2017 VCE Mathematical Methods 2 examination report

General comments

There were some excellent responses to the 2017 Mathematical Methods 2 examination and most students were able to attempt the four questions in Section B. Question 1 was very well answered. Some students found Questions 2f., 2g., 2h., 3g, 4h. and 4i. challenging.

Many students wrote their answers in the correct form. Exact answers needed to be given unless otherwise specified. Approximate answers were usually required in the probability questions. Most students answered everything that was required within a question. It is important for students to re-read questions as often they require more than one piece of information in the response.

Advice to students

- Technology should be updated well before the examination.
- Technology is best put in mode radians. Occasionally the mode may need to be changed according to the question, such as in Question 2d.
- Check that answers make sense. By looking at the graph in Question 1bi., the gradient was negative and likewise in Question 1d., the *a* value was positive.
- Define functions on the technology at the start of each question in Section B. This saves time, especially when dealing with probability questions that involve hybrid functions, such as Question 3.
- If the functions have been defined at the start of the question, it is acceptable to use the function name, such as $f(x)$, throughout the question rather than writing out the entire expression. This avoids missing out on marks if brackets are not inserted when finding the area between two curves, for example, in Question 1dii. and Question 4c. It also saves time and avoids transcription errors.
- Show a method for questions worth more than one mark. A small number of students did not show their method in the probability question, Question 3.
- Take time when sketching graphs, like in Question 3a. If it is a linear graph use a ruler. Check that the points have been positioned correctly if a grid has been given. Sketch along an axis if the function is defined for those points.
- Use brackets with questions involving logarithms, such as Question 4b., $y = \log_2(x+2) - 1$.
- Learn the correct wording to describe transformations. Not knowing this led to students making errors in Question 4g.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding, resulting in a total more or less than 100 per cent.

Section A

The table below indicates the percentage of students who chose each option. The correct answer is indicated by the shading.

Question	% A	% B	% C	% D	% E	% No answer	Comments
1	4	1	92	2	1	0	
2	1	5	12	80	1	0	
3	2	2	83	12	1	0	
4	3	10	6	7	75	0	
5	47	20	9	16	7	1	95% confidence interval is (0.039, 0.121). The sample proportion is in the middle of the confidence interval. $0.039+\frac{0.121-0.039}{2}=0.080$
6	5	3	88	3	1	0	
7	19	32	12	29	7	1	$(p-1)x^2+4x=5-p$ $(p-1)x^2+4x-5+p=0$ The discriminant is negative for no real solutions. $16-4(p-1)(p-5)<0$ $-4p^2+24p-4<0$ Divide by -4 and change the inequality. $p^2-6p+1>0$
8	64	14	6	11	5	0	
9	78	4	6	8	4	0	
10	9	23	6	47	14	0	$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ $x'=2x,\ x=\frac{x'}{2}$ $y'=\frac{1}{3}y,\ y=3y'$ $y=3\sin\left(2\left(x+\frac{\pi}{4}\right)\right)$ $3y'=3\sin\left(2\left(\frac{x'}{2}+\frac{\pi}{4}\right)\right)$ $y'=\sin\left(x'+\frac{\pi}{2}\right)$ $y'=\cos(x')$
11	10	5	10	72	3	0	

Question	% A	% B	% C	% D	% E	% No answer	Comments
12	11	18	45	15	11	1	$\sin(2x)=\frac{\sqrt{3}}{2}$ $2x=\frac{\pi}{3},\frac{2\pi}{3}...$ $x=\frac{\pi}{6},\frac{\pi}{3}...$ $x=-\frac{5\pi}{6},-\frac{2\pi}{3},\frac{\pi}{6},\frac{\pi}{3}$ $\text{sum}:-\frac{5\pi}{6}+-\frac{2\pi}{3}+\frac{\pi}{6}+\frac{\pi}{3}=-\pi$
13	12	14	17	10	46	1	$h(x)=\frac{1}{x-1}$ $(h(x))^2\neq h\left(x^2\right)$ $\left(\frac{1}{x-1}\right)^2=\frac{1}{x^2-2x+1}\neq\frac{1}{x^2-1}$
14	14	10	9	62	4	1	
15	14	58	7	9	11	0	
16	12	15	41	17	13	2	$X\sim\text{Bi}(5,p)$ $\Pr(X=0)=\binom{5}{0}p^0(1-p)^5=\frac{1}{243},\ p=\frac{2}{3}$ $\Pr(X>3)=\Pr(X\geq4)=0.4609$, correct to four decimal places

Question	% A	% B	% C	% D	% E	% No answer	Comments
17	3	37	21	21	17	0	$\text{Area} = \int_a^b f(x)dx - \int_b^c f(x)dx + \int_c^d f(x)dx$ $= 2\int_a^b f(x)dx - \int_b^c f(x)dx$ $= 2\int_a^b f(x)dx - 2\int_b^{b+c} f(x)dx$, as $b + c = 0,$ since $f(-x) = f(x)$
18	10	16	25	38	9	2	$np = \sqrt{np(1-p)}$ $n^2p^2 = np(1-p)$ $np\left(np - 1 + p\right) = 0, np \neq 0$ $np = 1 - p, p = \frac{1}{n+1}$ $\frac{1}{n+1} \leq 0.01,\ n \geq 99$
19	6	17	9	59	8	1	
20	8	47	18	18	9	1	$\cos(x) = \sqrt{3}\sin(x)$ $\tan(x) = \frac{1}{\sqrt{3}},\ x = \frac{\pi}{6}$ $B\left(\frac{\pi}{6},\ \frac{\sqrt{3}}{2}\right)$ $A_{\text{triangle}} = \frac{\pi}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\pi}{8}$ $A_{\text{shaded}} = \int_0^{\frac{\pi}{6}} \sqrt{3}\sin(x)dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(x)dx = \sqrt{3} - 1$ $A_{\text{shaded}} : A_{\text{triangle}}$ $\sqrt{3} - 1 : \frac{\sqrt{3}\pi}{8}$

Section B

Question 1a.

Marks	0	1	2	Average
%	7	19	74	1.7

$f: R \to R,\ f(x) = x^3 - 5x$, solve $f'(x) = 0$ for x, $x = \pm\frac{\sqrt{15}}{3}$, turning points

$\left(\frac{-\sqrt{15}}{3}, \frac{10\sqrt{15}}{9}\right), \left(\frac{\sqrt{15}}{3}, \frac{-10\sqrt{15}}{9}\right)$

This question was answered well. Exact answers were required to obtain full marks. Some students gave their answers as (–1.29, 4.3) and (1.29, –4.3). Others gave only the x values. Some mixed up the signs.

Question 1bi.

Marks	0	1	2	Average
%	16	9	75	1.6

gradient $\frac{f(1) - f(-1)}{1 - -1} = -4,\ y + 4 = -4(x - 1),\ y = -4x$

This question was answered well. A common incorrect answer was $y = 4x$. By inspection of the graph, the gradient was negative. Some students made arithmetic errors when calculating the gradient and/or *y*-intercept.

Question 1bii.

Marks	0	1	Average
%	22	78	0.8

$d = \sqrt{(x_2 - x_1)^2 + \left(f(x_2) - f(x_1)\right)^2} = \sqrt{(1 - -1)^2 + \left(f(1) - f(-1)\right)^2} = \sqrt{68} = 2\sqrt{17}$

The distance formula was used well. Some students applied this by hand and made arithmetic errors. Others had an incorrect distance formula using a multiplication operation between the two brackets instead of a plus. A common incorrect answer was $\sqrt{64}$.

Question 1ci.

Marks	0	1	2	Average
%	17	20	64	1.5

$d = \sqrt{(x_2 - x_1)^2 + \left(g(x_2) - g(x_1)\right)^2} = \sqrt{(1 - (-1))^2 + \left(g(1) - g(-1)\right)^2} = 2\sqrt{k^2 - 2k + 2}$

As in Question 1bii. some students did their solutions by hand and made arithmetic errors, especially sign errors. This would have been time consuming. $\sqrt{2^2 + (2 - 2k)^2} = 2 + 2 - 2k$ was sometimes given. Some incorrect answers contained x. When defining $g(x) = x^3 - kx$ on the technology, a multiplication sign must be inserted between k and x.

Question 1cii.

Marks	0	1	Average
%	37	63	0.7

Solve $2\sqrt{k^2-2k+2}=k+1$ for k, $k=1$ or $k=\frac{7}{3}$

Students who answered Question 1ci. correctly were generally able to answer this question. Some students gave only one value for *k*.

Question 1di.

Marks	0	1	Average
%	38	62	0.6

Solve $g(x)=x$ for x, $a=\sqrt{k+1}$, as $a>0$

A common incorrect answer was $a=\pm\sqrt{k+1}$. By inspection of the graph, the answer was positive. Some students found *k* in terms of *a*, instead of *a* in terms of *k*.

Question 1dii.

Marks	0	1	2	Average
%	40	18	42	1

$$\int_0^{\sqrt{k+1}}\left(x-g(x)\right)dx=\frac{(k+1)^2}{4}$$

$\int_0^{\sqrt{k+1}}\left(x-x^3-kx\right)dx$ was a common error, leaving out the brackets in $\int_0^{\sqrt{k+1}}\left(x-(x^3-kx)\right)dx$. To avoid these errors it would have been better to use the expression $\int_0^{\sqrt{k+1}}\left(x-g(x)\right)dx$. Some students overcomplicated the question by breaking up the areas into different sections. The easiest approach was to use ‘upper function subtract lower function’. There was evidence that students substituted $k+1$ instead of $\sqrt{k+1}$ and this resulted in the answer of $\frac{-(k-3)(k+1)}{4}$.

Question 2a.

Marks	0	1	Average
%	11	89	0.9

$h(t)=65-55\cos\left(\frac{\pi t}{15}\right)$, range $[-55+65,55+65]=[10,120]$, minimum height is 10 m and maximum height is 120 m

This question was well answered.

Question 2b.

Marks	0	1	Average
%	16	84	0.9

Period = $\frac{2\pi}{\left(\frac{\pi}{15}\right)} = 30$. He was in the capsule for 30 minutes.

A common incorrect answer was 15 minutes.

Question 2c.

Marks	0	1	2	Average
%	29	42	29	1

$h'(t) = \frac{11\pi}{3}\sin\left(\frac{\pi t}{15}\right)$, solve $h'(t) = \frac{11\pi}{3}$ for t, t = 7.5 minutes

Most students were able to find the derivative. There were occasions when $\frac{y_2 - y_1}{x_2 - x_1}$ was attempted (average rate of change). Some students had their technology in degree instead of radian mode, giving $h'(t) = \frac{11\pi^2 \sin\left(\frac{\pi t}{15}\right)}{540}$. Many could not find the maximum rate of change. A common incorrect answer was 15 minutes. Many found the value of *t* for the maximum value of *h*. Others gave a general solution or two *t* values.

Question 2d.

Marks	0	1	Average
%	64	36	0.4

$\tan(\theta) = \frac{65}{500}$, $\theta = 7.41°$, correct to two decimal places

Many students knew to get $\tan^{-1}\left(\frac{65}{500}\right)$ but they did not specify 'degree' for their technology. A common incorrect answer was $\theta = \tan^{-1}\left(\frac{55}{500}\right) = 6.28°$. Some used $\theta = \tan\left(\frac{65}{500}\right)$ instead of $\theta = \tan^{-1}\left(\frac{65}{500}\right)$. Others used $\theta = \sin^{-1}\left(\frac{65}{500}\right)$.

Students should be familiar with the relevant functionality for the context and select it appropriately.

Question 2e.

Marks	0	1	Average
%	10	91	0.9

$\frac{dy}{dx} = \frac{-x}{\sqrt{3025 - x^2}}$

This question was answered well. Some students wrote $\frac{dy}{dx}=\frac{x}{\sqrt{3025-x^2}}$. There was no need to rationalise the denominator.

Question 2f.

Marks	0	1	2	3	Average
%	67	21	4	8	0.5

$m_{P_2B}=\frac{-u}{\sqrt{3025-u^2}}$ or $\frac{-\sqrt{3025-u^2}-65}{500-u}$, solve $\frac{-u}{\sqrt{3025-u^2}}=\frac{-\sqrt{3025-u^2}-65}{500-u}$ for u

$u=12.9975...=13.00$, $v=118.4421...=118.44$, correct to two decimal places

Many students were able to find the gradient of the line segment in terms of *u*, using their answer from Question 2e. Others used *h*(*t*) or $y=\sqrt{3025-x^2}$ instead of $y=\sqrt{3025-x^2}+65$. Many students were unable to find the second gradient expression where they were required to use rise over run for the line segment P_2B.

Question 2g.

Marks	0	1	Average
%	93	7	0.1

$\alpha=\tan^{-1}\left(\frac{12.9975...}{\sqrt{3025-(12.9975...)^2}}\right)=13.67°$, correct to two decimal places

Some students used radians instead of degrees.

Question 2h.

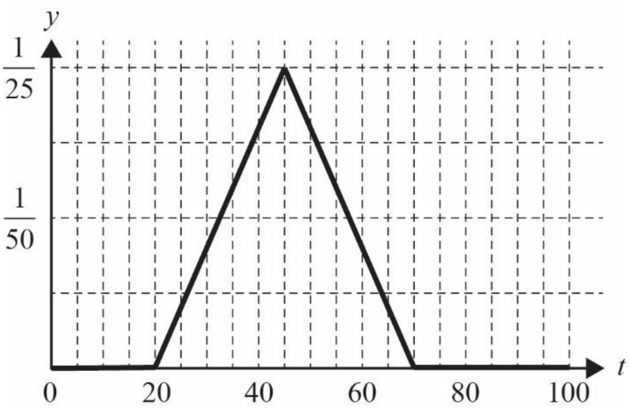
Marks	0	1	2	Average
%	94	5	2	0.1

Angle difference = $\theta=90-(13.669...-7.406...)=83.737...°$, $\frac{83.737...}{360}\times 30=6.978...=7$ minutes, to the nearest minute

This question was not answered well. Some students wrote 7 minutes without showing any working. As indicated in the instructions on the examination, for questions worth more than one mark, appropriate working must be shown.

Question 3a.

Marks	0	1	2	3	Average
%	15	12	44	29	1.9



Many students did not draw their graphs along the *t*-axis, ignoring *f*(*t*) = 0. Some had an open circle at (45, 0.04). Others had an open circle over a closed circle at (45, 0.04). Many students did not use rulers to draw the line segments. Some graphs looked like parabolas.

Question 3b.

Marks	0	1	2	Average
%	24	6	70	1.5

$$\int_{25}^{55} \left(f(t) \right) dt = \frac{4}{5}$$

This question was answered well. Some students had the incorrect terminals. 44 instead of 45 was occasionally given, for example, $\int_{25}^{44} \left(f(t) \right) dt + \int_{44}^{55} \left(f(t) \right) dt$. Others used 20 as the lower limit instead of 25.

Question 3c.

Marks	0	1	2	Average
%	21	30	49	1.3

$$\Pr\left(T \le 25 | T \le 55\right) = \frac{\Pr\left(T \le 25\right)}{\Pr\left(T \le 55\right)} = \frac{\int_{20}^{25} f(t) dt}{\int_{20}^{55} f(t) dt} = \frac{1}{41}$$

Many students were able to use the conditional probability formula. A common incorrect answer was $\frac{1}{40}$.

Question 3d.

Marks	0	1	2	Average
%	59	11	30	0.7

$\int_a^{70}\left(f(t)\right)dt = 0.7$ or $\int_{20}^{a}\left(f(t)\right)dt = 0.3$ or $\int_a^{45}\left(f(t)\right)dt = 0.2,\ a = 39.3649$, correct to four decimal places

A number of correct approaches were used. $\int_{20}^{a} f(t)dt = 0.7$, *a* = 50.6351 was a common incorrect answer. $\int_a^{75}\frac{1}{625}(70-t)dt = 0.7$ was occasionally given. Some students attempted to use the inverse normal as a method.

Question 3ei.

Marks	0	1	2	Average
%	26	16	58	1.3

$X \sim \text{Bi}\left(7, \frac{8}{25}\right),\ \Pr(X > 3) = 0.1534$, correct to four decimal places

Many students recognised that the distribution was binomial and gave the correct *n* and *p* values. Some used $\Pr(X \geq 3)$.

Question 3eii.

Marks	0	1	2	Average
%	31	11	59	1.3

$\Pr(X \geq 2 \mid X \geq 1) = \frac{\Pr(X \geq 2)}{\Pr(X \geq 1)} = \frac{0.7113...}{0.93277...} = 0.7626$, correct to four decimal places

Many students were able to set up the conditional probability. Some wrote $\Pr(X \geq 2 \mid X \geq 1) = \frac{\Pr(X > 2)}{\Pr(X > 1)}$. Others rounded incorrectly, giving 0.7625 as the answer.

Question 3f.

Marks	0	1	2	Average
%	60	8	32	0.7

$q(p) = \binom{7}{2}p^2(1-p)^5 + \binom{7}{3}p^3(1-p)^4 = 7p^2(p-1)^4(2p+3)$

Of those who attempted this question, some students did not realise that the binomial distribution was required.

Question 3gi.

Marks	0	1	2	Average
%	64	13	23	0.6

Solve $q'(p) = 0$ for p, $p = 0.3539,\ q = 0.5665$, correct to four decimal places

Some students knew to solve $q'(p)=0$ if they had an equation in Question 3f. Others found only *p*. Some gave exact values for their answers.

Question 3gii.

Marks	0	1	2	Average
%	91	3	7	0.2

$\int_d^{70} f(t)dt = 0.35388...$ $d = 48.9676.... =$ 49 minutes, to the nearest minute

Some students used *q* instead of *p* in their equation, solving $\int_d^{70} f(t)dt = 0.56646...$ for *d*. Others solved $\int_{20}^{d} f(t)dt = 0.35388...$, obtaining *d* = 41 minutes.

Question 4a.

Marks	0	1	2	Average
%	24	28	48	1.3

$f(x)=2^{x+1}-2$, the graph of $y=2^x$ has been translated one unit to the left and two units down to get the graph of *f*, $c=-1,\ d=-2$ or $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} c \\ d \end{bmatrix}$, $x'=x+c, x=x'-c$, $y'=y+d, y=y'-d$,

$y'-d=2^{x'-c}, y'=2^{x'-c}+d$, $c=-1,\ d=-2$

This question was answered reasonably well. Some students made sign errors.

Question 4b.

Marks	0	1	2	Average
%	12	25	64	1.5

Let $y=2^{x+1}-2$, inverse swap *x* and *y*, $x=2^{y+1}-2$, $x+2=2^{y+1}$, $y=\log_2(x+2)-1$

$f^{-1}:(-2,\infty)\to R,\ f^{-1}(x)=\log_2(x+2)-1$

There are other acceptable expressions for the inverse. Some students found the rule but did not give the domain. Some students did not use brackets, leaving their answer as $y=\log_2 x+2-1$.

Question 4c.

Marks	0	1	2	3	Average
%	22	10	20	49	2

Solve $f(x)=f^{-1}(x),\ x=-1$ or $x=0$, $2\int_{-1}^{0}(x-f(x))\,dx=\int_{-1}^{0}\left(f^{-1}(x)-f(x)\right)dx=\frac{-2}{\log_e(2)}+3$

$\int_{-1}^{0}\left(f(x)-f^{-1}(x)\right)dx$ and $\int_{-1}^{0}\left(f^{-1}(x)-2^{x+1}-2\right)dx$ were common incorrect expressions.

$\int_{-1}^{0}\left(f^{-1}(x)-2^{x+1}-2\right)dx$ should be written as $\int_{-1}^{0}\left(f^{-1}(x)-(2^{x+1}-2)\right)dx$. To avoid this type of error it is better to use the expression $\int_{-1}^{0}\left(f^{-1}(x)-f(x)\right)dx$. An exact answer was required. 0.1196 was often given.

Question 4d.

Marks	0	1	2	Average
%	26	13	62	1.4

$f'(0)=2\log_e(2)$, $f^{-1\prime}(0)=\frac{1}{2\log_e(2)}$

This question was answered reasonably well.

Question 4e.

Marks	0	1	Average
%	35	65	0.7

$g_k(x)=2e^{kx}-2$, solve $g_k(x)=f(x)$ for k, $k=\log_e(2)$

This question was answered reasonably well.

Question 4f.

Marks	0	1	Average
%	42	58	0.6

$g_k^{-1}(x)=\frac{1}{k}\log_e\left(\frac{x+2}{2}\right)$

Some students used their answer to Question 4e. $g_k^{-1}(x)=\frac{1}{\log_e(2)}\log_e\left(\frac{x+2}{2}\right)$ was a common incorrect response.

Question 4gi.

Marks	0	1	Average
%	69	31	0.3

$g_1(x)=2e^{x}-2$, $g_k(x)=2e^{kx}-2$, dilation of a factor of $\frac{1}{k}$ from the y-axis

Many students were unable to describe the transformation correctly, for example, 'dilation of a factor $\frac{1}{k}$ in the *y*-axis'. Others put their answer in terms of $\log_e(2)$ instead of *k*. Some gave two transformations.

Question 4gii.

Marks	0	1	Average
%	70	30	0.3

$g_1^{-1}(x) = \log_e\left(\frac{x+2}{2}\right)$, $g_k^{-1}(x) = \frac{1}{k}\log_e\left(\frac{x+2}{2}\right)$, dilation of a factor of $\frac{1}{k}$ from the *x*-axis

Students who answered Question 4gi. correctly tended to answer this question well.

Question 4h.

Marks	0	1	2	Average
%	77	21	2	0.3

$g_k(x) = 2e^{kx} - 2$, $g_k^{-1}(x) = \frac{1}{k}\log_e\left(\frac{x+2}{2}\right)$, $L_1 : y = 2kx$, $L_2 : y = \frac{x}{2k}$,

$2k = \tan(60°) = \sqrt{3}$ or $2k = \tan(30°) = \frac{1}{\sqrt{3}}$ $k = \frac{\sqrt{3}}{2}$ or $k = \frac{1}{2\sqrt{3}}$

This question was not answered well. Some students found one answer only. Others gave approximate answers.

Question 4ii.

Marks	0	1	2	Average
%	95	2	3	0.1

Solve $2k = \frac{1}{2k}$ for *k*, $k = \frac{1}{2}$, $k > 0$, $p = \frac{1}{2}$

Many students tried to solve $g_k(x) = g_k^{-1}(x)$ and then attempted to find the discriminant.

Question 4iii.

Marks	0	1	Average
%	98	2	0

As $k > \frac{1}{2}$ the graphs of g_k and g_k^{-1} will intersect in the third quadrant, $\lim_{k\to\infty}\int_{-2}^{0}\left(x - f(x)\right)dx = 4$, $b = 4$, as g_k^{-1} has a vertical asymptote with equation $x = -2$ and g_k has a horizontal asymptote with equation $y = -2$, the area will approach 4 as *k* increases.

This question was not answered well.