Neap:

Trial Examination 2017

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 20 pages.

Formula sheet.

Answer sheet for multiple-choice questions.

Instructions

Write your **name** and **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in the booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The maximal domain of the function with rule $f(x) = \log_e(2 - 3x)$ is

A. $R\backslash\left\{\frac{2}{3}\right\}$

B. R

C. $\left(-\infty, \frac{2}{3}\right)$

D. $(0, \infty)$

E. $\left(\frac{2}{3}, \infty\right)$

Question 2

For $f(x) = 4 - x^2$, the average rate of change with respect to x for the interval $[0, 2]$ is

A. -4

B. -2

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

Question 3

The function with the rule $f(x) = 5\tan\left(\frac{2x}{3}\right)$ has a period of

A. $\frac{\pi}{2}$

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{2}$

D. 3π

E. 5π

Question 4

Let $f(x) = e^x$ and $g(x) = (x + 1)^2$.

If $h(x) = f(g(x))$, then the range of h is

A. $[-1, \infty]$

B. R

C. $(1, \infty)$

D. $(0, \infty)$

E. $[1, \infty)$

Question 5

Let $f : R \to R$, $f(x) = x^3 - bx^2 + 3x + 9$.

The graph of the curve $y = f(x)$ will have two stationary points when

A. $b \in (-3, 3)$

B. $b \in [-3, 3]$

C. $b \in (-\infty, -3) \cup (3, \infty)$

D. $b \in (-\infty, -3] \cup [3, \infty)$

E. $b \in R$

Question 6

The probability density function for the continuous random variable *X* is given by

$$f(x) = \begin{cases} -2x + 2 & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

E(*X*) is equal to

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. 1

D. $\frac{4}{3}$

E. 2

Question 7

Let $\int_{-1}^{0} \frac{4}{1-2x} dx = p$.

The value of $\frac{e^p}{2}$ is

A. 3

B. $\log_e(3)$

C. $\log_e\left(\frac{9}{2}\right)$

D. $\frac{9}{2}$

E. e^3

Question 8

Let $g : R^+ \to R,\ g(x) = \frac{4}{x} - 2$.

The average value of the function g over the interval [1, 3] is

A. $-\frac{4}{3}$

B. 0

C. $\log_e(3) - 1$

D. $2\log_e(3) - 1$

E. $\log_e\left(\frac{9}{e^2}\right)$

Question 9

The graph of the curve $y = f(x)$ has a local maximum at the point (6, 2).

The graph of the curve $y = -f(2x + 4) - 1$ must have a local minimum at the point

A. (10, 3)
B. (8, –3)
C. (16, –1)
D. (1, –3)
E. (2, –1)

Question 10

For a given event space, two events A and B are such that $\Pr(A) = \frac{1}{4}$ and $\Pr(B) = \frac{1}{8}$.

If A and B are mutually exclusive, then $\Pr(A' \cap B')$ equals

A. $\frac{1}{8}$

B. $\frac{1}{32}$

C. $\frac{5}{8}$

D. $\frac{21}{32}$

E. $\frac{31}{32}$

Question 11

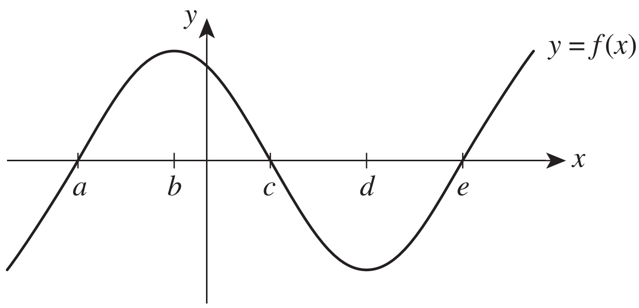
From a random sample of size n, a 95% confidence interval for the population proportion p is found to be (0.1, 0.2). A second random sample of size $4n$ is taken and found to have the same sample proportion as the first sample.

The 95% confidence interval for p for this second sample is

A. (0.1, 0.2)
B. (0.125, 0.175)
C. (0.1125, 0.1875)
D. (0, 0.4)
E. (0.4, 0.8)

Question 12

Part of the graph of $y = f(x)$ of the polynomial function f is shown below.



$f(x)$ is strictly increasing for

A. $(-\infty, b) \cup (d, \infty)$
B. $(a, c) \cup (e, \infty)$
C. (b, d)
D. R
E. $R \backslash (b, d)$

Question 13

The area of the region enclosed by the graph with equation $y = \sqrt{4-2x} - 5$ and the x- and y-axes is

A. $-\dfrac{10\sqrt{10}+53}{3}$

B. $\dfrac{27}{2}$

C. $\dfrac{22}{3}$

D. $\dfrac{21}{2}$

E. $\dfrac{125}{6}$

Question 14

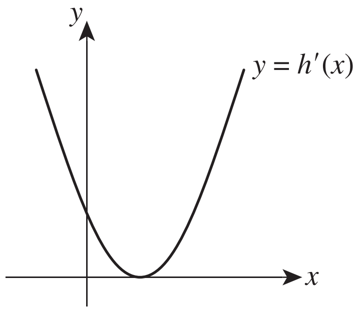
The continuous random variable X is normally distributed with a mean of 50 and a standard deviation of 8.

If $\Pr(X > b) = 0.025$, then $\Pr\left(X > \frac{b}{2}\right)$ correct to four decimal places is equal to

A. 0.0125
B. 0.9775
C. 0.9840
D. 0.9875
E. 1.0000

Question 15

The graph of the gradient function $h'(x)$ is shown below.



Which of the following statements **could** be true?

A. The graph of $y = h(x)$ is a straight line.
B. The graph of $y = h(x)$ has a turning point on the x-axis.
C. The graph of $y = h(x)$ has two x-intercepts.
D. The graph of $y = h(x)$ passes through the origin (0, 0).
E. The graph of $y = h(x)$ has a stationary point of inflection located on the y-axis.

Question 16

If $y = f(x)e^{g(x)}$ then $\frac{dy}{dx}$ is equal to

A. $f'(x)e^{g'(x)}$

B. $f'(x)e^{g(x)} + f(x)e^{g'(x)}$

C. $f'(x)e^{g(x)} + f(x)g'(x)e^{g'(x)}$

D. $e^{g(x)}(f'(x) + f(x)g'(x))$

E. $f'(x)e^{g(x)}$

Question 17

The equation of the tangent to the curve with an equation $y = e^{ax} + ax$ at the point where the curve crosses the y-axis is

A. $y = 2ax + 1$

B. $y = (ae^{a} + a)x - (a - 1)e^{a}$

C. $y = (ae^{a^2} + a)x - (a^2 - 1)e^{a^2}$

D. $y = (a + 1)x + 1$

E. $y = ax + 1$

Question 18

The equation $\{x : \sin^2(x) - 2\sin(x) + k = 0\}$ has no real solution when

A. $k \in (1, \infty)$

B. $k \in [1, \infty)$

C. $k = 1$

D. $k \in (-\infty, -3) \cup (1, \infty)$

E. $k \in (-3, \infty)$

Question 19

The discrete random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$\Pr(X = x)$	$0.4p^2$	$1 - 0.6p$	$0.2p$	q

Given that $\Pr(X = 3 | X > 1) = \dfrac{1}{2p + 1}$, then

A. $p = 0.5, q = 0.1$

B. $p = 0.2, q = 0.1$

C. $p = 0, q = 1$

D. $p = 1, q = 0$

E. $p = 0.2, q = 0.2$

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Question 20

The curve with equation $y = a\sin(bx) + c$ is mapped onto the curve with equation $y = 2\sin\left(x + \frac{\pi}{2}\right) - 3$ by the transformation $T : R^2 \rightarrow R^2$ defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\pi}{2} \\ 3 \end{bmatrix}$$

The values for a, b and c could be

A. $a = -2,\ b = \frac{1}{2},\ c = 6$

B. $a = 2,\ b = \frac{1}{2},\ c = 6$

C. $a = -2,\ b = 2,\ c = 6$

D. $a = -2,\ b = \pi,\ c = 6$

E. $a = 2,\ b = 2,\ c = 6$

END OF SECTION A

SECTION B

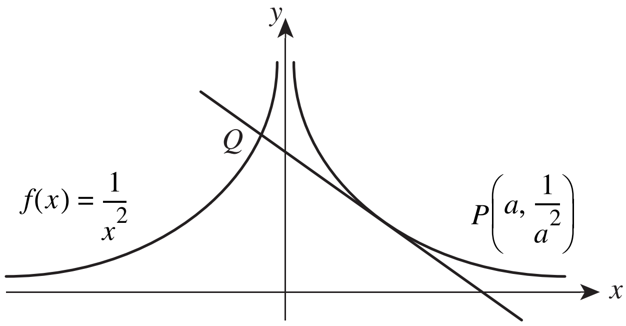
Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (8 marks)

Let $f : R\backslash\{0\} \to R, f(x) = \frac{1}{x^2}$. The point $P\left(a, \frac{1}{a^2}\right)$ is on the graph of f, where $a > 0$, as shown below.

The tangent at P cuts the graph of f at a second point Q as indicated.



a. Find the rule for the gradient function $f'(x)$. 1 mark

b. **i.** Find the equation of the tangent line at the point P. 1 mark

ii. Hence, show that the point Q has the coordinates $\left(-\frac{a}{2}, \frac{4}{a^2}\right)$. 2 marks

c. Find the minimum distance between the points P and Q. 4 marks

Question 2 (11 marks)

The number of worms in a garden bed varies according to the rule $n(t) = 1000 + 200\sin\left(\frac{\pi t}{6}\right)$, where n is the number of worms and t is the number of months after 1 January 2017.

a. Find the period and amplitude of the function n. 2 marks

b. What is the maximum population of worms? 1 mark

c. For how many months each year is the population greater than 1150 worms? Give your answer correct to two decimal places. 2 marks

The rule for the worm population can also be expressed with the function m where $m(t) = 1000 + 200\cos\left(\frac{\pi}{6}(t + d)\right)$ and $d > 0$.

d. **i.** Find the smallest value that d could take. 1 mark

ii. Find all values that d could take. 2 marks

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e. At the beginning of 2018, a new fertiliser is applied to the garden and the population can now be modelled by the function w where $w(t) = n(t) + q(t - 12)$ where $t > 12$ and $q \in R$.

For what value(s) of q will the worm population always be increasing? 3 marks

Question 3 (19 marks)

A school has a total of 1500 students of which 150 are known to ride to school everyday. The school takes a number of independent random samples of its students. Each sample contains 20 students. Let X be the random variable that represents the number of students in a sample that ride to school.

a. **i.** Find $\Pr(X < 3)$. Give your answer correct to four decimal places. 2 marks

ii. Given that less than 3 students in a sample ride to school, find the probability that none of the students in the sample ride to school. Give your answer correct to four decimal places. 2 marks

b. Find the smallest sample size that would need to be used to ensure the probability at least one student in the sample rides to school is at least 80%. 2 marks

Let $\hat{P}$ be the random variable of the distribution of sample proportions of students who ride to school.

c. **i.** Find the expected value of $\hat{P}$. 1 mark

ii. Find the standard deviation of $\hat{P}$ with a sample size of 20. Give your answer correct to four decimal places. 1 mark

iii. Find the new sample size required to halve the standard deviation of $\hat{P}$. 1 mark

For those students that do ride to school, the distance that each student rides is known to be normally distributed with a mean of 2 km and a standard deviation of 400 m.

d. **i.** Find the probability that a particular student who rides to school rides between 1.5 km and 2 km. Give your answer correct to four decimal places. 2 marks

ii. Hence, find the probability that a student rides at least 2 km, given that they ride at least 1.5 km. Give your answer to four decimal places. 3 marks

The school embarks on a campaign to increase the number of students who ride to school each day. At the end of the campaign they survey 100 students and find that 20 students ride to school.

e. An approximate 95% confidence interval for the population proportion that corresponds to this sample proportion is found to be $\left(\frac{a}{25}, \frac{b}{25}\right)$ where a and b are integers.

Find a and b. Use an integer multiple of the standard deviation in your calculations. 2 marks

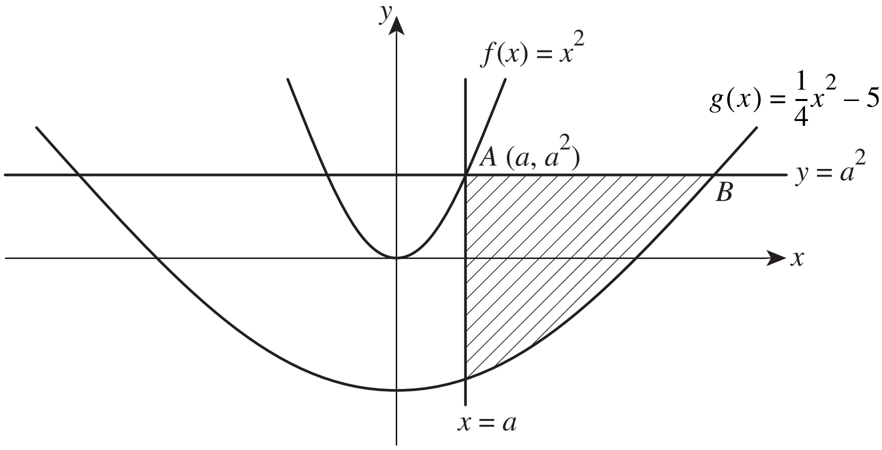
f. Of the 20 students in the sample that ride to school, 12 ride further than 2 km and 6 ride between 1.5 km and 2 km, and the distance they ride is normally distributed.

Find the new mean and standard deviation for the distance these students ride to school. Give your answer correct to four decimal places. 3 marks

Question 4 (7 marks)

Consider the function $f(x) = x^2$ and $g(x) = \frac{1}{4}x^2 - 5$.

Parts of the graphs of f and g are shown below, as is the line with equation $y = a^2$, where $a > 0$. Points A and B are the intersection points of the line with the curve as indicated.



a. Complete the correct sequence of transformations required to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. 2 marks

- dilation factor of __________ from the x-axis or alternatively a dilation factor of __________ from the y-axis
- translation of ____

b. Show that the point B has the coordinates $\left(2\sqrt{a^2 + 5}, a^2\right)$. 1 mark

___

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___

c. **i.** Find an expression in terms of a for the shaded area which is defined as the region bounded by the curve $y = g(x)$ and the lines $y = a^2$ and $x = a$. 2 marks

ii. Hence, find the coordinates of the point A that gives the minimum area of the region bounded by the curve $y = g(x)$ and the lines $y = a^2$ and $x = a$. 2 marks

Question 5 (15 marks)

a. **i.** State the largest value of k for which the function $h:(-\infty, k] \to R,\ h(x) = \frac{1}{2}x^2 + x - \frac{7}{2}$ has an inverse. 2 marks

ii. Find the rule for the inverse function $h^{-1}(x)$. 2 marks

iii. State the domain and range for $h^{-1}(x)$. 2 marks

b. Find the coordinates of the point of intersection of the curves $y = h(x)$ and $h^{-1}(x)$. 2 marks

Let $g(x) = h(x + b)$.

c. **i.** Find the rule for the function $g(x)$. 1 mark

ii. Find the rule for the inverse function $g^{-1}(x)$. 2 marks

d. For what value(s) of b will the graphs of $y = g(x)$ and $y = g^{-1}(x)$ have an intersection point at the origin (0, 0)? 2 marks

e. For what value(s) of b will the graphs of $y = g(x)$ and $y = g^{-1}(x)$ have no intersection points? 2 marks

END OF QUESTION AND ANSWER BOOKLET