Solutions

PER ASPERA AD ASTRA

Girton

Teacher's Name: Students Circle

Algreen-Ussing (Mr) Itter (Mrs) James (Mr)

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SEMESTER 2 EXAMINATIONS - NOVEMBER 2017

Year 11 Mathematical Methods Examination

Reading time: 10 Minutes

Writing time: 120 Minutes

Marks Allocated:

Sections within Booklet	Number of Questions	Number of Marks
Exam 2		
Multiple Choice	20	20
Extended Answer	5	60

Specific Instructions

Multiple Choice: Calculator and summary book allowed. Answer on multiple choice answer sheet
Extended Answer: Calculator and summary book allowed.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Supplies and Equipment

Supplies: Please ensure you have the correct supplies/instruments for taking the examination before you enter the examination venue (e.g. pencils, pens, calculator, ruler, etc). There will be no sharing allowed. No other paper, etc. will be allowed to come in with you unless instructed as Specific Instructions. A clear bottle containing only water is permissible.

At the Conclusion: Please wait quietly for specific instruction as to how you will be dismissed. Leave your examination paper on your table. Pick up unwanted papers around you, push your chair under the table, and put your rubbish in the bin on your way out of the examination room.

Sem 2, 2017

Year 11 Mathematical Methods Multiple Choice Answer Sheet

Name: Solutions. **Teacher:**__________________

Clearly circle the best answer for each question. If you wish to alter your choice, rewrite the question number and the new answer next to it.

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

Short Answer	**/40**
Multiple Choice	**/20**
Extended Answer	**/60**
Total	**/120**

Multiple Choice

Place answers on provided multiple choice answer sheet.

Question 1

If **A** = $\begin{bmatrix} 8 & 2 \\ 11 & 3 \end{bmatrix}$ then $\mathbf{A}^{-1}$ =

A $\frac{1}{2}\begin{bmatrix} -8 & 11 \\ 2 & -3 \end{bmatrix}$

B $\frac{1}{2}\begin{bmatrix} 3 & -2 \\ -11 & 8 \end{bmatrix}$

C $2\begin{bmatrix} -8 & 11 \\ 2 & -3 \end{bmatrix}$

D $2\begin{bmatrix} 3 & -2 \\ -11 & 8 \end{bmatrix}$

E $-\frac{1}{2}\begin{bmatrix} -8 & 11 \\ 2 & -3 \end{bmatrix}$

Question 2

If $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & -11 \\ -5 & 2 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 16 & 21 \\ 35 & 70 \end{bmatrix}$, and **AX** + **B** = **C** then **X** is

A $\begin{bmatrix} 1 & 8 \\ 5 & 4 \end{bmatrix}$

B $\begin{bmatrix} 11 & 88 \\ 55 & 44 \end{bmatrix}$

C $\begin{bmatrix} 13 & 32 \\ 40 & 68 \end{bmatrix}$

D $\begin{bmatrix} 7 & -2 \\ -5 & 3 \end{bmatrix}$

E $\begin{bmatrix} 21 & 13 \\ 8 & 7 \end{bmatrix}$

Question 3

The matrix which describes the composition of mappings

- dilation of factor 3 from the x-axis
- reflection in the line $y = x$
- reflection in the x-axis

is

A $\begin{bmatrix} 3 & 0 \\ -1 & 3 \end{bmatrix}$

B $\begin{bmatrix} 0 & 0 \\ -1 & 3 \end{bmatrix}$

C $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 0 & 0 \\ -3 & 1 \end{bmatrix}$

E $\begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix}$

Question 4

A transformation described by the 2×2 matrix $\begin{bmatrix} 0 & 5 \\ -1 & 0 \end{bmatrix}$ maps the point with coordinates (a, b) to the point $(10, 20)$, where (a, b) is

A $(4, 2)$

B $(-20, 2)$

C $(-2, 20)$

D $(-2, 4)$

E $(-20, -2)$

Question 5

$\dfrac{\left(2x^2y^4\right)^3}{3\left(xy^2\right)^4} \div \dfrac{32xy}{9y^6}$ is equal to

A. $\dfrac{3xy^9}{4}$

B. $\dfrac{3x^5y^9}{4}$

C. $\dfrac{9xy^9}{16}$

D. $\dfrac{9x^5y^9}{16}$

E. $\dfrac{256x^3}{27y}$

Question 6

$3\log_a(2)+\log_a(5)-\log_a(10)$ is equal to

A. $\log_a\left(a^2\right)$
B. $\log_a(1)$
C. $\log_a(3)$
D. $\log_a(4)$
E. $\log_a(16)$

Question 7

Let $f:[0,\infty)\to R, f(x)=5^{2x}+3$.
The range of *f* is

A. $(-\infty,3]$
B. $(3,\infty)$
C. $[3,\infty)$
D. $(4,\infty)$
E. $[4,\infty)$

Question 8

For the function: $g(x)=\log_{10}(x-1)+2$, the maximal domain is:

A. $(-\infty,\infty)$
B. $(-1,\infty)$
C. $(0,\ \infty)$
D. $(1,\ \infty)$
E. $(2,\ \infty)$

Question 9

The x-intercept of the graph of $y=\log_a(3x+2)$ is

A. $-\frac{2}{3}$
B. $-\frac{1}{3}$
C. a
D. $\frac{2a}{3}$
E. $\frac{2}{3}$

Question 10

The population P of a regional centre is modelled by the function

$$P(t) = 34\,500 \times 10^{-0.1t} + 23\,000$$

where t is measured in years and $t = 0$ represents 1 January 2017.
According to this model, the population of this regional centre over the long term will approach

A. 11 500
B. 23 000
C. 34 500
D. 46 000
E. 57 500

Question 11

To convert the angle $\frac{\pi}{12}$ to degrees, it must be multiplied by

A. π

B. $\frac{180^\circ}{\pi}$

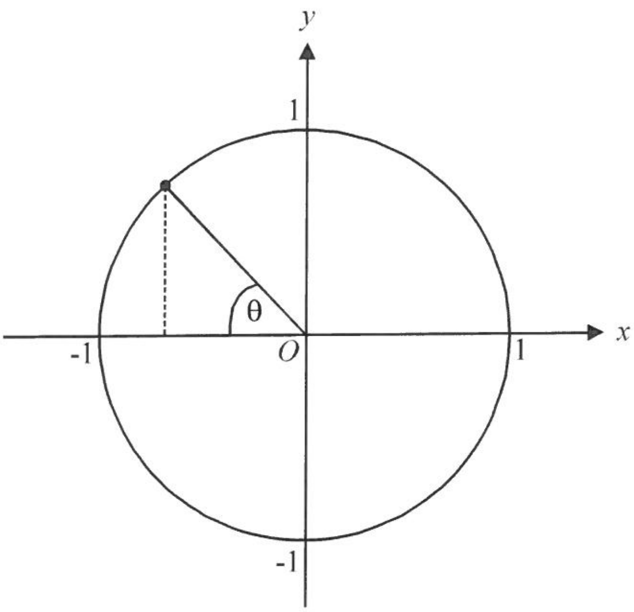
C. $\frac{\pi}{360^\circ}$

D. $\frac{\pi}{180^\circ}$

E. $\frac{15^\circ}{\pi}$

Question 12

The diagram below shows a unit circle.



The length of the vertical dotted line shown, represents

A. $\cos(-\theta)$
B. $\cos(\theta)$
C. $\sin(\pi-\theta)$
D. $\cos(\pi-\theta)$
E. $\sin(2\theta)$

Question 13

The temperature T (in degrees Celsius) in a controlled environment is given by the function,

$$T(t)=21+7\sin\left(\frac{\pi t}{12}\right), \quad t\geq 0$$

where t is measured in hours and $t=0$ corresponds to 9am on Monday.

At 8pm on the same Monday, the temperature in degrees Celsius, is closest to

A. 21.3
B. 22.8
C. 25.9
D. 26.6
E. 27.2

Question 14

A graph which has an amplitude of 2 and a period of π could have a rule given by

A. $y=\sin(\pi x)+2$
B. $y=2\sin(\pi x)$
C. $y=\sin(2x)+2$
D. $y=2\sin(2x)+4$
E. $y=\pi\sin\left(\frac{x}{2}\right)$

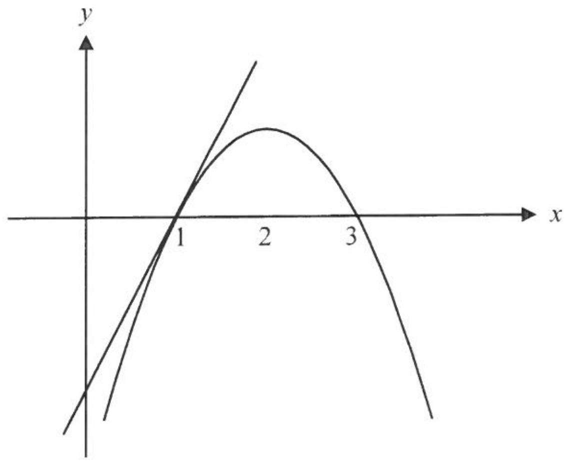
Question 15

The graph of $y=3\cos(2x)-1$ for $x \in R$, has a minimum value of

A. 2
B. 0
C. -1
D. -3
E. -4

Question 16

The graph of the function $y=-x^2+4x-3$ is shown below.



A tangent is drawn to the graph at the point $(1,0)$. The gradient of the tangent is

A. -3
B. 0
C. 1
D. 2
E. 3

Question 17

Given that $g(x)=\frac{2}{x}+\sqrt[3]{x}$, then $g'(8)$ is equal to

A. $\frac{5}{96}$
B. $\frac{9}{4}$
C. 3
D. 5.8
E. 16.2

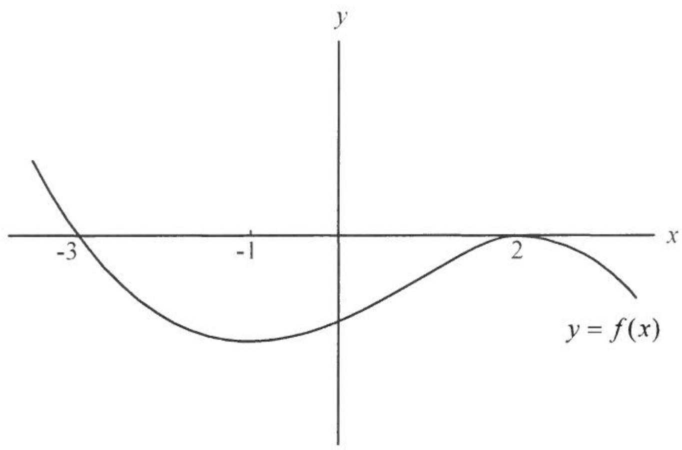
Question 18
Grain is being poured into a silo. The height *h*, in metres, of grain in the silo *t* hours after it has started to be poured in, is given by

$$h(t)=-t^3+12t, \qquad t\in[0,2]$$

The rate, in metres per hour, at which the height of the grain in the silo is changing when $t=1$ is

A. 9
B. 11
C. 13
D. 15
E. 17

Question 19
The graph of the function *f* is shown below.



Which one of the following statements is **not** true?

A. $f'(-1)=0$
B. $f(2)=0$
C. $f'(2)=0$
D. $f(-3)=0$
E. $f'(-3)=0$

Question 20

The instantaneous rate of change of the function $y = \frac{3}{x^2}$ when $x = -1$ is

A. -6
B. -3
C. 0
D. 3
E. 6

END OF MULTIPLE CHOICE

Extended answer

Working out required where appropriate

Place answers in the space provided on the exam paper.

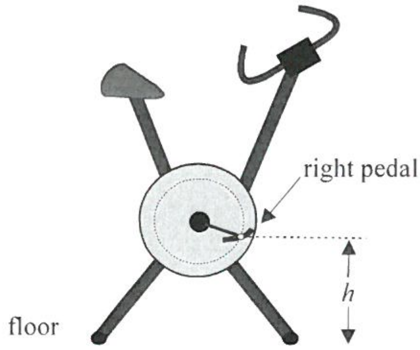
Question 1 (12 marks)

An athlete straps her feet into the pedals of an exercise bike.

For the first part of her training session, the height h, in centimetres, of the right pedal above the floor, t seconds after she begins pedalling is given by

$$h(t) = 12\cos(4\pi t) + 24, \quad t \in [0,120].$$

The exercise bike is shown below.



a. What is the starting height, in centimetres, of the right pedal above the floor? 1 mark

$t = 0, \quad h(0) = 36$ cm

b. What is the mean height, in centimetres, of the right pedal above the floor? 1 mark

24 cm

c. Find the maximum height above the floor that the right pedal reaches. Express your answer in centimetres. 1 mark

36 cm

d. How long does it take for the right pedal to complete one revolution? Express your answer in seconds. 1 mark

Period $= \frac{2\pi}{4\pi}$

$= \frac{1}{2}$ second

e. How many complete revolutions does the right pedal complete in this first part of the athlete's training session? 1 mark

240 revolutions

f. What is the height above the floor of the right pedal 7.25 seconds after the athlete begins pedalling? Express your answer in centimetres. 1 mark

$h(7.25) = 12$ cm

g. When will the right pedal be 18 centimetres above the floor for the first time? 2 marks

$18 = 12\cos(4\pi t) + 24$

$t = 1/6$ seconds

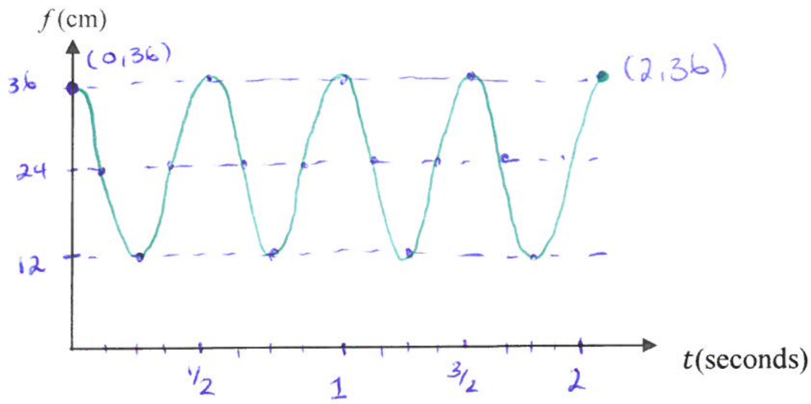
The athlete stops and has a drink before resuming pedalling.

The height, in centimetres, of the right pedal of the exercise bike above the floor t seconds after she begins pedalling during this next part of her training session is given by the function

$$f(t) = 12\cos(4\pi t) + 24$$

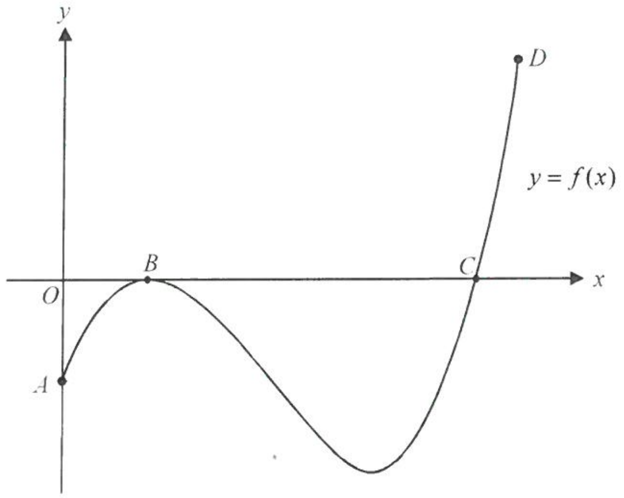
but this time she only pedals for 2 seconds before stopping; that is, $t \in [0, 2]$.

h. On the set of axes below, sketch the function f. Label the endpoints clearly. 4 marks



Question 2 (9 marks)

Let $f(x)=(x-1)^2(x-5)$ where $0 \le x \le 6$.
The graph of f is shown below.



The points A and D are the endpoints of the graph and points B and C are the x-intercepts of the graph.

a. Find the coordinates of

1 mark

i. point A.

$(0, -5)$

ii. point B. 1 mark

$(1, 0)$

iii. point C. 1 mark

$(5, 0)$

iv. point D. 1 mark

$(6, 25)$

b. Find the range of f. 3 marks

$f(x) =$ $\left[-256/27, 25\right]$

or $[-9.48, 25]$

c. Show that $f(x) = f'(x)$ at point B. 2 marks

Point B $x = 1$ $f(x) = (x^2 - 2x + 1)(x - 5)$

$f(1) = 0$

$= x^3 - 5x^2 - 2x^2 + 10x + x - 5$

$= x^3 - 7x^2 + 11x + 5$

$\therefore f(1) = f'(1)$ $f'(x) = 3x^2 - 14x + 11$

$f'(1) = 3 \times 1^2 - 14 \times 1 + 11$

$= 0$

Question 3 (14 marks)

Plants grown from genetically modified seeds are observed.
The average height p, in centimetres, of a batch of genetically modified seedlings t days after they were planted out is given by

$$p(t) = 2.8 \times 5^{0.013t} + 4, \ t \in [0, 120]$$

a. For how many days were the plants observed? 1 mark

120 days

b. What was the average height of the seedlings when they were planted out? 1 mark

$t = 0$, $P(0) = 2.8 \times 1 + 4$

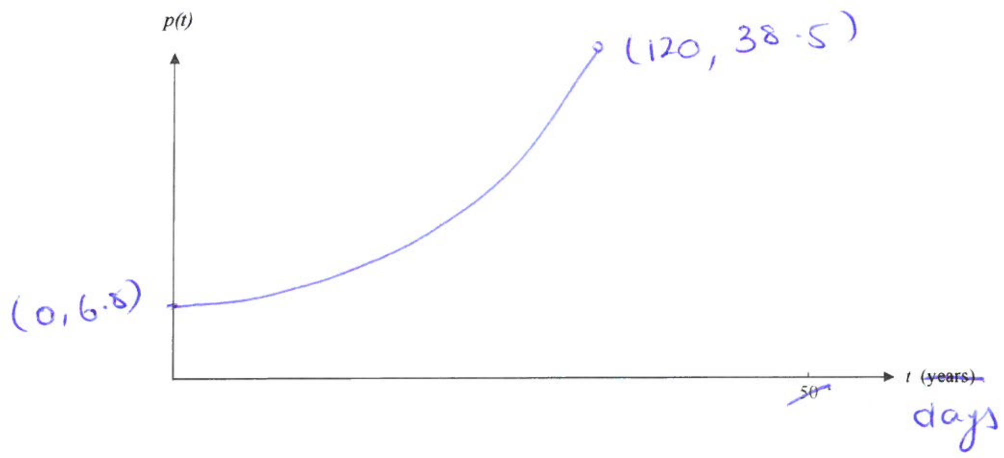
$= 6.8$ cm

c. What was the average height of the seedlings 100 days after they had been planted out? Give your answer correct to two decimal places. 1 mark

$P(100) = 2.8 \times 5^{0.013 \times 100} + 4$

$= 26.69$ cm

d. On the set of axes below, sketch the graph of the function p. Indicate clearly on the graph the coordinates of any endpoints (correct to 1 decimal place where necessary). 3 marks



e. Find the number of days it took for the seedlings to reach an average height of 10cm after having been planted out. Give your answer correct to the nearest whole number. 2 marks

$10 = 2.8 \times 5^{0.013t} + 4$

$t = 36$ days

Find the average rate of change of $p(t)$ between day 10 and day 60.

Give your answer in cm/day correct to two decimal places. 2 marks

$\frac{f(60) - f(10)}{50} = 0.127$ cm / day

$= 0.13$ cm / day

Typically, these type of plants grown from genetically modified seed, reach a suitable height for harvesting after the seedlings have been planted out for 100 days.

f. How many days after the seedlings have been planted out would they have reached **half** this suitable height for harvesting? Express your answer correct to two decimal places. 2 marks

26.69 after 100 days

$13.35 = 2.8 \times 5^{0.013t} + 4$

57.60 days.

A batch of the same type of plants, but which had grown from non-genetically modified seed, was also observed. The average height, in centimetres, of these non-genetically modified plants is given by

$$n(t) = 2.8 \times 5^{kt} + 4, \; t \in [0, 120]$$

where t represents the number of days since the seedlings were planted out and k is a positive, real, constant.
The average height of these plants was 9 centimetres, thirty days after the seedlings had been planted out.

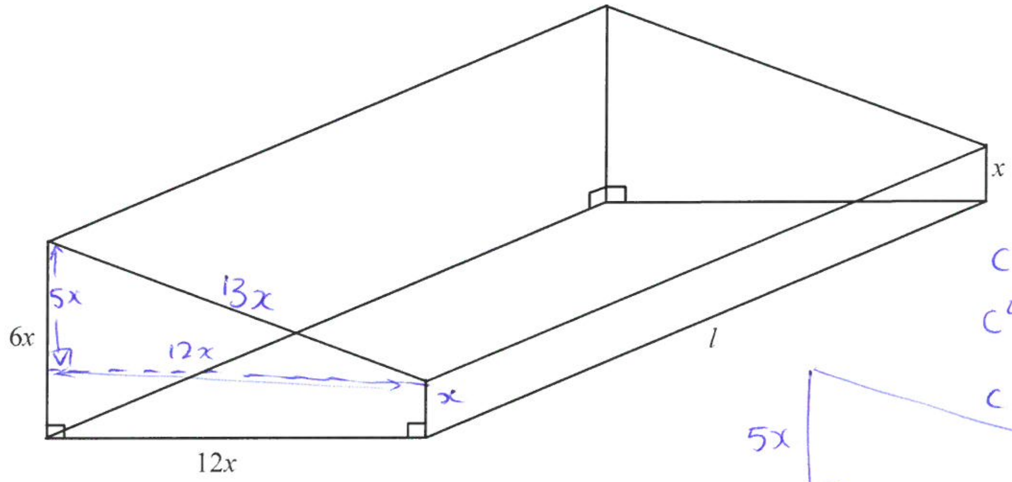
g. Find the value of k. Give your answer correct to three decimal places. 2 marks

$9 = 2.8 \times 5^{30k} + 4$

$k = 0.012$

Question 4 (16 marks)

A display cabinet is in the shape of a triangular prism resting on top of a rectangular prism.
The framework of the cabinet is to be made from 800cm of timber. The framework, including dimensions in cm, is shown in the diagram below.



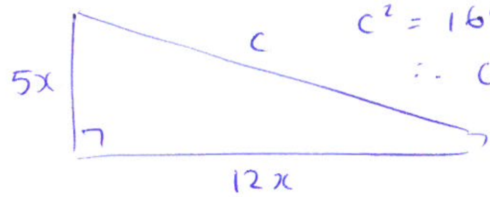
Pythag.

$c^2 = (5x)^2 + (12x)^2$

$c^2 = 25x^2 + 144x^2$

$c^2 = 169x^2$

$\therefore c = 13x$



a. Show that $l = 200 - 16x$.

$4l + 2x + 24x + 12x + 26x = 800$

$4l + 64x = 800$

$4l = 800 - 64x$

$l = 200 - 16x$

3 marks

b. Hence, show that an expression for the volume V, in terms of x, of the cabinet is given by

$$V = 8400x^2 - 672x^3.$$

$V =$ Area of base $\times$ Height.

Base is Trapezium $\Rightarrow A_{Trap} = h \times \frac{a+b}{2}$

$V = \left(h \times \frac{a+b}{2}\right) \times$ height

$= 12x \times \frac{x+6x}{2} \times (200-16x)$

$= 42x^2 \times (200-16x)$

$= 8400x^2 - 672x^3$

2 marks

c. Find the value of x for which the volume of the cabinet is a maximum.

$dV/dx = 16800x - 2016x^2$

$0 = 16800x - 2016x^2 \quad (\div 32)$

$0 = 525x - 63x^2$

$0 = x(525 - 63x)$

$x = 0$ and $x = 525/63$

$x = 25/3$

2 marks

d. Find the value of l, in cm, when the volume of the cabinet is a maximum.

When $x = 25/3$ $\quad l = 200 - 16x$

$= 200 - 16 \times 25/3$

$= 600/3 - 400/3$

$= 200/3$ or 66.67 cm

2 marks

e. Find the maximum volume of the cabinet in cm^3.

When $x = 25/3$

$V = 8400 \times (25/3)^2 - 672 \times (25/3)^3$

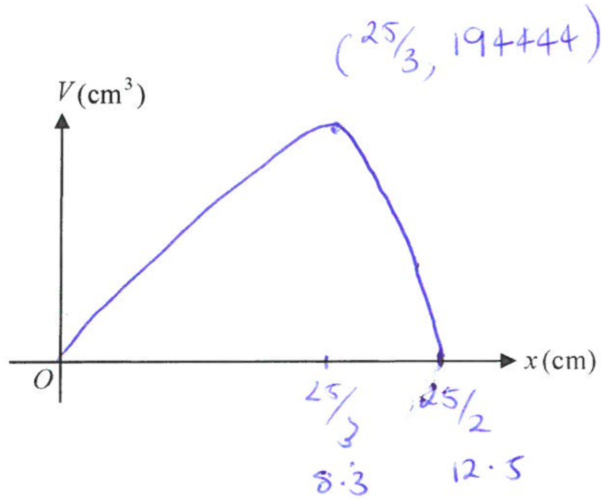
$= 194444.44\ cm^3$

2 marks

f. On the set of axes below sketch the graph of the volume function given in part **b.**

$$V = 8400x^2 - 672x^3$$

Indicate clearly any intercepts, endpoints and turning points.



3 marks

g. Write down the domain and the range of the volume function.

Domain $(0, 25/2)$

Range $(0, 194444]$

2 marks

Question 5 (9 marks)

The matrix equation $X' = AX + B$ defines a transformation where $A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

The curve with equation $y = x^2$ undergoes this transformation.

a. Show that the image produced is given by $y = (x+3)^2 + 2$. 4 marks

$X' = AX + B$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 2x \\ 4y \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$x' = 2x - 3$ $\qquad y' = 4y + 2$

$\therefore x = \frac{x'+3}{2}$ $\qquad y = \frac{y'-2}{4}$

$\Rightarrow y = x^2$

$\frac{y'-2}{4} = \left(\frac{x+3}{2}\right)^2$

$y - 2 = 4 \times \frac{(x+3)^2}{4}$

$y - 2 = (x+3)^2$

$y = (x+3)^2 + 2$

b. Show that the equation of the normal line to the curve with equation $y = (x+3)^2 + 2$ is $y = \frac{x}{2} + 5$ when $x = -4$ 3 marks

When $x = -4$, $y = (-4+3)^2 + 2$ $\qquad (-4, 3)$.

$= 3$

$y = x^2 + 6x + 9 + 2$

$= x^2 + 6x + 11$

$dy/dx = 2x + 6$

$x = -4$, $dy/dx = -2$

$\therefore$ gradient of normal $= 1/2$

$y = mx + c$

$y = \frac{1}{2}x + c$

$3 = \frac{1}{2} \times {}^{-}4 + c$

$3 = -2 + c$

$\therefore c = 5$

$y = \frac{1}{2}x + 5$

(equation of Normal)

c. Find the coordinates of the points of intersection of $y = \frac{x}{2} + 5$ and $y = (x+3)^2 + 2$

2 marks

ac	b
24	11
8×3	$8+3$

P.O.I. $\frac{x}{2} + 5 = x^2 + 6x + 11$

$x + 10 = 2x^2 + 12x + 22$

$0 = 2x^2 + 11x + 12$

$0 = 2x^2 + 8x + 3x + 12$

$= 2x(x+4) + 3(x+4)$

$= (x+4)(2x+3)$

P.O.I when $x = -4$ and $-3/2$

$\Rightarrow y = \frac{x}{2} + 5$ $(-4, 3)$ and $(-3/2, 17/4)$

END OF EXAM