



# THE SCHOOL FOR EXCELLENCE (TSFX)

## UNITS 3 & 4 MATHEMATICAL METHODS 2017

### WRITTEN EXAMINATION 1

Reading Time: 15 minutes

Writing Time: 1 hour

### QUESTION AND ANSWER BOOKLET

**Student Name:**

#### Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### **Materials Supplied**

- Question and answer book of 9 pages.
- A separate sheet of miscellaneous formulas.
- Working space is provided throughout the book.

#### **Instructions**

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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### Instructions

- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

#### QUESTION 1 (5 marks)

(a) Find  $\frac{dy}{dx}$  if  $y = x^3 \log_e(4x)$ .

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2 marks

(b) If  $f(x) = \cos^2(3x)$  find  $f'\left(\frac{\pi}{12}\right)$ .

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3 marks



**QUESTION 4** (6 marks)

- (a) Describe the transformations required (in the correct order) to change the graph with equation  $f(x) = 2x^2 - 4x - 6$  to the graph with equation  $g(x) = x^2$ .

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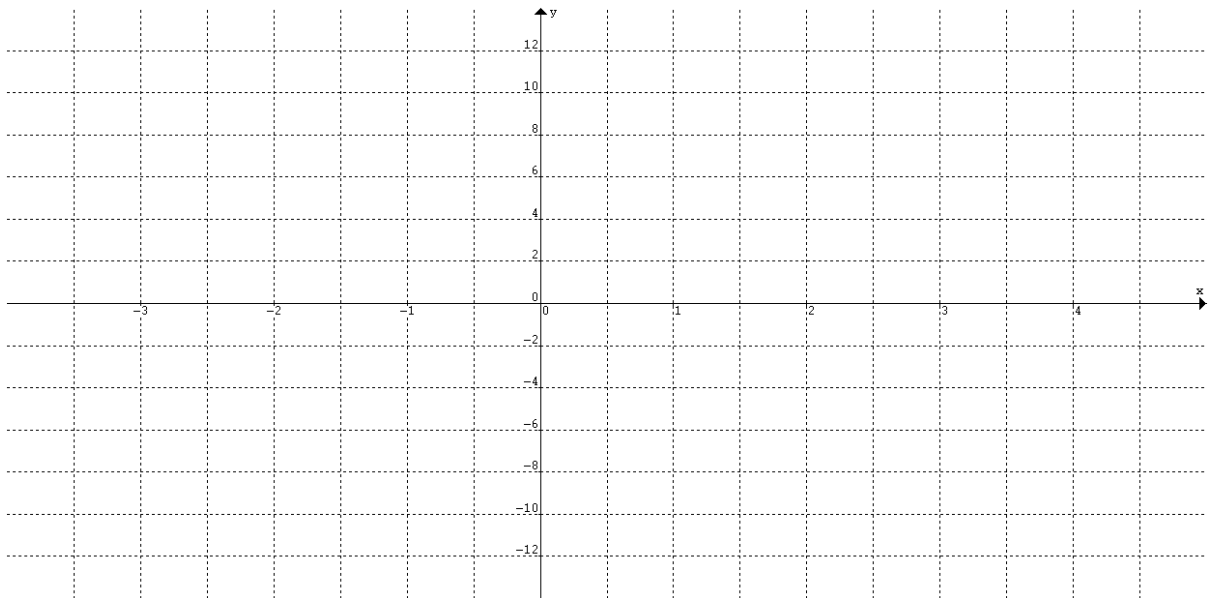
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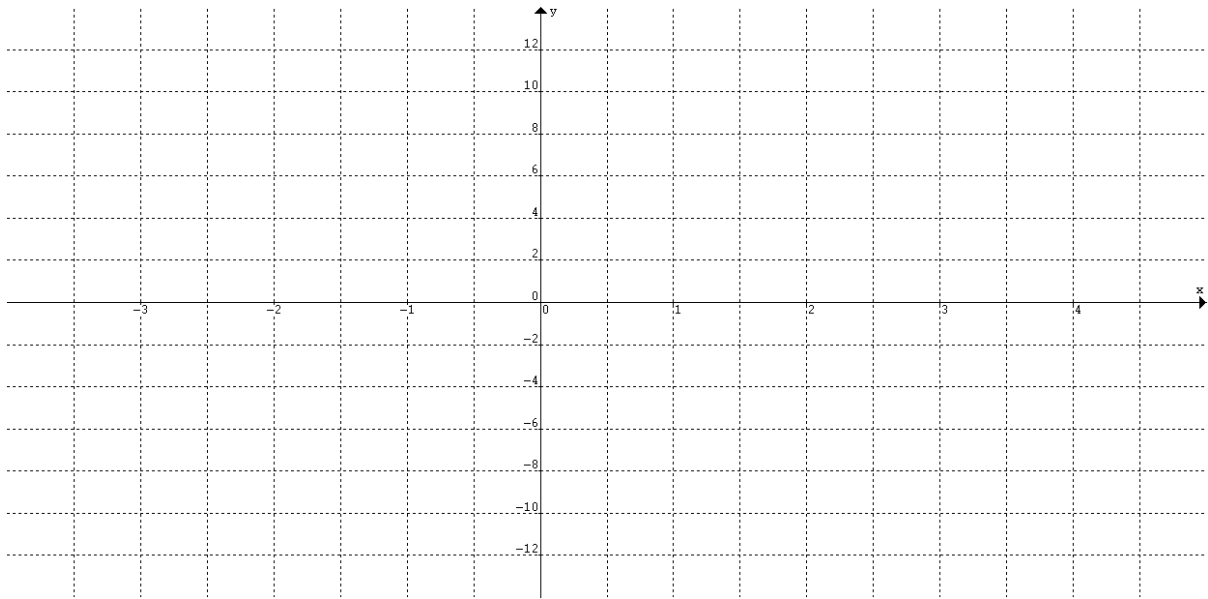
2 marks

- (b) Sketch the graph of  $y = h(x)$  where  $h(x) = \begin{cases} f(x), & (-2, -1] \cup [3, 4] \\ -f(x), & (-1, 3) \end{cases}$  on the axes below, showing the coordinates of all of key points.



2 marks

- (c) Sketch the graph of  $y = h'(x)$  on the axes below, showing the coordinates of any intercepts with the axes.



2 marks

**QUESTION 5** (4 marks)

Consider the simultaneous linear equations where  $m \in R$ .

$$(m+1)x + 9y = 21$$

$$2x + (m-2)y = 7$$

- (a) Find the value(s) of  $m$  for which there is no unique solution for  $x$  and  $y$ .

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2 marks

- (b) (i) State the value of  $m$  for which there is an infinite set of solutions.

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1 mark

- (ii) If  $x = \lambda$  (where  $\lambda \in R$ ) is one solution for the value of  $m$  found in (b) part i, write down the corresponding value of  $y$  in terms of  $\lambda$ .

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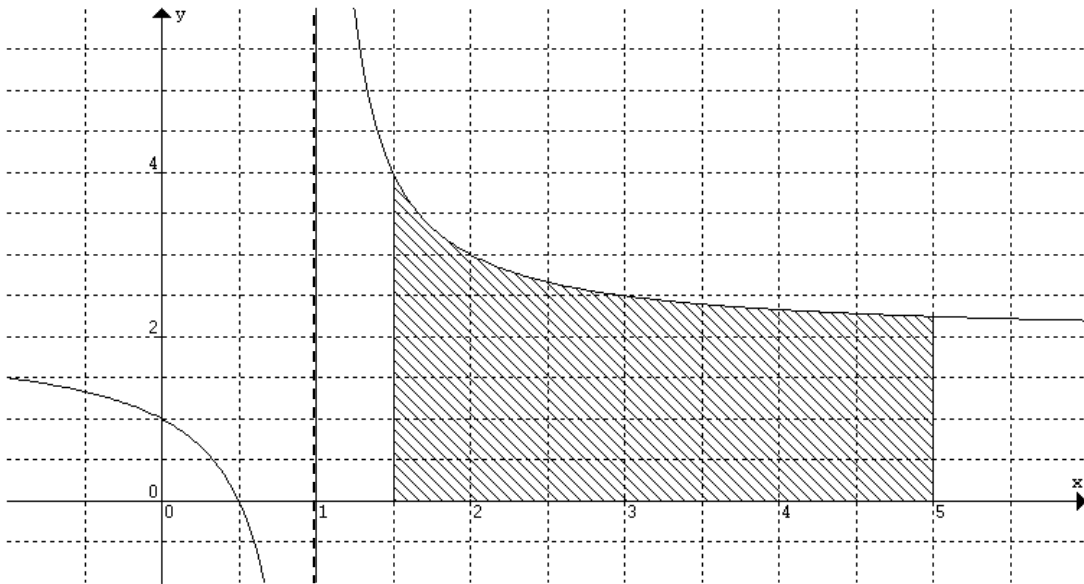
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1 mark

**QUESTION 6** (4 marks)

Part of the graph of  $y = 2 + \frac{1}{x-1}$  is shown below.



The shaded area between the curve from  $x = \frac{3}{2}$  to  $x = 5$  and the  $x$  axis is given by the value  $A + B \log_e 2$ , where  $A$  and  $B$  are positive integers. Find the values of  $A$  and  $B$ .

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4 marks



**QUESTION 7** (5 marks)

The proportion,  $X$ , of arsenic found in a randomly selected sample of a toxic chemical is a random variable having a probability distribution function given by

$$f(x) = \begin{cases} ax^4(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

where  $a$  is a positive integer.

- (a) Show that  $a = 30$ .

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1 mark

- (b) Find the mean of the distribution.

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2 marks

- (c) Find the mode of the distribution.

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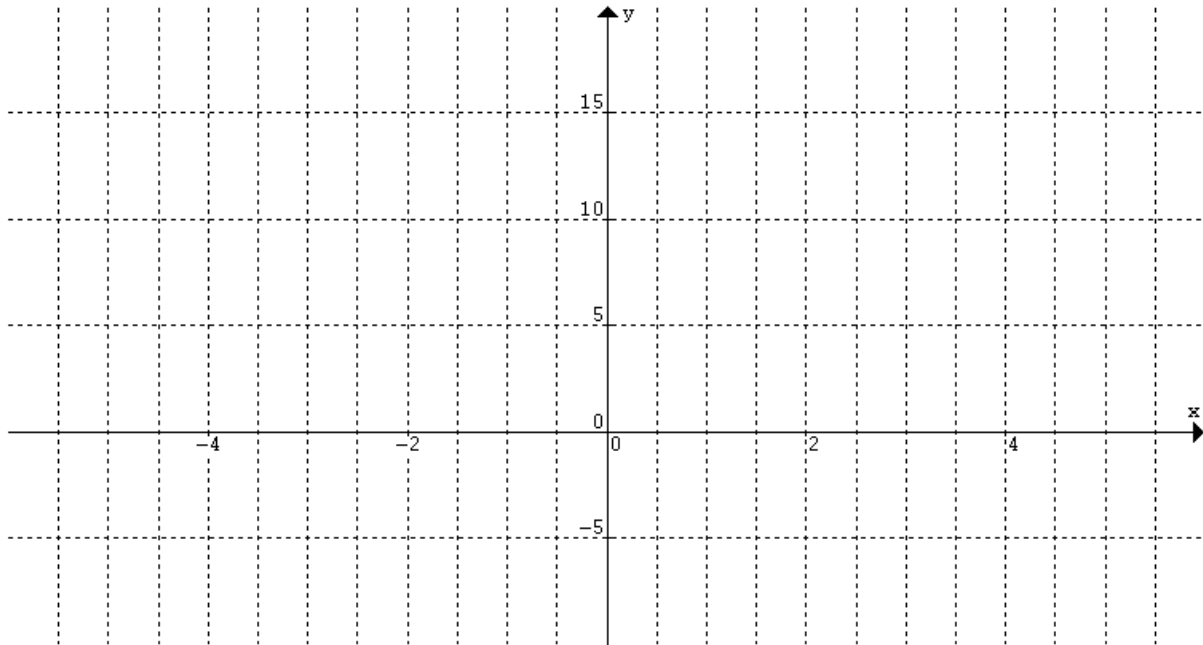
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2 marks

**QUESTION 8** (3 marks)

Consider the functions  $f(x) = x^2 - 4x$  and  $g(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ . The domain for both functions is  $R$ .

(a) Sketch the graph of  $f(g(x))$  on the axes below.



2 marks

(b) State the domain of  $f'(g(x))$ .

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1 mark

**QUESTION 9** (6 marks)

The number of deaths following heart surgery performed by a particular surgeon in the previous month was 9 from a total of 15 operations. The national average death rate from the same type of heart surgery was 13%.

- (a) Calculate the sample proportion and state what this value represents.

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2 marks

- (b) Calculate a 95% confidence interval to estimate the population proportion.

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2 marks

- (c) Can the Normal approximation be used for the distribution of the sample proportion? Give a reason for your answer.

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2 marks

**END OF QUESTION AND ANSWER BOOKLET**