2016 VCE Mathematical Methods 2 examination report

General comments

In 2016 students achieved a range of results, and the multiple-choice section of the paper was completed very well. Students handled the new content in the revised study confidently. Students found two of the 'show that' questions in Section B (Questions 4d. and 4fii.) quite difficult, but some students offered good solutions.

Some students gave approximate answers when exact answers were required, in particular in Questions 1 and 4 of Section B. Exact answers must always be given unless otherwise specified. Approximate answers are usually required in probability questions.

Advice to students

- Ensure that the appropriate mode is used with technology. Occasionally students might need to change the mode according to the question. Some students had their technology in mode degrees instead of radians, and this affected their marks for Question 1.
- Check that answers make sense. In Question 3 the mean and the median values should have been similar. Some students wrote down the correct formula but did not delete the *x* on their technology from the previous question, solving $\int_0^m (x \times f(x))dx = \frac{1}{2}$, getting $m = 75.58$.
- Define functions at the start of each question in Section B and include domain constraints. General solutions were often seen in Question 1, and solutions outside the domain were seen in Question 3hii. and Question 4.
- Knowing how to get the equation of the tangent line using technology saves time and avoids errors.
- Correct mathematical notation should be used in written responses. Calculator syntax appeared in the probability section, Question 3. If an equation is required, as in Question 1, it must contain an equal-to sign; for example, if $y = -x + 2\pi$ is the answer, $-x + 2\pi$ is unacceptable.

Students should re-read questions to make sure they answer all parts of a question; for example, Question 2bii. and Question 4bi. required two pieces of information and Question 4c. required three pieces of information. In Question 3, many students did not know convert hours to minutes or vice versa.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding, resulting in a total more or less than 100 per cent.

Section A – Multiple-choice questions

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	92	1	2	4	1	0	
2	3	90	2	4	1	0	
3	4	3	77	10	6	0	
4	2	3	8	85	2	0	
5	10	2	2	75	12	0	
6	67	8	9	8	7	1	
7	2	9	74	6	8	1	
8	8	77	4	6	4	0	
9	11	16	9	41	21	1	$\frac{d\left(xe^{kx}\right)}{dx} = (kx+1)e^{kx}$ $\int\left((kx+1)e^{kx}\right)dx = xe^{kx} + c_1$ $\int\left(kxe^{kx}\right)dx + \int\left(e^{kx}\right)dx = xe^{kx} + c_1$ $\int\left(kxe^{kx}\right)dx = xe^{kx} - \int\left(e^{kx}\right)dx + c_1$ $\int\left(xe^{kx}\right)dx = \frac{1}{k}\left(xe^{kx} - \int\left(e^{kx}\right)dx\right) + c$
10	23	6	10	52	8	1	
11	7	14	20	47	12	1	$\frac{dy}{dx} = 2f(x) - f(y) = (y-x)f(xy)$ $f(x) = \frac{1}{x}$ $\text{LHS} = \frac{1}{x} - \frac{1}{y}$ $\text{RHS} = (y-x)\times\frac{1}{xy} = \frac{1}{x} - \frac{1}{y} = \text{LHS}$

Question	% A	% B	% C	% D	% E	% No Answer	Comments
12	5	19	3	52	22	0	
13	3	15	6	7	69	0	
14	7	21	20	14	37	1	Area of the rectangle = $x(4-x^2)$ Let $A(x)=x(4-x^2)$ Solve $A'(x)=-3x^2+4=0,\ x=\frac{2\sqrt{3}}{3}$ $A\left(\frac{2\sqrt{3}}{3}\right)=\frac{16\sqrt{3}}{9}$
15	2	6	84	5	3	0	
16	3	6	6	6	78	1	
17	56	13	11	13	6	1	
18	8	62	17	8	5	1	
19	16	25	22	20	15	1	$a+4b+0.2=1$ $a+4b=0.8\ldots$(1) $\mathrm{E}(X)=-a+5b^2+0.8\ldots$(2) Substitute $0.8-4b$ for a into (2). $\mathrm{E}(X)=5b^2+4b$, $0\le b\le 0.2$, maximum value of b occurs when a is zero, $4b+0.2=1,\ b=0.2$ The smallest value for $\mathrm{E}(X)$ is 0 and the largest value is 1.
20	11	23	30	18	17	1	Reflect the graph of *f* in the *y*-axis. $\int_0^3 f(x)dx=\int_{-3}^0 f(-x)dx=5$ Then dilate the graph by a factor of 3 from the *x*-axis. $3\int_{-3}^0 f(-x)dx=15$ Then translate the graph 5 units up. $3\int_{-3}^0 (f(-x))dx+3\times 5=30$

Section B

Question 1a.

Marks	0	1	2	Average
%	10	33	57	1.5

$f:[0,8\pi]\to R, f(x)=2\cos\left(\frac{x}{2}\right)+\pi$, Period = $\frac{2\pi}{\frac{1}{2}}=4\pi$, Range = $[-2+\pi, 2+\pi]$

This question was answered well. However, some students included round brackets instead of square brackets for the range. Range = $[2+\pi, -2+\pi]$ was occasionally seen. Some students gave approximate answers instead of exact answers.

Question 1b.

Marks	0	1	Average
%	13	87	0.9

$f'(x)=-\sin\left(\frac{x}{2}\right)$

This question was answered well. Some students did not write an equation, leaving their answer as $-\sin\left(\frac{x}{2}\right)$. Others made errors when using the chain rule. Some had their technology in degree mode rather than radian mode.

Question 1c.

Marks	0	1	Average
%	33	67	0.7

$f'(\pi)=-1, f(\pi)=\pi, y-\pi=-(x-\pi), y=-x+2\pi$

This question was answered well. Students were not required to show any working. The answer could be obtained directly using technology. Some left their answer as $-x+2\pi$.

Question 1d.

Marks	0	1	2	Average
%	48	5	47	1.0

$f'(x)=1, x=3\pi \text{ or } x=7\pi, f(3\pi)=\pi, f(7\pi)=\pi, y-\pi=1(x-3\pi), y-\pi=1(x-7\pi), y=x-2\pi, y=x-6\pi$

Once students found $x=3\pi \text{ or } x=7\pi$ the rest of the question could be completed using technology. Some students gave only one of the equations of the tangents.

Question 1e.

Marks	0	1	2	3	Average
%	48	34	9	10	0.8

$T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ b \end{bmatrix}, x' = x - \pi, x = x' + \pi, y' = ay + b, y = \frac{y' - b}{a}$

$y = 2\cos\left(\frac{x}{2}\right) + \pi,\ \frac{y' - b}{a} = 2\cos\left(\frac{x' + \pi}{2}\right) + \pi,\ y' = 2a\cos\left(\frac{x' + \pi}{2}\right) + a\pi + b, y' = -2a\sin\left(\frac{x'}{2}\right) + a\pi + b,$

$-2a = -1, a = \frac{1}{2},\ a\pi + b = 0,\ b = -\frac{\pi}{2}$

This question was not answered well. Many students were able to take the equations out of the matrices and rearrange them. Some attempted to draw the graphs but were unable to describe the transformations.

Question 1f.

Marks	0	1	2	Average
%	49	4	47	1.0

Solve $f(x) = 2f'(x) + \pi$ for *x*, $2\cos\left(\frac{x}{2}\right) + \pi = -2\sin\left(\frac{x}{2}\right) + \pi, \tan\left(\frac{x}{2}\right) = -1,\ \frac{x}{2} = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

$x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}$

Some students gave $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$ as the answer. Others tried solving $2f'(x) + \pi = 0$ instead of $2f'(x) + \pi = f(x)$.

Question 2ai.

Marks	0	1	Average
%	25	75	0.7

$f(x) = -\frac{1}{3}(x+2)(x-1)^2, g(x) = \int\left(-\frac{1}{3}(x+2)(x-1)^2\right)dx = -\frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + c$

$g(0) = -\frac{0}{12} + \frac{0}{2} - \frac{0}{3} + c = 1, c = 1,\ g(x) = -\frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + 1$

Often the *dx* and + c were missing from students' answers. Some students made sign errors when writing the equation for *g*; for example, $g(x) = \frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + 1$.

Question 2aii.

Marks	0	1	Average
%	18	82	0.8

Solve $g'(x)=0$ for *x*, $x=-2$ or $x=1$

This question was answered well. Some students gave three *x* values. Others gave the coordinates of the stationary points, which was not necessary.

Question 2bi.

Marks	0	1	Average
%	15	85	0.9

$y=3-\frac{4x}{3}$, the coordinates of *B* are (0, 3)

This question was answered well. Some students did not give the coordinates of *B* and only wrote 3. Others gave the coordinate as (3, 0).

Question 2bii.

Marks	0	1	2	Average
%	27	18	55	1.3

$m=\frac{3}{4}$, $g(2)=\frac{1}{3}$, $y-\frac{1}{3}=\frac{3}{4}(x-2)$, $y=\frac{3}{4}x-\frac{7}{6}$, the coordinates of *C* are $\left(0,-\frac{7}{6}\right)$

Some students found the correct equation but then did not give the coordinates of *C*. Others gave approximate answers. Exact answers were required.

Question 2biii.

Marks	0	1	2	Average
%	26	25	49	1.2

Area of a triangle = $\frac{1}{2}bh=\frac{1}{2}\times\left(\frac{7}{6}+3\right)\times 2=\frac{25}{6}$ or area = $\int_0^2\left(\left(3-\frac{4x}{3}\right)-\left(\frac{3x}{4}-\frac{7}{6}\right)\right)dx=\frac{25}{6}$

Many students knew the formula for the area of the triangle or other suitable methods but had incorrect substitutions.

Question 2ci.

Marks	0	1	2	Average
%	53	6	40	0.9

$m=-\frac{4}{3}$, solve $g'(x)=-\frac{4}{3}, x=-1$, $g(-1)=\frac{25}{12}$, the coordinates of *D* are $\left(-1, \frac{25}{12}\right)$

This question was not answered well. Some students found $x=-1$ but then gave the incorrect *y* coordinate.

Question 2cii.

Marks	0	1	2	3	Average
%	62	6	5	27	1.0

A $\left(2, \frac{1}{3}\right)$, tangent at $x = -1$, $y = \frac{3}{4} - \frac{4}{3}x$, solve $y = \frac{3}{4} - \frac{4}{3}x$ and $y = \frac{3}{4}x - \frac{7}{6}$ for *x* and *y*,

E $\left(\frac{23}{25}, -\frac{143}{300}\right)$, Length = $\sqrt{\left(\frac{23}{25} - 2\right)^2 + \left(-\frac{143}{300} - \frac{1}{3}\right)^2} = \frac{27}{20}$

This question was not answered well.

Question 3a.

Marks	0	1	2	Average
%	16	8	76	1.6

$X \sim \text{Bi}(22, 0.1)$, $\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - 0.9^{22} = 0.9015$, correct to four decimal places

This question was answered well. Most students recognised that it was binomial and gave the correct *n* and *p* values. Some used $\Pr(X > 1)$ instead of $\Pr(X \geq 1)$.

Question 3b.

Marks	0	1	2	Average
%	36	27	37	1.0

$\Pr(X < 5 | X \geq 1) = \frac{\Pr(X \geq 1 \cap X < 5)}{\Pr(X \geq 1)} = \frac{\Pr(X \geq 1 \cap X \leq 4)}{\Pr(X \geq 1)} = \frac{0.839389...}{0.901522...}$ = 0.9311, correct to four decimal places

Many students recognised that this was a conditional probability question but had the incorrect numerator or denominator. Some included 5 in their calculations, getting 0.9798. Others rounded too soon and gave 0.9312 as the answer.

Question 3c.

Marks	0	1	2	Average
%	29	23	48	1.2

$Y \sim N(190, 6^2)$, $\Pr(Y \leq 180) = 0.0478$, correct to four decimal places

Some students thought 3 hours and 10 minutes was 3.1 hours and 6 minutes was 0.6 hours. Others had the standard deviation as 10 minutes. Some gave the answer without showing any working. Students are reminded that some working must be shown for questions worth more than one mark. $\Pr(Y > 180) = 0.9522$ was a common incorrect response.

Question 3d.

Marks	0	1	2	3	Average
%	46	29	4	21	1.0

$X_2 \sim \text{Bi}(100, 0.04779...)$, $\Pr\left(\hat{P} \geq 0.06 \mid \hat{P} \geq 0.05\right) = \dfrac{\Pr\left(\hat{P} \geq 0.05 \cap \hat{P} \geq 0.06\right)}{\Pr\left(\hat{P} \geq 0.05\right)} = \dfrac{\Pr(X_2 \geq 6)}{\Pr(X_2 \geq 5)} = \dfrac{0.34433...}{0.52340...}$

= 0.658, correct to three decimal places

Most students used the conditional probability formula but tried to use the normal distribution rather than the binomial distribution.

Question 3e.

Marks	0	1	2	Average
%	53	14	32	0.8

$Y_2 \sim N\left(180, \sigma^2\right), \dfrac{190-180}{\sigma} = 1.1749..., \sigma = 8.5107$, correct to four decimal places

Students made some rounding errors.

Question 3f.

Marks	0	1	Average
%	79	21	0.2

$\Pr\left(Y_2 < 180\right) = \dfrac{1}{2},\ \Pr\left(Y_2 > 180\right) \times \Pr\left(Y_2 > 180\right) \times \Pr\left(Y_2 < 180\right) = \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{8}$

Some students did not attempt this question.

Question 3g.

Marks	0	1	Average
%	59	41	0.4

(0.01, 0.11), correct to two decimal places

Students were not expected to write out the formula; the relevant computation could be done directly using technology. There were some rounding errors. A common incorrect interval was (0.01, 0.12).

Question 3hi.

Marks	0	1	Average
%	33	67	0.7

$\text{E}(X) = \int_0^{210} (x \times f(x))dx = 170.01$, correct to two decimal places

This question was answered well.

Question 3hii.

Marks	0	1	2	Average
%	27	15	58	1.3

Solve $\int_0^m (f(x))dx = \frac{1}{2}$ for m, $m = 176$ to the nearest integer or solve $\int_m^{210} (f(x))dx = \frac{1}{2}$ for m, $m = 176$ to the nearest integer. As function f is a close approximation of the probability density function, answers to the nearest integer were accepted.

Some students wrote down the correct formula but did not delete the x on their technology from the previous computation for E(X), solving $\int_0^m (x \times f(x))dx = \frac{1}{2}$, getting m = 75.58. Students should check their answers to see if they make sense as 75.58 is very different from 170.01. The dx was often missing from students' working. Some students gave two solutions; they did not consider the domain.

Question 4a.

Marks	0	1	2	Average
%	42	6	52	1.1

$$\frac{2x+1}{x+2} = 2 + \frac{-3}{x+2}$$

This question was answered well and could be done directly using technology. Some students left their answer as $2x + \frac{1}{x+2}$, which was incorrect.

Question 4bi.

Marks	0	1	2	Average
%	19	26	56	1.4

$f(x) = \frac{2x+1}{x+2}$, let $y = \frac{2x+1}{x+2}$, inverse swap x and y, $x = \frac{2y+1}{y+2}$, $y = \frac{-3}{x-2} - 2$, range of f is $R \setminus \{2\}$

Domain of f^{-1} is $R \setminus \{2\}$, $f^{-1} : R \setminus \{2\} \to R, f^{-1}(x) = \frac{-3}{x-2} - 2$

This question was answered well. Equivalent forms were acceptable. Some students did not give the domain. Others attempted to solve for y by hand and were unsuccessful.

Question 4bii.

Marks	0	1	Average
%	38	62	0.6

Solve $f(x)=x,\ x=-1 \text{ or } x=1$, Area = $\int_{-1}^{1}(f(x)-x)\,dx=-3\log_e(3)+4 \text{ units}^2$

This question was answered well. Some students gave an approximate answer when an exact answer was required. The base was sometimes missing from students' responses.

Question 4biii.

Marks	0	1	Average
%	38	62	0.6

Area = $\int_{-1}^{1}\left(f(x)-f^{-1}(x)\right)dx=-6\log_e(3)+8$

Some students realised that the answer to this question was double that of the previous result.

Question 4c.

Marks	0	1	2	3	Average
%	69	4	14	14	0.7

$P\left(x,\dfrac{2x+1}{x+2}\right)$, distance $y=\sqrt{x^2+\left(\dfrac{2x+1}{x+2}\right)^2}$, $-2<x<\infty$, solve $\dfrac{dy}{dx}=\sqrt{x^2+\left(\dfrac{2x+1}{x+2}\right)^2}=0$

$c=\sqrt{3}-2$, $d=-\sqrt{3}+2$, distance = $2\sqrt{2}-\sqrt{6}$

This question was not answered well. Some students did not consider the domain and gave two sets of values for *c* and *d* or chose the incorrect value for *c*. Others had the correct answers for *c* and *d* but did not work out the minimum distance. Some did not give exact values.

Question 4d.

Marks	0	1	2	Average
%	90	4	6	0.2

Let $x_2>x_1>-k$, where $k>1$, $g(x_2)-g(x_1)=\dfrac{(x_2-x_1)(k^2-1)}{(x_1+k)(x_2+k)}$ and $x_2-x_1>0,\ k^2-1>0$

If $x_2>x_1>-k$, then $-x_2<-x_1<k$, so $k+x_1>0$ and $k+x_2>0$

Hence $g(x_2)-g(x_1)>0$ and $g(x_2)>g(x_1)$.

Many students did not attempt this question. Some just substituted in specific values, which was not acceptable.

Question 4ei.

Marks	0	1	2	Average
%	60	16	24	0.6

Solve $g(x)=-x$ for x, $x=-k+\sqrt{k^2-1}$, $y=k-\sqrt{k^2-1}$, $X\left(-k+\sqrt{k^2-1},k-\sqrt{k^2-1}\right)$

Some students did not consider the domain and chose the incorrect value for *x*. Others gave two sets of coordinates. Some did not include a multiplication sign between *k* and *x* when using technology.

Question 4eii.

Marks	0	1	2	Average
%	51	11	38	0.9

Solve $-k+\sqrt{k^2+1}=-\frac{1}{2}$ for k, $k=\frac{5}{4}$ or solve $g\left(-\frac{1}{2}\right)=\frac{1}{2}$, $k=\frac{5}{4}$

Some students did not use their answer to Question 4ei. but were successful in solving $g\left(-\frac{1}{2}\right)=\frac{1}{2}$

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Question 4eiii.

Marks	0	1	2	Average
%	91	5	3	0.1

$YZ=2\sqrt{2}$, $s(k)=\left(\frac{1}{2}\times 2\sqrt{2}\times\left(\sqrt{\left(-k+\sqrt{k^2+1}\right)^2+\left(k-\sqrt{k^2+1}\right)^2}\right)\right)^2=4\left(\sqrt{k^2-1}-k\right)^2$, solve $s(k)>1$,

$1<k\le\frac{5}{4}$

This question was not answered well. Some students had $1\le k\le\frac{5}{4}$.

Question 4fi.

Marks	0	1	2	Average
%	61	20	18	0.6

$A(k)=\int_{-1}^{1}\left(g(x)-x\right)dx=\left(k^2-1\right)\log_e\left(\frac{k-1}{k+1}\right)+2k$

Most students were able to set up the integral. $A(k)=\int_{-1}^{1}\left(f(x)-x\right)dx$, which was independent of *k*, was sometimes given. Other students tried to use $A(k)=\int_{1}^{\infty}\left(f(x)-x\right)dx$. Some broke the area up into different sections and were successful, but this would have been time consuming.

Question 4fii.

Marks	0	1	2	Average
%	94	4	2	0.1

$g(x)$ is strictly increasing over the interval $[-1,1]$ and $g(x) > x$, $k > 1$, then $A(k) > 0$ and will always be in the region bounded by the triangle formed by the lines $x = -1$, $y = x$ and $y = 1$. The area of this triangle is 2 units^2, hence $0 < A(k) < 2$

Many students did not attempt this question.