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Year 12 *Trial Exam Paper*

2017

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring blank sheets of paper, notes of any kind or correction liquid/tape into the examination.
- Calculators are NOT permitted in this examination.

Materials provided

- Question and answer book of 11 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

Please provide **exact** answers to all questions where a numerical answer is required, unless otherwise stated.

In questions where more than one mark is available, show appropriate working.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \dfrac{\sin(x)}{x^3 - 3x}$.

Find $\dfrac{dy}{dx}$.

2 marks

b. Let $f(x) = x^3 e^{1-4x}$.

Evaluate $f'(-1)$.

2 marks

Question 2 (4 marks)

Let $f:\left(-\infty, \frac{5}{4}\right] \to R$, where $f(x) = 2 + \sqrt{5-4x}$.

a. Find $f'(x)$.

1 mark

b. Find the equation of the tangent to $y = f(x)$ at $x = -1$.
Give your answer in the form $ax + by = c$.

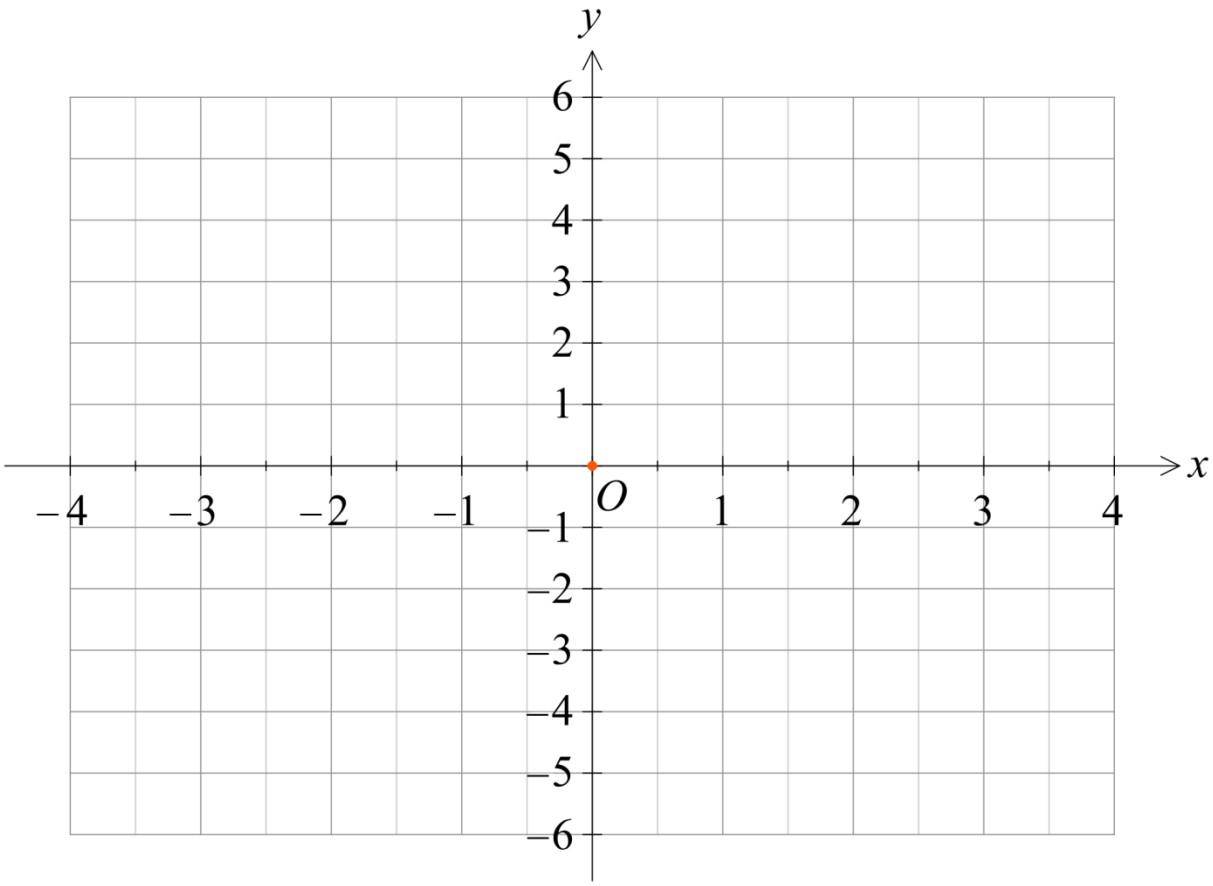
3 marks

Question 3 (6 marks)

Let $f : R \setminus \left\{-\frac{1}{2}\right\}$, where $f(x) = 3 - \frac{4}{2x+1}$.

a. Sketch the graph of $y = f(x)$. Label the axes intercepts with their coordinates and label any asymptotes with the appropriate equation.

3 marks



b. For the area enclosed by the graph of $y = f(x)$, the lines $x = 1$ and $x = 3$ can be expressed in the form $a + \log_e\left(\frac{b}{c}\right)$. Find the values of a, b and c.

3 marks

Question 4 (4 marks)

Six seeds are planted in a pot. The probability that any one seed will germinate is $\frac{1}{4}$.

a. What is the probability that none of the six seeds planted in the pot germinate?

Give your answer in the form $\left(\frac{a}{b}\right)^c$.

1 mark

b. The probability that exactly two seeds germinate in one pot can be expressed as $\frac{k}{4096}$.

Find the value of k.

2 marks

c. A warehouse contains a large number of pots. Into each of these pots, exactly six seeds are planted. Four of these pots are then randomly selected. What is the probability that no seeds have germinated?

Express your answer in the form $\left(\frac{a}{b}\right)^c$, where a, b and c are positive integers.

1 mark

Question 5 (8 marks)

Let $f:[-1,2] \to R$, where $f(x) = \frac{1}{x+2} + \frac{1}{4-x}$.

a. Find $f'(x)$.

1 mark

_____

b. Find the range of $f(x)$.

2 marks

_____

_____

_____

_____

_____

Let $g:[-1,a] \to R,\ g(x) = \dfrac{k}{x^2 - 2x - 8}$.

c. Find the value of k such that $f(x) = g(x)$.

2 marks

d. State the largest value of a such that the inverse function, g^{-1}, exists.

1 mark

e. Find the rule for $g^{-1}(x)$.

2 marks

Question 6 (8 marks)

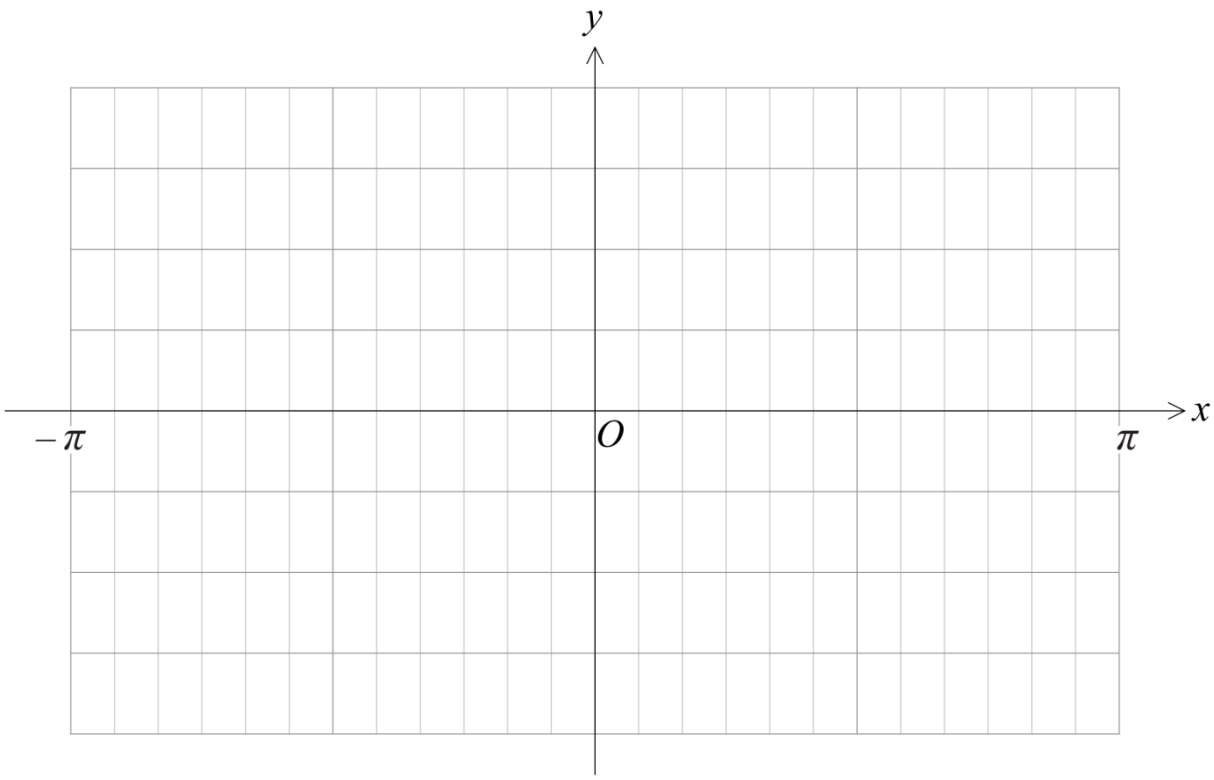
a. Solve the equation $2\cos(2x)-1=0$ for $x\in[-\pi,\pi]$.

2 marks

Let $f:[-\pi,\pi]\to R$, where $f(x)=2\cos(2x)-1$.

b. Sketch the graph of $y=f(x)$.
Label all axes intercepts and endpoints with their coordinates.

3 marks

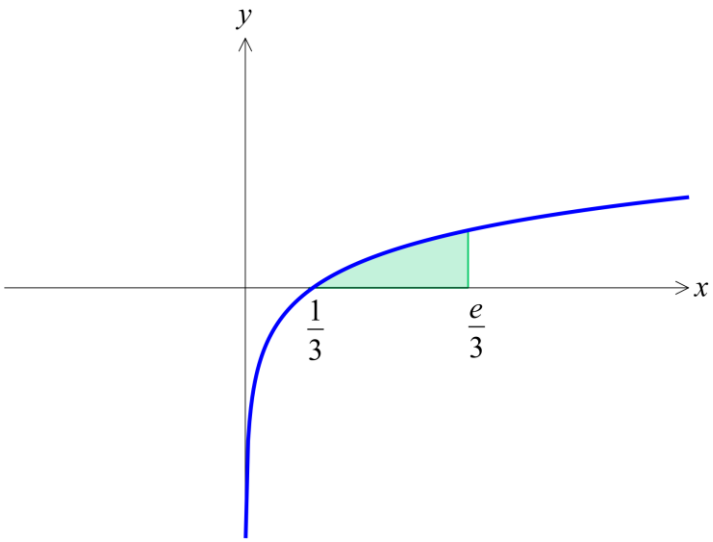


c. Calculate the average value of f over the interval $\frac{-\pi}{4} \leq x \leq \frac{\pi}{6}$.

3 marks

Question 7 (3 marks)

The graph of the function $f : R^+ \to R$, where $f(x) = \log_e(3x)$ is shown below.



a. If $y = x\log_e(3x) - x$, find $\frac{dy}{dx}$.

1 mark

b. Hence, find the exact area of the shaded region.

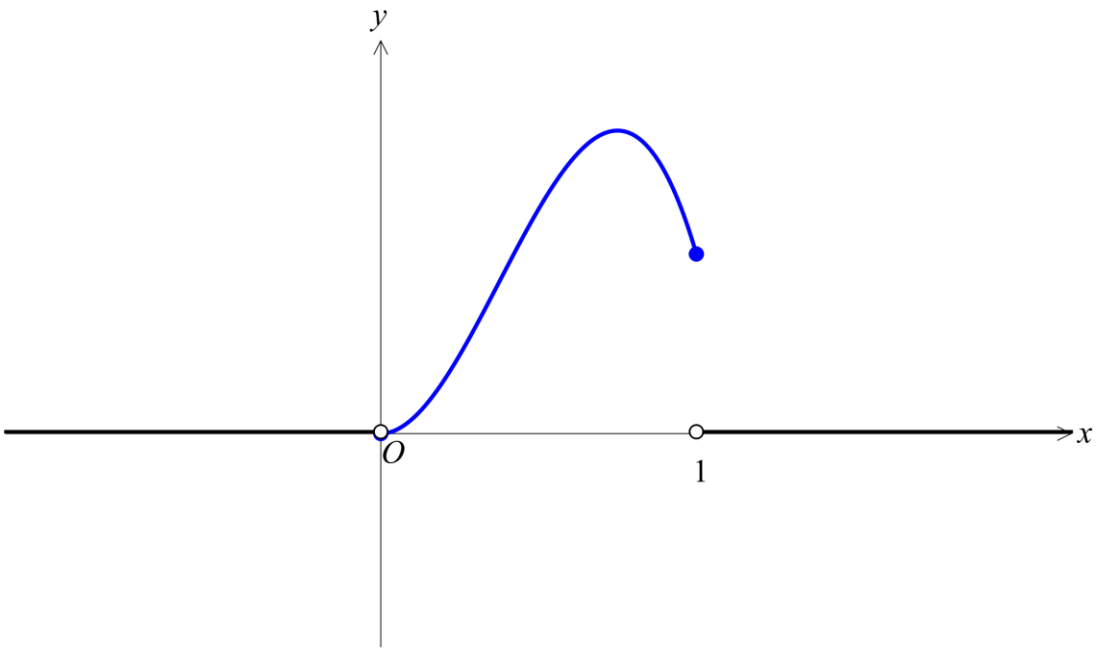
2 marks

Question 8 (3 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} ax^2 - bx^3 & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

where a and b are positive real numbers. Part of the graph $y = f(x)$ is shown below. The graph has a turning point at $x = \frac{3}{4}$.



Find the value of a and b.

END OF QUESTION AND ANSWER BOOK