TSSM™
Creating VCE Success

2016 Trial Examination

STUDENT NUMBER

Letter

Figures

Words

MATHEMATICAL METHODS
Written examination 1

Reading time: 15 minutes
Writing time: 60 minutes

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape, notes of any kind, or a calculator of any kind.

Materials supplied
- Question and answer book of 12 pages.

Instructions
- Print your **name** in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{1}{3}(2x - 1)^6$.

Find $\frac{dy}{dx}$.

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<div align="right">1 mark</div>

b. Let $f(x) = \frac{\sin(x)}{x^2}$

  i. Find $f'(x)$

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_____

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<div align="right">2 marks</div>

  ii. Find $f'(\pi)$

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<div align="right">1 mark</div>

<div align="right">**TURN OVER**</div>

Question 2 (4 marks)

a. Let $f'(x) = \frac{2x - x^4}{x^2}$, for $x \neq 0$

Given that $f(1) = -2$, $find$ $f(x)$.

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3 marks

b. Hence, find $f(e^2)$

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1 mark

Question 3 (4 marks)

Consider the function $f:[0, \infty) \to R, \ f(x) = \frac{1}{2}(x-1)(k+x)^2, \ where \ k \in R$
The graph of $y = f(x)$ passes through the point (2, 0).

a. Show that $k = -2$.

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                                                            1 mark

The rule of the function $f(x)$ can be written as $f(x) = \frac{1}{2}(x^3 - 5x^2 + 8x - 4)$

b. Find the x-coordinates of the stationary points of the function.

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                                                            2 marks

c. Find the range of the function.

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                                                            1 mark

TURN OVER

Question 4 (5 marks)

Solve the following equations for x.

a. $log_2(x) - log_2(\sqrt{x} - 1) = 2$

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3 marks

b. $e^{-2x} - 2e^{-x} = 0$

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2 marks

Question 5 (3 marks)

Let the random variable X be normally distributed with mean 139.5 and standard deviation 6.2
Let Z be the standard normal random variable, such that $Z \sim N(0, 1)$.

a. Find $\Pr(X > 151.9)$.

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<div align="right">1 mark</div>

b. Find b such that $\Pr(X < b) = \Pr(Z > 1)$

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<div align="right">2 marks</div>

Question 6 (4 marks)

For events P and Q, $\Pr(P \cap Q) = 0.1$, $\Pr(P) = 0.3$

a. Find $\Pr(Q)$ if P and Q are independent events.

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<div align="right">1 mark</div>

<div align="right">**TURN OVER**</div>

b. Find $\Pr(P|Q')$, if $\Pr(Q) = 0.2$

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2 marks

c. Explain why P and Q cannot be mutually exclusive events.

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1 mark

Question 7 (3 marks)

The tangent to the curve $y = \frac{a}{x} - 1$ at the point $x = 4$ passes through the point (3, -2).
Find the value of a.

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TURN OVER

Question 8 (2 marks)

A transformation $T: R^2 \rightarrow R^2$, *is defined by* $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Find the image of the function with the rule $f(x) = -\sqrt{x-1}$ under the above transformation.

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2 marks

Question 9 (5 marks)

Consider the function $f: (0, 2\pi) \rightarrow R, \ f(x) = 2\sin(x) + 1$

a. Solve the equation $f(x) = 0$.

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<div align="right">2 marks</div>
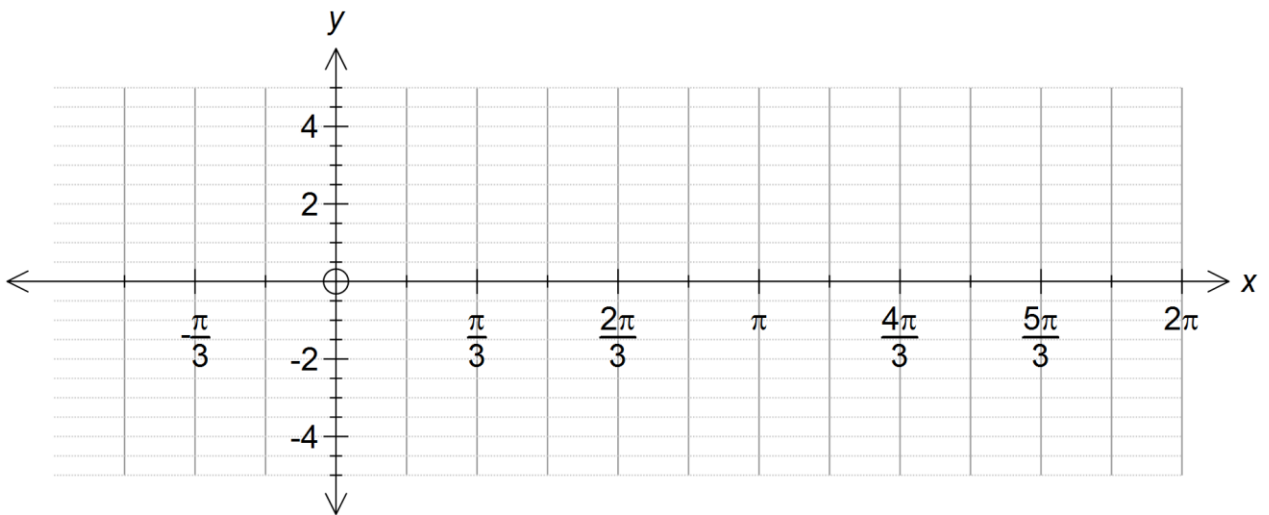
b. Sketch the graph of $y = f(x)$ on the axes below, labelling end-points, axes intercepts and turning points.

<div align="right">3 marks</div>

<div align="right">**TURN OVER**</div>

Question 10 (6 marks)

A new packet of chips is being tested in the market. It is estimated that 60% of consumers will like the new packet of chips. A sample of 96 people tasted the new packet of chips.

a. Find the mean of the sample proportion.

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1 mark

b. Find the standard deviation of the sample proportion.

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2 marks

c. Use $\Pr(Z \geq 2.2) = 0.0139$ to find the probability that at least 71% of consumers will indicate they like the new packet of chips.

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3 marks

END OF QUESTION AND ANSWER BOOK