MATHEMATICAL METHODS

Written examination 1

TSSM™
Creating VCE Success

2016 Trial Examination

SOLUTIONS

Question 1

a. $y = \frac{1}{3}(2x-1)^6 \rightarrow \frac{dy}{dx} = \frac{6}{3}(2x-1)^5 \times 2 = 4(2x-1)^5$

1 mark

b.

i. $f'(x) = \frac{x^2\cos(x) - 2x\sin(x)}{x^4} = \frac{xcos(x) - 2\sin(x)}{x^3}$

2 marks

ii. $f'(\pi) = \frac{\pi\cos(\pi) - 2\sin(\pi)}{\pi^3} = -\frac{1}{\pi^2}$

1 mark

Question 2

a. $f'(x) = \frac{2}{x} - x^2$

$f(x) = 2\ln(x) - \frac{x^3}{3} + c$

$-2 = -\frac{1}{3} + c \rightarrow c = -\frac{5}{3}$

$f(x) = 2\ln(x) - \frac{x^3}{3} - \frac{5}{3}$

3 marks

b. $f(e^2) = \frac{7}{3} - \frac{e^6}{3}$

1 mark

Question 3

a. $0 = \frac{1}{2}(2-1)(k+2)^2$
$k+2=0$
$k=-2$

1 mark

b. $f'(x) = 0 \rightarrow \frac{1}{2}(3x^2 - 10x + 8) = 0$
$x = 2, \quad \frac{4}{3}$

2 marks

c. $Range: [-2, \infty)$

1 mark

Question 4

a. $log_2\left(\frac{x}{\sqrt{x}-1}\right) = 2$
$\frac{x}{\sqrt{x}-1} = 4$
$x = 4\sqrt{x} - 4$
$Let \ a = \sqrt{x}$
$a^2 - 4a + 4 = 0 \rightarrow a = 2$
$\sqrt{x} = 2 \rightarrow x = 4$

3 marks

b. $e^{-x}(e^{-x} - 2) = 0$
$e^{-x} = 0, \quad e^{-x} = 2$
$x = -\ln(2)$

2 marks

Question 5

a. $\Pr(X > 151.9) = 0.025$

1 mark

b. $\Pr(Z > 1) = \Pr(X > 145.7) = \Pr(X < 133.3)$
$b = 133.3$

2 marks

Question 6

a. $\Pr(\mathrm{P} \cap \mathrm{Q}) = \Pr(\mathrm{P}) \times \Pr(\mathrm{Q})$
$0.1 = 0.3 \times \Pr(\mathrm{Q}) \rightarrow \Pr(\mathrm{Q}) = \frac{1}{3}$

1 mark

b. $\Pr(\mathrm{P}|\mathrm{Q}') = \frac{\Pr(P \cap Q')}{\Pr(Q')} = \frac{\Pr(P) - \Pr(P \cap Q)}{1 - \Pr(Q)} = \frac{0.3 - 0.1}{1 - 0.2} = \frac{0.2}{0.8} = \frac{1}{4}$

2 marks

c. Because $\Pr(\mathrm{P} \cap \mathrm{Q}) = 0.1 \neq 0$

1 mark

Question 7

$\frac{dy}{dx} = -\frac{a}{x^2} \rightarrow m_t = -\frac{a}{16}$
$Point\ is\ (4, \frac{a}{4} - 1)$
$y - \left(\frac{a}{4} - 1\right) = -\frac{a}{16}(x - 4)$
$-2 - \left(\frac{a}{4} - 1\right) = -\frac{a}{16}(3 - 4)$
$-2 - \frac{a}{4} + 1 = \frac{a}{16} \rightarrow a = -\frac{16}{5}$

3 marks

Question 8

$x' = 2x - 1,\ \ y' = -y + 2$
$x = \frac{x'+1}{2},\ \ y = -y' + 2$
$-y' + 2 = -\sqrt{\frac{x'+1}{2} - 1}$
$y' = \sqrt{\frac{x'-1}{2}} + 2$
$y = \sqrt{\frac{x-1}{2}} + 2$

2 marks

Question 9

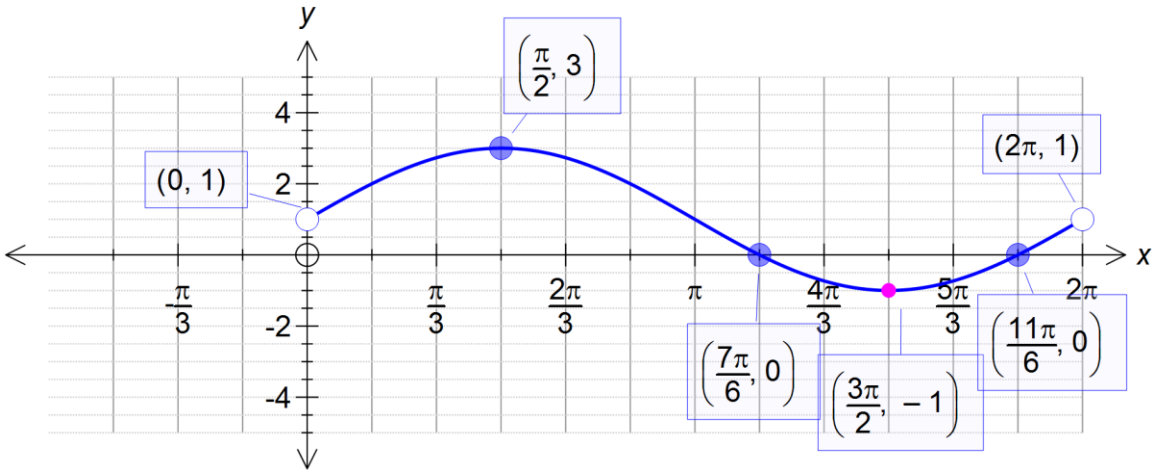
a. $2\sin(x) + 1 = 0$

$\sin(x) = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

2 marks

b.



1 mark for end points, 1 mark for turning points, 1 mark for axes intercepts

Question 10

a. 0.6

1 mark

b. $SD(\hat{p}) = \sqrt{\frac{0.6 \times 0.4}{96}} = \frac{1}{20} = 0.05$

2 marks

c. $\Pr(\hat{p} \geq 0.71) = \Pr\left(Z \geq \frac{0.71 - 0.6}{0.05}\right) = \Pr\left(Z \geq \frac{11}{5}\right) = \Pr(Z \geq 2.2) = 0.0139$

3 marks