MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

*(*TSSM's *2013 trial exam updated for the current study design)*

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: D

Explanation:

Domain of sum of functions is given by $dom\ of\ f \cap dom\ of g$

Question 2

Answer: B

Explanation:
Swap *x* and *y* and use CAS to make *y* the subject
Note that range of f^{-1} = domain of f
And domain of f^{-1} =range of f

Question 3

Answer: E

Explanation:

$\frac{-2k}{x-1} = 3x$
$3x^2 - 3x + 2k = 0$
$\Delta = 0$ $(as\ the\ line\ is\ a\ tangent, it\ will\ cut\ the\ graph\ at\ only\ one\ point)$
$k = \frac{9}{24}$

Question 4

Answer: B

Explanation:

$\begin{vmatrix} k & 2 \\ 1 & k-1 \end{vmatrix} = 0 \ \ gives\ k = -1, 2$
$k \neq 2,\ \ so\ k = -1$

Question 5

Answer: A

Explanation:

Eliminate incorrect solutions from formula (binomial)

Question 6

Answer: D

Explanation:

$9 + 3x = 0 \ \ gives\ x = -3 \ \ as\ the\ horizontal\ asymptote.$

Question 7

Answer: D

Explanation:

$Use\ CAS\ to\ define\ the\ functions\ and\ find\ g(f(x))$
Note that $g(f(x))$ is defined only when $Range\ of\ f\ is\ a\ subest\ of\ domain\ of\ g$

Question 8

Answer: C

Explanation:

$Solve(x = 2 - 3log_e(1-y), y)\ on\ CAS\ \ to\ get\ f^{-1}(x) = 1 - e^{-\frac{x}{3}+\frac{2}{3}}$

Question 9

Answer: A

Explanation:
Use chain rule.

Question 10

Answer: C

Explanation:

Only statement ii is correct

Question 11

Answer: C

Explanation:

$Substitute\ (0,6)\ in\ y = alog_e(x+4)\ to\ get\ a = \frac{6}{log_e(4)}$

Question 12

Answer: A

Explanation:
$\int 2sin\left(\frac{5x}{2}\right)dx$ on CAS

Question 13

Answer: D

Explanation:

$Period = \pi$ and it is reflected in the *x*-axis.

Question 14

Answer: D

Explanation:

$\Pr(X < 2.3) = \Pr(Z < -2)$ *which is the same as* $\Pr(Z > 2)$

Question 15

Answer: E

Explanation:
Use CAS to find the value of *a*, $\int_0^a 3x^2 dx = 0.125$

Question 16

Answer: E

Explanation:

$np = 20,\ np(1-p) = 16.$ *Solve the two equations to get* $p = 0.2$ *and* $n = 100$

Question 17

Answer: B

Explanation:

Solve on CAS: $solve\left(\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}, x\right) | 0 \le x \le 4\pi$ and then add the two solutions.

Question 18

Answer: A

Explanation:

$Average\ ROC = \frac{1000e - 1000}{10 - 0}$ on CAS.

Question 19

Answer: D

Explanation:

$f'(2) = 0 \; implies \; a = 6 \;\; and \; f(2) = 5 \; implies \; b = 21$

Question 20

Answer: A

Explanation:

$2\pi r(r + h) = 726\pi \;\; implies \; h = \frac{363 - r^2}{r}$
$V = 363\pi r - \pi r^3, \; V' = 0 \;\; implies \;\; r = 11$
$Max \; volume = V(11) = 2662\pi$

Question 21

Answer: C

Explanation:

$E(X) = (0.3 + 0.4 + 1.6 + 0.8) = 3.1$

Question 22

Answer: B

Explanation:

$Period = \frac{2\pi}{\left(\frac{\pi}{15}\right)} = 30 \; seconds$

SECTION 2: Analysis Questions

Question 1

a. $f'(x) = 3ax^2 + 2bx + c$

$f'(2) = 0$ *implies* $12a + 4b + c = 0$

$f(2) = -1$ *implies* $8a + 4b + 2c - 5 = -1$

Solve the above two equations to get $a = \frac{c-4}{4}$ *and* $b = 3 - c$

M1+A2

3 marks

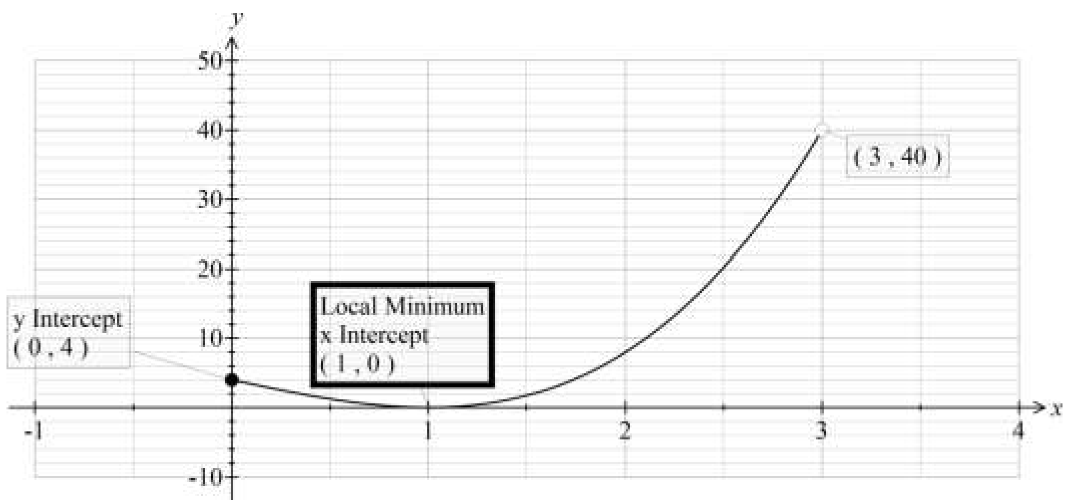
b. $f(-1) = 9$ *gives* $-a + b - c - 5 = 9$

Substitute *a* and *b* from part **a.** in the above equation to get $c = -\frac{40}{9}$ $a = \frac{-19}{9}, b = \frac{67}{9}$

M1+A1

2 marks

c.



1 mark for y-intercept, 1 mark for turning point, 1 mark for showing the range

3 marks

d. On CAS: $\frac{d}{dx}(g(x)) = 6x^2 - 6$

A1

1 mark

e. The point is (2, 8)

$m_T = g'(2) = 18$
$m_N = -\frac{1}{18}$
Equation of the normal is:
$y - 8 = -\frac{1}{18}(x - 2)\ or\ \ y = -\frac{1}{18}x + \frac{73}{9}$

M2+A1
3 marks

Question 2

a. $\binom{4}{1}(0.6)^1(0.4)^3$
$= 0.1536$

A1
1 mark

b. $1 - \binom{4}{4}(0.4)^4$
$= 0.9744$

M1+A1
2 marks

c. $\Pr(X < 5) = 0.8$
$\Pr(Z < 0.8416) = 0.8$
$\frac{5-\bar{x}}{1.1} = 0.8416$
$\bar{x} = 4.07$

M+A1
2 marks

d. $Solve\ \ k\int_0^4 t(4-t)dt = 1\ on\ CAS\ to\ get\ k = \frac{3}{32}$

M2
2 marks

e. $Pr(X \geq 2) = \frac{3}{32}\int_2^4 t(4-t)dt = 0.5 = \frac{1}{2}$

M1+A1
2 marks

f. Let $X \sim Bi(4, \frac{1}{2})$

$Pr(X \leq 3) = 0.9375$ (using $binomcdf\left(4, \frac{1}{2}, 0, 3\right)$)

M1+A1

2 marks

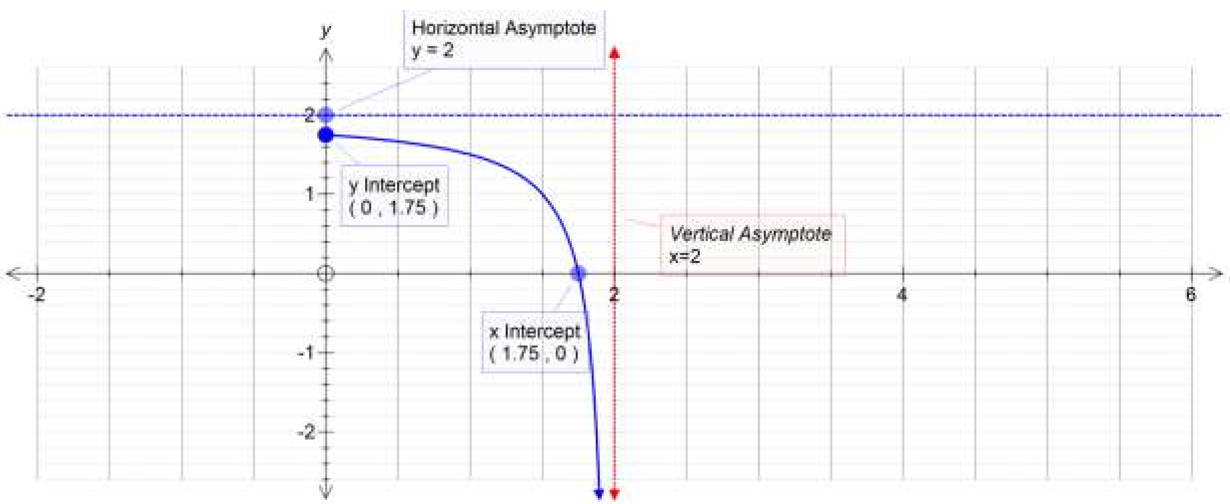
g. $Pr(X < n) = 0.15.$ *Solve* $\frac{3}{32}\int_0^n t(4-t)dt = 0.15$ *to get* $n = 0.9776hr = 59\ mins$

M1+A1

2 marks

Question 3

a.



1 mark for asymptotes, 1 mark for intercepts, 1 mark for drawing in the required domain

3 marks

b. $a = \frac{7}{4}$

A1

1 mark

c. $f'(x) = -\frac{1}{2(x-2)^2}$

Range is $(-\infty, -\frac{1}{8})$

A1+A1

2 marks

d. $\int_0^b \left(\frac{1}{2x-4}+2\right)dx = \frac{1}{2}log_e\left(\frac{|b-2|}{2}\right)+2b$

M1+A1
2 marks

e. $Find\ the\ area\ using\ \int_c^d f(y)dy$

$\int_{-3}^{0} \frac{4y-7}{2(y-2)}dy = -\frac{1}{2}log_e\left(\frac{5}{2}\right)+6$

M2+A1
3 marks

f. $0 = log_e(3-a)\ which\ gives\ a = 2$

A1
1 mark

g. $A\left(1,\frac{3}{2}\right)\ and\ B(4, log_e 2)$

$Distance\ between\ A\ and\ B = \sqrt{(4-1)^2+(ln2-\frac{3}{2})^2} = 3.1066\ units$

M2+A1
3 marks

Question 4

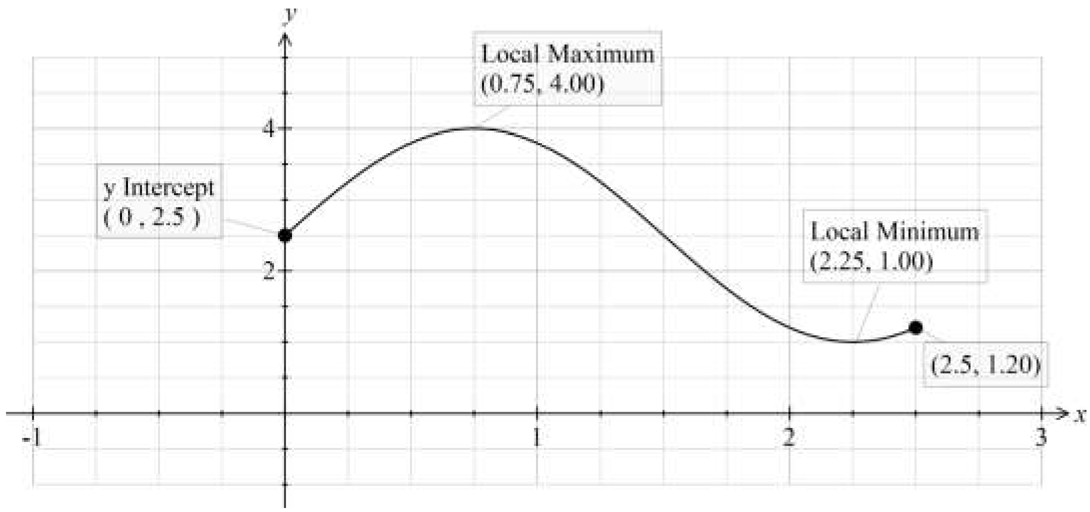
a. $B - A = 1\ \ and\ B + A = 4$

$Solve\ for\ A\ and\ B\ to\ get\ A = \frac{3}{2}\ \ and\ B = \frac{5}{2}$

$h(0.75) = 4\ \ gives\ n = \frac{2}{3}$

A3
3 marks

b.



A3

3 marks

c. Solve $h'(x) = \pi cos\left(\frac{2\pi}{3}x\right)$

Greatest value of $h'(x) = \pi$

$Solve\ \pi = \pi cos\left(\frac{2\pi}{3}x\right)\ \ to\ get\ x = 0$

M2+A1

3 marks

Question 5

a. $27000 = 2(x^2 + 2xh)$

$h = \frac{13500 - x^2}{2x}$

M1+A1

2 marks

b. $x > 0,\ 13500 - x^2 > 0\ \ which\ gives\ x < 30\sqrt{15}$

$0 < \ x < 30\sqrt{15}$

M1+A1

2 marks

c. $V = x^2\left(\frac{13500-x^2}{2x}\right)$

$V = 6750x - \frac{x^3}{2}$

$V'(x) = 6750 - \frac{3x^2}{2},\ V'(x) = 0\ \ implies\ x = 30\sqrt{5}$

$Substitute\ x = 30\sqrt{5}\ \ in\ part\ \boldsymbol{a}.to\ get\ h = 30\sqrt{5}\ c$

M2+A1
3 marks

d. Substituting for *x* in the volume equation in part **c.**

$V\left(30\sqrt{5}\right) = 135000\sqrt{5}\ cm^3$

A2
2 marks