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2017

Mathematical Methods

Trial Examination 2
(2 hours)

SECTION A Multiple-choice questions

Instructions for Section A

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Question 1 An ordered pair in $A = \left\{(x, y): y = (ax-b)^{1.5}, x \in R\right\}$ is

A. $\left(0, -b^{1.5}\right)$

B. $\left(\frac{1}{a}, -b^{1.5}\right)$

C. $\left(\frac{b-4}{a}, 4\right)$

D. $\left(\frac{a-8}{b}, \frac{b-4}{a}\right)$

E. $\left(\frac{b+4}{a}, 8\right)$

Question 2 The solution(s) to the equation $(x-1)^{\frac{3}{2}}\left(e^{-ax}-1\right)\left(\log_e(2+x)\right) = 0$ for x is/are

A. $-1, 0$ or 1

B. $-1, 0$ and 1

C. -1 or 1

D. 1

E. 0 or 1

Question 3 $y = f(x)$ is transformed to $y = g(x)$ where $g(x) = 0.2f(0.1x + 0.2) + 0.1$, and $y = g(x)$ is transformed to $y = h(x)$ where $h(x) = 5g(10x - 2) - 0.5$, then

A. $h(x) = f(x + 2.2) - 0.6$

B. $h(x) = f(x - 1.8) + 0.4$

C. $h(x) = f(x)$

D. $h(x) = f(x + 1.8) - 0.4$

E. $h(x) = f(x - 2.2) + 0.6$

Question 4 The equation of the inverse of $y = e^{\left(\frac{x}{10}\right)} - \log_e\left(\frac{x}{10}\right)$ is

A. $y = \log_e\left(\frac{x}{10}\right) - e^{\left(\frac{x}{10}\right)}$

B. $y = \log_e(10x) - e^{(10x)}$

C. $\log_e(10y) - e^{(10y)} = x$

D. $\log_e\left(\frac{y}{10}\right) - e^{\left(\frac{y}{10}\right)} = x$

E. $\log_e\left(\frac{y}{10}\right) - e^{\left(\frac{y}{10}\right)} = -x$

Question 5 The graphs of $y = mx$ and $y = \tan(x)$ intersect at exactly one point in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ if

A. $m \le 1$ only

B. $m \in R^+$ only

C. $m \in R^-$ only

D. $m > -\infty$ only

E. $-1 \le m \le 1$ only

Question 6 If $f(x-1)+f(x+1)=0$ for all $x\in R$, $f(x)$ could be

A. $\sin\left(\frac{\pi x}{4}\right)$

B. $\cos\left(\frac{\pi x}{4}\right)$

C. $\tan\left(\frac{\pi x}{4}\right)$

D. $\cos\left(\frac{\pi x}{2}\right)$

E. $\tan\left(\frac{\pi x}{2}\right)$

Question 7 If $x^3+ax^2+bx+c=(x-p)^3$ for $x\in R$, then

A. $a^2=b+c$

B. $a^2=3b$

C. $a^2+3b=c$

D. $b^2=4ac$

E. $c=2b^2$

Question 8 $ax+by+z=1$ and $cx+dy+z=0$ are simultaneous equations in x, y and z, a possible solution set $(x,\ y,\ z)$ to the equations is

A. $\left(0,\frac{1}{b-d},\frac{d}{d-b}\right)$

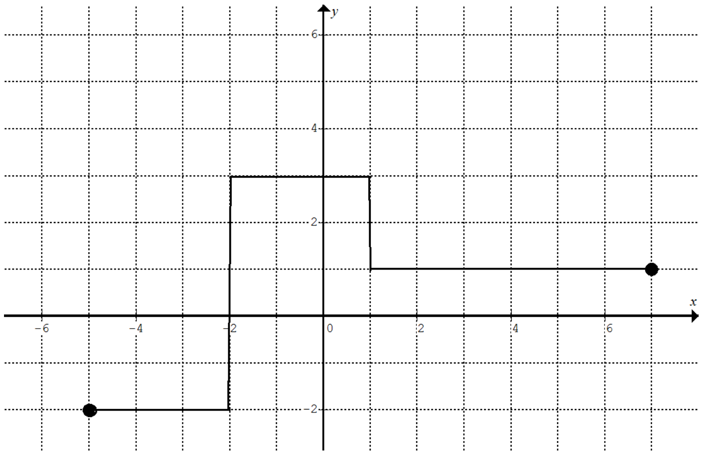
B. $\left(0,\frac{1}{b-d},\frac{d}{b-d}\right)$

C. $\left(\frac{1}{a-c},0,\frac{c}{a-c}\right)$

D. $\left(\frac{1}{c-a},0,\frac{c}{a-c}\right)$

E. $\left(\frac{d}{bc-ad},\frac{c}{bc-ad},0\right)$

Question 9



The average value of the function shown in the graph above is closest to

A. 0.7

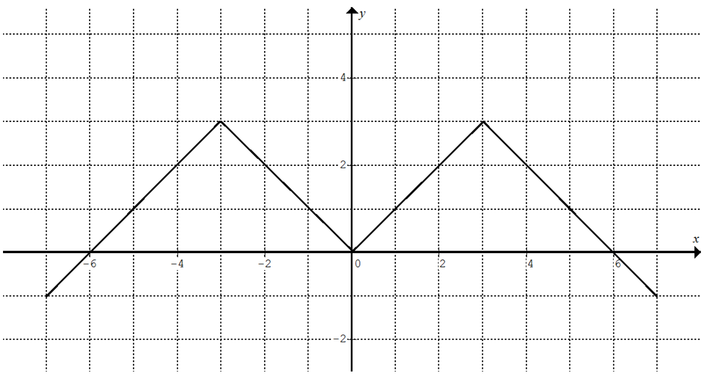
B. 1.0

C. 1.3

D. 1.6

E. 1.9

Question 10



The graph of $y = f'(x)$ is shown above. Which one of the following statements is always true?

A. $f(x) > 0$

B. $f(x) - f(-x) = 0$

C. $f(x) + f(-x) = 0$

D. $f(x) - f(-x) = 1$

E. $f(x) + f(-x) = 1$

Question 11 The graphs of $y = e^x + c$ and $y = e^{2x}$ intersect at two distinct points.
The maximal interval for c is

A. $(-1, 0]$

B. $(-0.1, 0.1)$

C. $(-0.25, -0.1)$

D. $(-0.25, 0)$

E. $[-0.24, -0.1)$

Question 12 The gradient of the tangent to the curve $y = a\sin(x)\cos\left(\frac{x}{2}\right)$ at $x = \frac{1}{2}$ is 0.84, where $a \in R$.

The gradient of the tangent to the curve $y = \frac{a}{2}\sin(2x)\cos(x)$ at $x = \frac{1}{4}$ is

A. 0.21

B. 0.42

C. 0.84

D. 1.68

E. 3.36

Question 13 If $\int_0^a \left(e^{bx} - e^{\frac{bx}{2}}\right)dx = \frac{1}{b}$ where $a, b \in R^+$, then ab is equal to

A. $\log_e(0.25)$

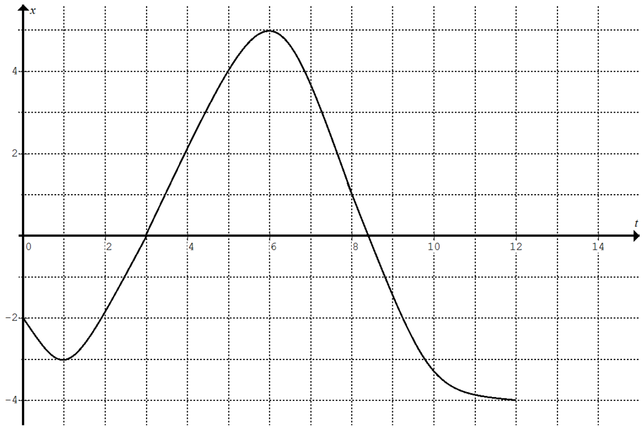
B. $\log_e(0.5)$

C. $\log_e 2$

D. $\log_e e$

E. $\log_e 4$

Question 14 A particle moves in a straight line. Its position is x m relative to the origin O at time t s. The *x*-*t* graph of the motion for $0 \le t \le 12$ is shown below.



The number of times that *the instantaneous rate of change of* x = *the average rate of change of* x in the interval $0 \le t \le 12$ is

A. 0

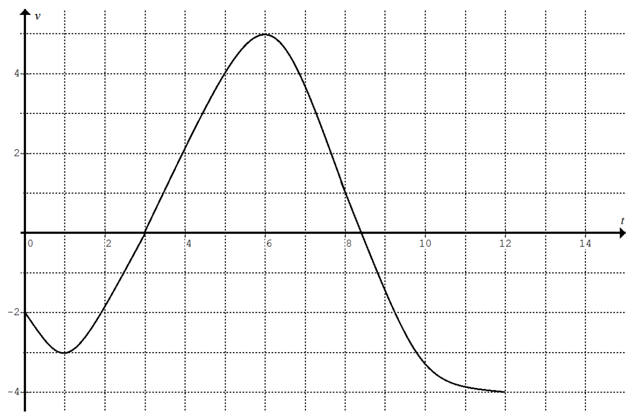
B. 1

C. 2

D. 3

E. 4

Question 15 A particle moves in a straight line. Its velocity is v m s^{-1} at time t s, positive value for forward motion and negative for backward. The *v*-*t* graph of the motion for $0 \le t \le 12$ is shown below.



The average speed (m s^{-1}) of the particle is closest to

A. -0.5

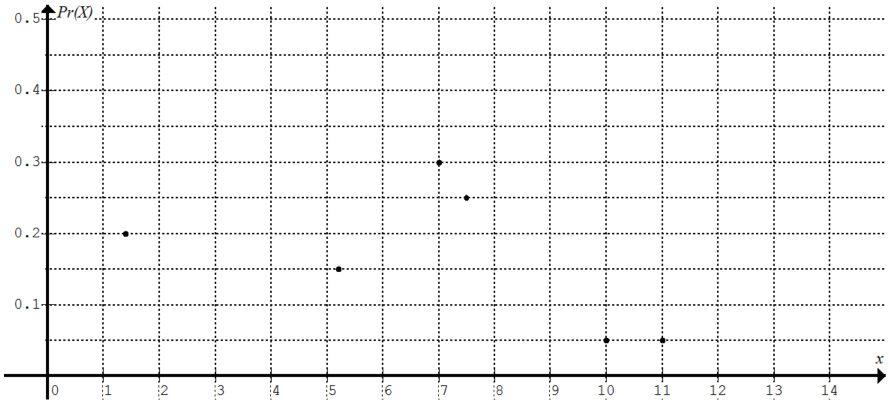
B. 0

C. 0.5

D. 1.5

E. 2.5

Question 16



The probability distribution of random variable X is shown in the graph above.
The value of $\Pr(3 < X < 11 \mid X > 5.5)$ is closest to

A. 0.90

B. 0.65

C. 0.61

D. 0.60

E. 0.59

Question 17 The random variable, X, has a probability density function given by

$$f(x) = \begin{cases} \frac{a}{2}(1 - \cos x), & 0 \le x \le 2\pi \\ 0, & \text{elsewhere} \end{cases}$$

$\Pr\left(\frac{\pi}{2} < X < \frac{3\pi}{2}\right)$ is equal to

A. $\frac{\pi + 2}{2\pi}$

B. $\frac{\pi - 2}{2\pi}$

C. $\frac{\pi + 1}{2\pi}$

D. $\frac{\pi - 1}{2\pi}$

E. $\frac{\pi - 1}{\pi}$

Question 18 A fair dice is rolled twice and the uppermost numbers are recorded as an ordered pair, e.g. outcome $(4,1)$ means getting 4 in the first roll and 1 in the second roll.
Which one of the following is **not** a random variable on the sample space of this probability experiment?

A. Difference of the numbers

B. Different uppermost numbers

C. Number of odd numbers

D. Sum of the uppermost numbers

E. Number of numbers less than 3

Question 19 Random variable X has a binomial probability distribution. The probability of $X = 0$ and the probability of $X = 1$ are shown in the following table. The other probabilities are **not** shown.

x	0	1
$\Pr(X = x)$	0.0001	$4(0.9)(0.001)$

The mean and standard deviation of X are respectively

A. 0.4 and 0.6

B. 0.4 and 0.36

C. 3.6 and 0.6

D. 3.6 and 0.36

E. 0.36 and 0.6

Question 20 60% of a city population favours renewable energy. Random samples of 150 persons each are to be selected and interviewed. Out of 20 such random samples the number of samples with sample proportion favouring renewable energy less than 0.56 is closest to

A. 1

B. 3

C. 7

D. 13

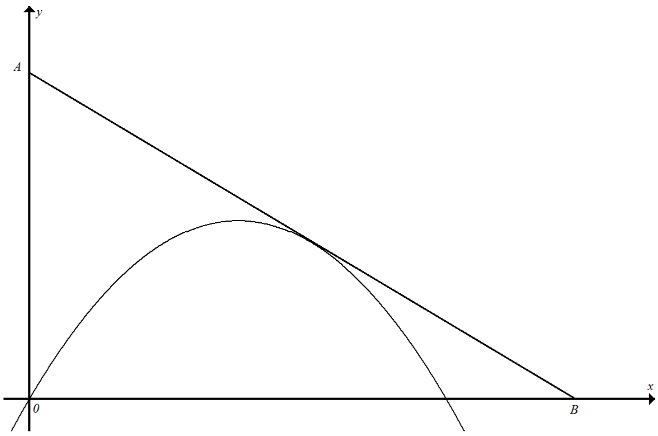
E. 17

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise stated.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Question 1



The diagram above shows two graphs. The curve is a parabola of equation $y = 2ax - x^2$ where $a > 0$.
The vertex of the parabola is $V(p, q)$. The straight line is a tangent to the parabola. The tangent cuts the *y*-axis at $A(0, c)$ and the *x*-axis at B, and $c > q$.

a. Find the values of p and q in terms of parameter a. 2 marks

b i. Show that $(m - 2a)^2 - 4c = 0$, where m is the gradient of the tangent. 2 marks

b ii. Hence show that $m = 2\left(a - \sqrt{c}\right)$ with clear reasoning. 2 marks

Let $a = 1$ in the following questions.

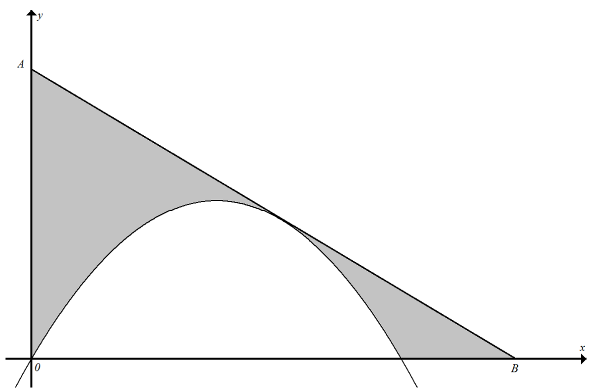
c i. Show that the equation of the tangent is $2(\sqrt{c}-1)x + y = c$. 1 mark

c ii. Hence find the coordinates of B in terms of parameter c. 1 mark

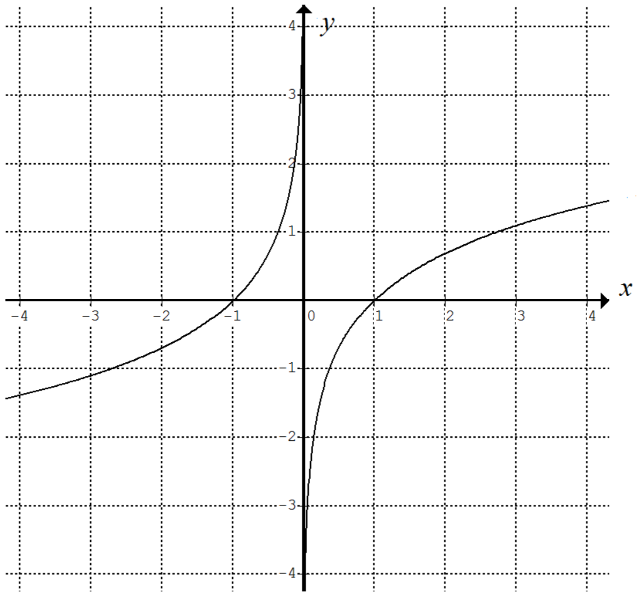
d. Show that the area of ΔAOB is $\dfrac{c^2(\sqrt{c}+1)}{4(c-1)}$. 2 marks

e. Find the exact value of the minimum area of ΔAOB and the exact value of c when it occurs. 3 marks

f. Find exact value of the minimum area of the shaded region in the following diagram. 2 marks



Question 2 The graphs of $f(x)=\log_e x$ and $g(x)=\log_e(-x)^{-1}$ are shown below.



a. Write down the transformation matrix which changes $g(x)=\log_e(-x)^{-1}$ to $f(x)=\log_e x$. 1 mark

b. Determine the exact area of the region bounded by $y=\pm 1$ and the two curves. 2 marks

c. Find $g'(x)$. 1 mark

d. $P(\alpha, \log_e \alpha)$ is a point on the graph of $f(x)$ and Q is a point on the graph of $g(x)$. In terms of α, find the coordinates of Q if the tangents at the two points have the same gradient.

2 marks

e. Show that the length of line segment PQ is $2\sqrt{\alpha^2 + (\log_e \alpha)^2}$. 2 marks

f. Show that the length of PQ is minimum when $\alpha^2 + \log_e \alpha = 0$. 2 marks

g. Find the minimum length of PQ and the corresponding value of α , correct to 3 decimal places.
2 marks

h. Show that PQ is a common normal to both curves when its length is minimum. 2 marks

i. Find the area of the largest square that can slide through the gap between the two curves, correct to 3 decimal places.
1 mark

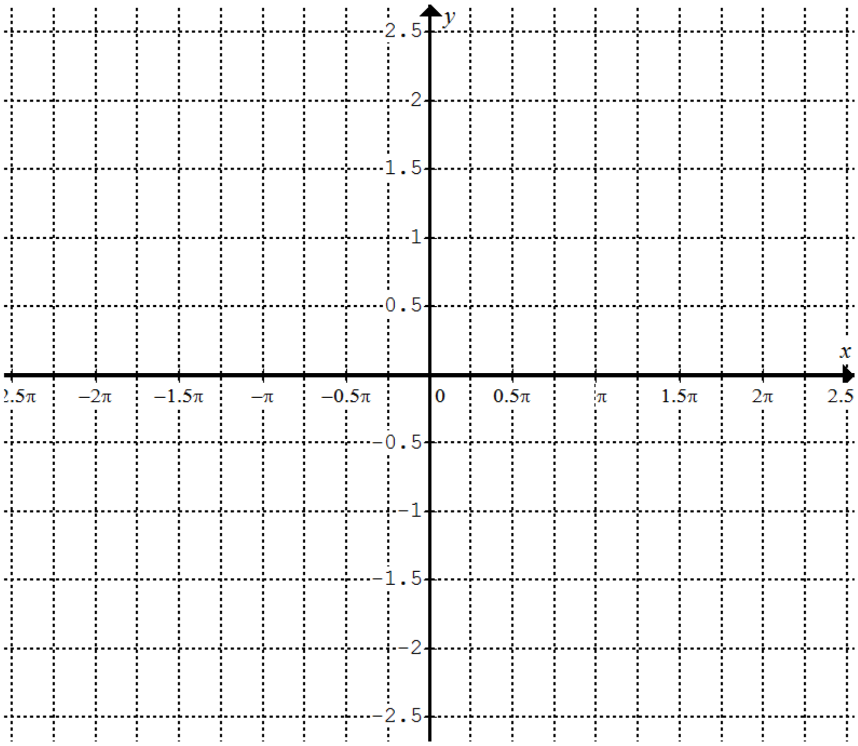
Question 3 Consider $f(x)=0.0012833x^4-0.0102664\pi^2x^2$, *assume that the given numerical values are exact.*

a. $f(x)$ can be expressed in the form $ax^2(x+b)(x-c)$ where a, b and c are positive real numbers. Find the exact values of a, b and c.

2 marks

b. Find the exact x-coordinates of the stationary points of $f(x)$. 3 marks

c. Sketch the graph of $y=f(x)$ for $-2\pi \le x \le 2\pi$ on the grid below. Showing stationary points, end points and axis intercepts. Label the graph as $y=f(x)$. 3 marks



d. Sketch $y=g(x)$ for $-2\pi \le x \le 2\pi$, where $g(x)=\cos\left(\frac{x}{2}\right)$, on the grid above.

Label the graph as $y=g(x)$. 1 mark

e. Given $g(x) \approx f(x)+1$ for $-2\pi \le x \le 2\pi$, determine the area of the region bounded by the graphs of $y = g(x)$ and $y = f(x)+1$, correct to 3 decimal places. 2 marks

f. Determine the x-intercepts of the graph of $y = f(x)+1$ for $-2\pi \le x \le 2\pi$, correct to 3 decimal places.

1 mark

g. The x-intercepts of the graph of $y = f(x)+1$ for $-2\pi \le x \le 2\pi$ coincide with the x-intercepts of the graph of $y = g(x)$ if $y = f(x)+1$ is dilated by factor m in the x direction.
Determine the value of m, correct to 3 decimal places. 2 marks

h. The equation of the dilated $y = f(x)+1$ is $y = \dfrac{0.0012833}{p}x^4 - \dfrac{0.0102664}{q}\pi^2 x^2 + 1$.
Find the values of p and q, correct to 3 decimal places. 2 marks

Question 4 A city has a large population of public transport users travelling to work.
The probability distribution of the time taken X (hours) to travel to work using public transport by each individual is given by $f(x)=\begin{cases} a\,x^2(x-1.5)^2e^x & \text{for } 0\le x\le 1.5 \\ 0 & \text{elsewhere} \end{cases}$

Round all numerical answers to 3 decimal places in this question.

a. Show that $a=1.793$. 1 mark

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b. Find the mean of X. 1 mark

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c. Find the median of X. 1 mark

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The proportion of the population going to work by public transports spending less than 0.500 hours travelling time is 0.137.

d i. Ten people going to work by public transports are randomly selected. Find the probability that less than 3 of them spending less than 0.500 hours travelling time.

1 mark

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d ii. Given at least one of them spending less than 0.500 hours travelling time, find the probability that less than 3 of them spending less than 0.500 hours travelling time.

2 marks

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___

___

e. For samples of size 100 from population going to work by public transports, estimate the mean and standard deviation of the sample proportion spending less than 0.500 hours travelling time.

2 marks

f. Let $\hat{P}$ be the random variable of the distribution of sample proportion spending less than 0.500 hours travelling time. Find $\Pr\left(\hat{P} < 0.1 \mid \hat{P} < 0.2\right)$.

2 marks

g. Find $\Pr\left(\hat{P} < 0.1 \mid \hat{P} < 0.2\right)$ for sample size of 10 and also for sample size of 200.
Comment on your answers.

2 marks

h. A random sample of size 100 from a large population going to work by public transports is taken from a second city. The sample proportion going to work by public transports spending less than 0.500 hours travelling time is 0.137. Find an approximate 95% confidence interval where $z \approx 1.96$.

2 marks

End of exam 2