# MATHEMATICAL METHODS

## Units 3 & 4 – Written examination 1

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***(*TSSM's *2011 trial exam updated for the current study design)***

***SOLUTIONS***

**Question 1**

**a.** $f'(x) = \frac{-4(2-x)^3}{(2-x)^4} = \frac{4}{x-2}$

M1+A1
2 marks

**b.** $x = \log_e(2-y)^4$
$x = 4\log_e(2-y)$
$\frac{x}{4} = \log_e(2-y)$
$e^{\frac{x}{4}} = 2-y$
$y = 2 - e^{\frac{x}{4}}$
$f^{-1}(0) = 1$

M2+A1
3 marks

**c.** $g'(x) = 2xe^{\sin(x)} + x^2\cos(x)e^{\sin(x)}$
$g'(x) = 2 \times \frac{\pi}{2} \times e^{\sin\left(\frac{\pi}{2}\right)} + \left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) e^{\sin\left(\frac{\pi}{2}\right)} = \pi e$

M1+A1
2 marks

**Question 2**

**a.** $\int \sqrt{x} - 2\sin\left(\frac{\pi x}{4}\right) dx = \frac{2x^{\frac{3}{2}}}{3} + \frac{8}{\pi}\cos\left(\frac{\pi x}{4}\right) + c$ where $c$ is a real constant (but not necessary)

M1+A1

2 marks

**b.** $\left[\frac{(2x-1)^4}{2\times 4}\right]_0^2 = e^{\log_e m}$

$\frac{81}{8} - \frac{1}{8} = m$

$m = 10$

M1+A1

2 marks

**Question 3**

**a.** Amplitude = 3

Period $= \frac{2\pi}{n} = 2\pi \times 5 = 10\pi$

A2

2 marks

**b.** $\frac{-\cos(2x)}{\cos(2x)} = \frac{\sin(2x)}{\cos(2x)}$

$\tan(2x) = -1,\ -\pi \le x \le \pi$

Reference $\angle$: $\frac{\pi}{4}$

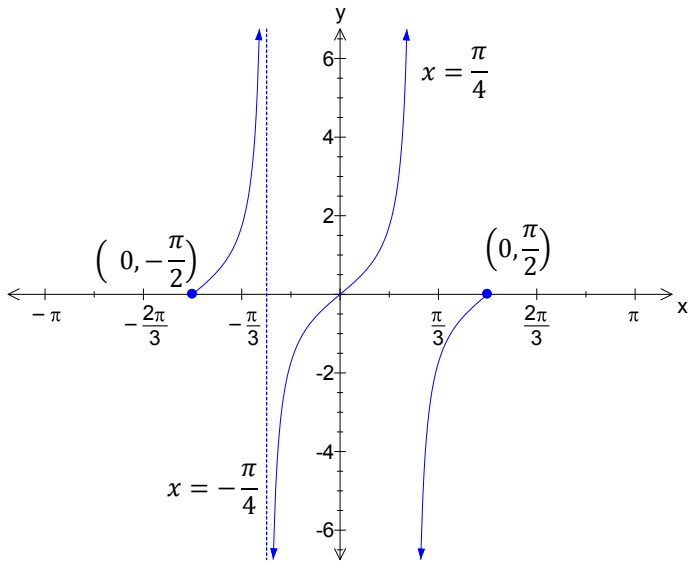
$2x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ or $2x = \frac{3\pi}{4} - \pi = -\frac{\pi}{4}$

$x = \frac{3\pi}{8}$ $\qquad$ $x = \frac{-\pi}{8}$

M2+A1

3 marks

**c.**




A2

2 marks

**Question 4**

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4x + 3 \\ 2y - 1 \end{bmatrix}$

$x' = -4x + 3$ and $y' = 2y - 1$

$x = \frac{3-x'}{4}$ and $y = \frac{y'+1}{2}$

$\frac{y'+1}{2} = 2\log_e(3 - x') + 1$

$y' = 4\log_e(3 - x') + 1$

$a = 4, b = -1, c = 3$ and $d = 1$

M2+A2
4 marks

**Question 5**

Let $y = 0$ for $x$ intercepts $\frac{1}{(x-2)^2} - 1 = 0$

$x = 1$ or $x = 3$

Then find the derivative $h'(x) = \frac{-2}{(x-2)^3}$

$h'(1) = 2$ , $h'(3) = -2$

Using points (1,0) and (3,0) respectively:

We get equations $y = 2x - 2$ and $y = -2x + 6$

M2+A1
3 marks

**Question 6**

**a.** $\hat{p} = 0.18$

A1
1 mark

**b.** $M = \sqrt{\frac{0.18 \times 0.82}{n}}$

A1
1 mark

**c.** If $n$ was halved, the value of $M$ will increase by a factor of $\sqrt{2}$

A2
2 marks

**Question 7**

**a.** $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} m\cos(x)dx = 1$

$m[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 1$

$m(-1-1) = 1$

$m = \frac{-1}{2}$

M2+A1
3 marks

**b.** $Pr(X > \pi) = \int_{\pi}^{\frac{3\pi}{2}} -0.5\cos(x)dx$

$-0.5[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -0.5(-1) = 0.5$

M1+A1
2 marks

**Question 8**

**a.** $Pr(X < 7) = 0.025$

$z = \frac{x-\mu}{\sigma} = \frac{7-11}{2} = -2 \Longrightarrow$ 2 standard deviations below the mean

$Pr(Z < -2) = Pr(X < 7) = 0.025$

A1
1 mark

**b.** $Pr(X < 8) = Pr(X > 14) = Pr\left(Z > \frac{14-11}{2}\right) = Pr\left(Z > \frac{3}{2}\right),\ m = \frac{3}{2}$

M1+A1
2 marks

**Question 9**

**a.**
$$\begin{array}{r} -2 \\ x+2\overline{)\,-2x+3} \\ \underline{-2x-4} \\ 7 \end{array}$$

$\frac{3-2x}{x+2} = \frac{7}{x+2} - 2$

M1+A1
2 marks

**b.** $\int \frac{3-2x}{x+2}dx = \int \frac{7}{x+2} - 2\ dx = 7\log_e|x+2| - 2x + c$

A1
1 mark