\cos\left(\frac{\pi t}{12}\right) = 1 \Rightarrow T = 16 \text{ is min}$	(A1) (A1)
(c)	$T(8) = 20 - 4\cos\left(\frac{\pi}{12} \times 8\right) = 22$	(A1)
(d)	$T(t) \geq T(8) \Rightarrow \cos\left(\frac{\pi t}{12}\right) \leq -\frac{1}{2}$  $\therefore T(t) \geq T(8)$ for $\frac{1}{3}$ of a period.	(M1) (A1)

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Question 2 (7 marks)

(a)i	$f\big(g(x)\big) = x$	(A1)
(a)ii	$g\big(f(x)\big) = x$	(A1)
(b)	area $OABC = 8 \times \big(0 - g(8)\big) = 8(2 + e^2)$	(A1)
(c)	$\int_0^8 g(x)\,dx = -4(3 + e^2)$	(A1)
(d)	area region 1 $= -\int_0^8 g(x)\,dx = 4(3 + e^2)$	(A1)
(e)	area region 2 $=$ area $OABC$ $-$ area region 1 $= 4(1 + e^2)$	(M1) (A1)

Question 3 (11 marks)

(a)	$x = -1 \; or \; x = \dfrac{1 - m^2}{1 + m^2}$	(A1)
(b)	Substitute $y = m(x + 1)$ in $x^2 + y^2 = 1$ gives $x^2 + m^2(x + 1)^2 = 1$. Substitute $x = \frac{1-m^2}{1+m^2}$ in $y = m(x + 1)$ gives $y = m\left(1 + \frac{1-m^2}{1+m^2}\right) = \frac{2m}{1+m^2}$	(M1) (M1) (M1)
(c)	slope $PB = \dfrac{0 - \frac{2m}{1+m^2}}{1 - \frac{1-m^2}{1+m^2}} = -\frac{1}{m}$ slope $AP \times$ slope $PB = -1 \Rightarrow \angle APB = \dfrac{\pi}{2}$	(M1) (M1)
(d)	area of $\Delta APB = \dfrac{1}{2} \times AB \times y - coordinate\ at\ P$ $= \dfrac{2m}{1 + m^2}$	(M1) (A1)
(e)	$\dfrac{d}{dm}\left(\dfrac{2m}{1 + m^2}\right) = \dfrac{2(1 - m^2)}{(1 + m^2)^2}$	(A1)
(f)	$\dfrac{d}{dm}\left(\dfrac{2m}{1 + m^2}\right) = 0 \Rightarrow m = 1$ since $m > 0$ $\therefore$ max area of $\Delta APB = 1$	(A1) (A1)

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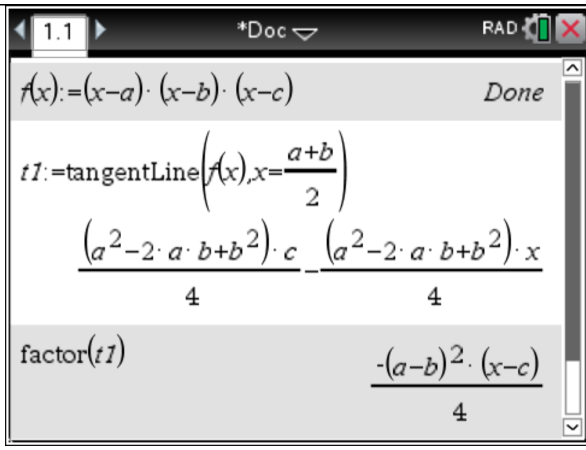
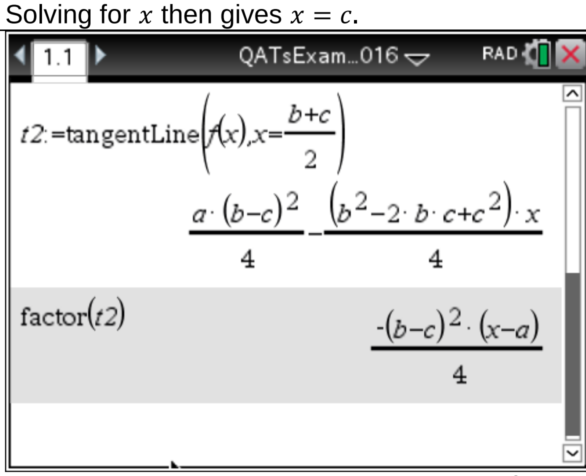
Question 4 (17 marks)

(a)	 Tangent line at $x=\frac{a+b}{2}$ has equation $y=\frac{(a^2-2ab+b^2)\,c}{4}-\frac{a^2-2ab+b^2}{4}x$ Factoring then gives $y=\frac{(a-b)^2(c-x)}{2}$.	(A1) (or equivalent representation) (A1) fully factored
(b)	At the $x-int, y=0 \Rightarrow \frac{(a-b)^2(c-x)}{2}=0$ Solving for x then gives $x=c$.	(A1)
(c)	 Tangent line at $x=\frac{b+c}{2}$ has equation $y=\frac{(b^2-2bc+c^2)\,a}{4}-\frac{(b^2-2bc+c^2)}{4}x$. Factoring then gives $y=\frac{(b-c)^2(a-x)}{2}$.	(A1) (or equivalent representation) (A1) fully factored
(d)	At the $x-int, y=0 \Rightarrow \frac{(b-c)^2(a-x)}{2}=0$ Solving for x then gives $x=a$.	(A1)
(e)	If tangent at $x=\frac{a+b}{2}$ is ‖ to tangent at $x=\frac{b+c}{2}$ then they have same slope $\Rightarrow -\frac{(b-a)^2}{2}=-\frac{(b-c)^2}{2}$	(M1) or equivalent equation

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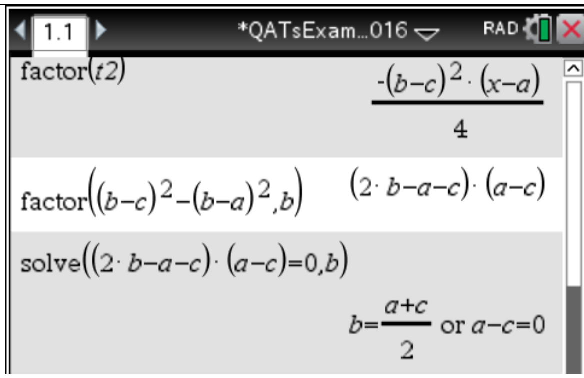
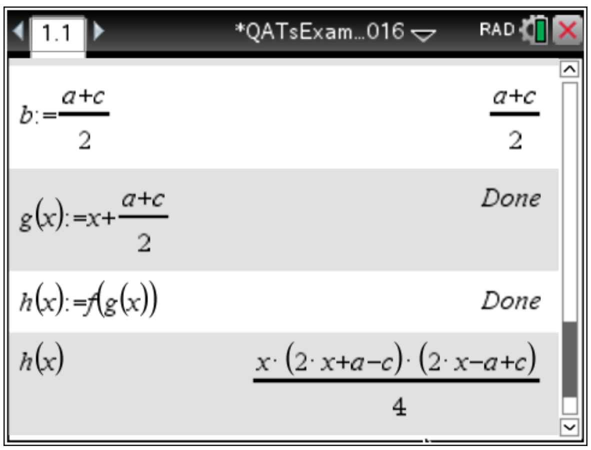
	 Cancelling and then factorising by the difference of perfect squares identity $(2b-a-c)(a-c)=0$ By the Null Factor Law $b=\frac{a+c}{2}$ since $a-c\neq 0$.	(M1) (M1)
(f)	 $h(x)=\frac{x(2x+a-c)(2x-a+c)}{4}$ $h(x)=x(x-\frac{a-c}{2})(x+\frac{a-c}{2})$ (or $h(x)=x(x+\frac{c-a}{2})(x-\frac{c-a}{2})$)	(A1) or any equivalent equation (A1) accept either
(g)	$h(-x) = -x(-x-\frac{a-c}{2})(-x+\frac{a-c}{2})$ $= -x(x+\frac{a-c}{2})(x-\frac{a-c}{2})$ $= -h(x)$	(M1) substitute for $-x$ (M1) demonstrate $h(x)$ is odd
(h)	A necessary and sufficient condition for the graph of the graph $y=h(x)$ to have a point of inflection at $x=v$ is $h''(v)=0$.	(M1) attempt solve $h''(v)=0$

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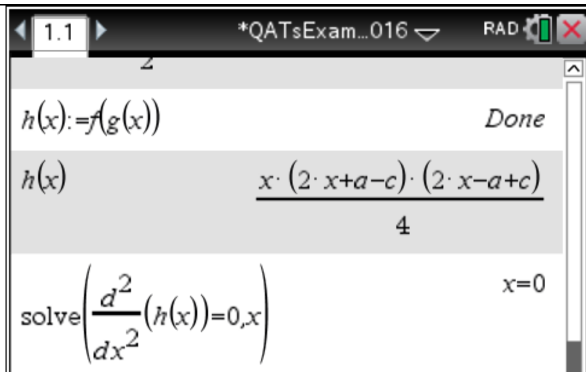
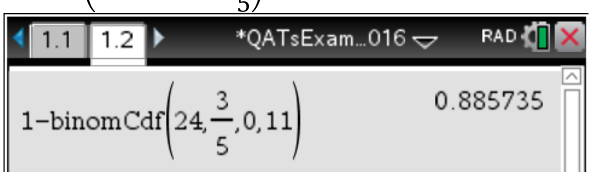
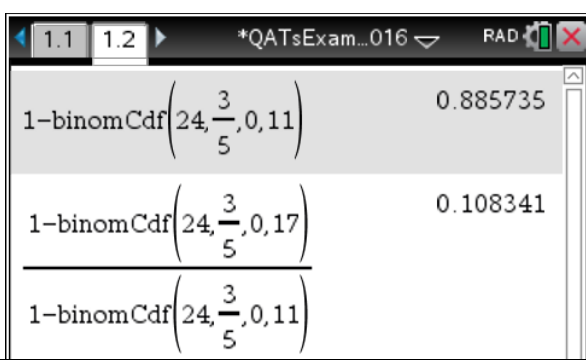
	 $\text{P.O.I.} = (0,0)$	(A1)
(i)	The graph $y = f(x)$ is the image of the graph $y = h(x)$ after the transformation $(x, y) \to \left(x + \frac{a+c}{2}, y\right)$. $\text{P.O.I.} = \left(\frac{a+c}{2}, 0\right)$	(M1) uses transformation to motivate answer (A1)

Question 5 (18 marks)

a(i)	$X \sim \text{Bi}\left(n = 24, p = \frac{3}{5}\right)$  $\Pr(X \geq 12) = 1 - \Pr(X \leq 11) = 0.8857$	(M1) (A1)
a(ii)	$\Pr(X \geq 18 \mid X \geq 12) = \frac{\Pr(X \geq 18 \cap X \geq 12)}{\Pr(X \geq 12)}$ $= \frac{\Pr(X \geq 18)}{\Pr(X \geq 12)}$ ≈ 0.108 	(M1) (M1) (A1)

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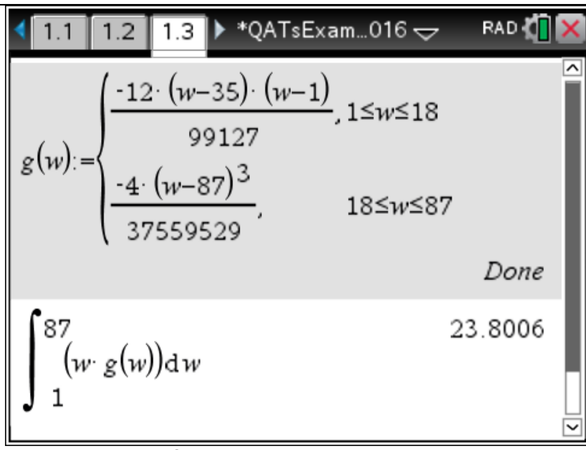
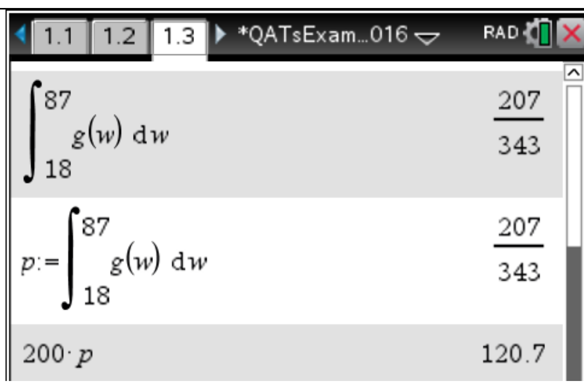
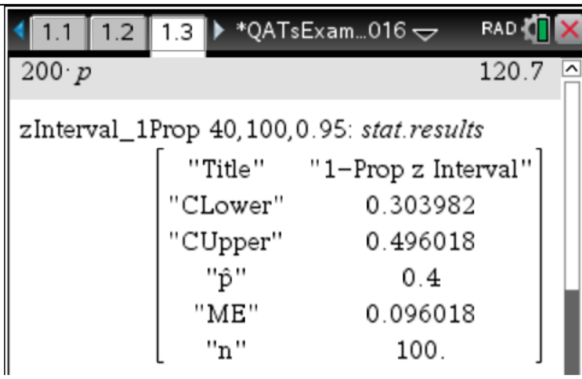
a(iii)	mean of $\hat{P} = p = \frac{3}{5}$ variance of $\hat{P} = \frac{p(1-p)}{n}$ $= \frac{1}{100}$	(A1) (M1) (A1)
a(iv)	$\Pr\left(\frac{3}{5} - 2 \times \frac{1}{10} \le \frac{X}{24} \le \frac{3}{5} + 2 \times \frac{1}{10}\right)$ $= \Pr(10 \le X \le 19)$ ≈ 0.965	(M1) (M1) (A1)
a(v)	$\Pr\left(\hat{P} \ge \frac{3}{4} \mid \hat{P} \ge \frac{3}{5}\right) = \frac{\Pr(X \ge 18)}{\Pr(X \ge 15)}$ ≈ 0.196	(M1) (A1)
b(i)	 $E(W) = \int_1^{87} wg(w)dw$ ≈ 23.8006	(M1) (A1)
b(ii)	 $\Pr(W > 18) = \int_{18}^{87} g(w)dw = \frac{207}{343}$ $Expected\ Value = 200 \times \frac{207}{343} \approx 121$	(A1) (A1)

<table>
<tr>
<td>c</td>
<td>


$95\%\ \text{C.I.} = (0.304, 0.496)$
</td>
<td>(A1)</td>
</tr>
</table>

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(x)$	variance	$\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\, dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{P} - z\sqrt{\dfrac{\hat{P}(1-\hat{P})}{n}}, \hat{P} + z\sqrt{\dfrac{\hat{P}(1-\hat{P})}{n}}\right)$

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