Year 2018
VCE
Mathematical Methods
Trial Examination 1

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Victorian Certificate of Education
2018

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 17 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{\cos(2x)}{2x}$, find $\frac{dy}{dx}$.

2 marks

b. Let $g(x) = e^{\sqrt{x}}$. Evaluate $g'(4)$.

2 marks

Question 2 (3 marks)

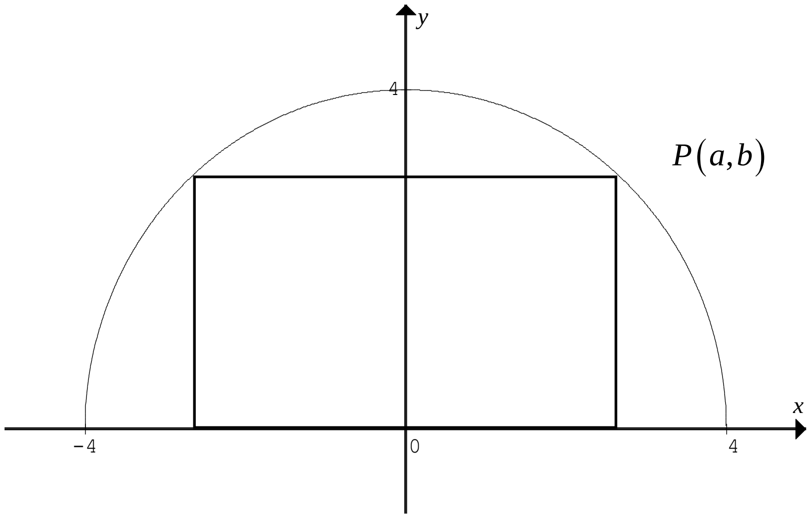
A binomial distribution of the random variable *X*, with four independent trials, is such that $\Pr(X=1)=2\Pr(X=2)$. If *p* is the probability of a success on any trial, find the value of *p*.

Question 3 (6 marks)

a. The tangent to the curve $y=\sqrt{16-x^2}$ at the point where $x=p$ makes an angle of 60^0 measured with the positive direction of the *x*-axis. Find the value of *p*.

3 mark

b. A rectangle has two vertices on the graph of $y=\sqrt{16-x^2}$, one at the point $P(a,b)$ where $a>0$ and two on the x-axis as shown in the diagram below.



i. Let A be the area of the rectangle, show that $A=2a\sqrt{16-a^2}$

1 mark

ii. Hence find the value of a, for which the area of the rectangle is a maximum.

2 marks

Question 4 (4 marks)

a. Solve the equation $\log_8(x+5)+\log_8(3x-1)=2$ for *x*.

2 marks

b. Solve the equation $32\times 64^x - 12\times 8^x + 1 = 0$ for *x*.

2 marks

Question 5 (4 marks)

Packets of smarties contain red smarties and other smarties of other colours.

a. A small packet of ten smarties contains three red smarties.
Let $\hat{P}$ represent the sample proportion of red smarties in a packet.

i. What values can $\hat{P}$ take.

1 mark

ii. Find $\Pr\left(\hat{P}=\frac{1}{5}\right)$

1 mark

b. A large jar of smarties contains over 600 smarties. One in four of these are red smarties. A random sample of 300 smarties is selected.
Find an approximate 95% confidence interval for $\hat{P}$, the sample proportion of red smarties. Use an integer multiple of the standard deviation in your calculations.

2 marks

Question 6 (5 marks)

Consider the functions with the rules $f(x) = \log_e(x+2)$ and $g(x) = 3 + 4x - x^2$, defined on their maximal domains.

i. Show that $f(g(x))$ does not exist.

2 marks

ii. If $g : D \to R$, $g(x) = 3 + 4x - x^2$, find the largest subset D of R, such that $f(g(x))$ is defined and determine the function $f(g(x))$.

3 marks

Question 7 (3 marks)

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$2\cos^2(k)$	$\cos(k)$

a. Find the possible values of k, given $0 \le k \le 2\pi$.

2 marks

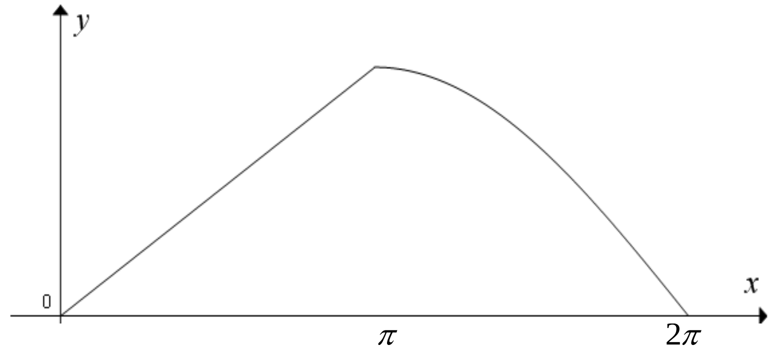
b. Find $\text{var}(X)$.

1 mark

Question 8 (5 marks)

The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} kx & 0\le x\le \pi \\ a\sin(nx) & \pi \le x \le 2\pi \end{cases}$$



The graph of $f(x)$ is shown.

i. Explain why $n=\frac{1}{2}$.

1 mark

ii. Find the values of a and k.

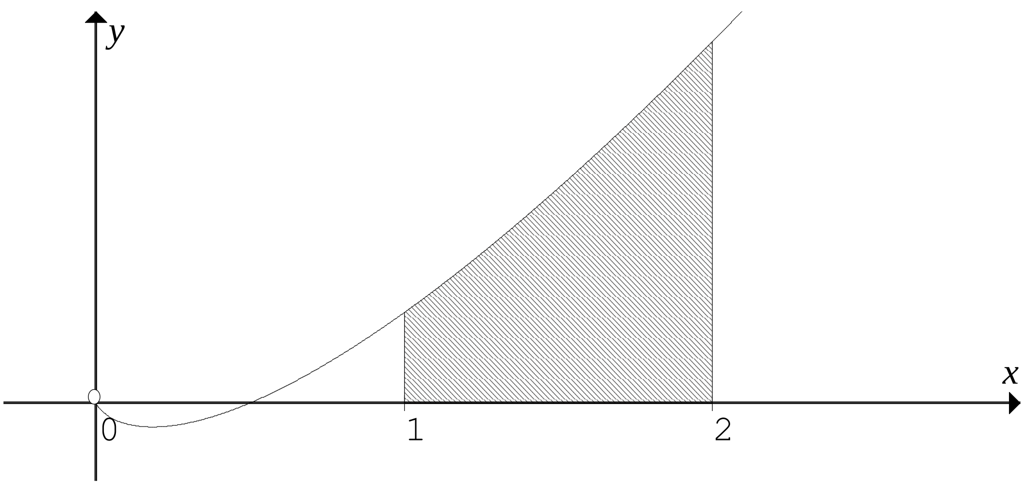
4 marks

Question 9 (6 marks)

a. Differentiate $x^2 \log_e(2x)$ with respect to x.

1 mark

b. Part of the graph of the function $f: R^+ \to R,\ f(x) = x\log_e(2x)$ is shown below.



i. Find the coordinates of the minimum turning point on the graph of f.

2 marks

ii. Find area of the shaded region.
Give your answer in the form $a\log_e(2)+b$, where $a,b \in R$.

3 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

EXTRA WORKING PAGE

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(\left(ax+b\right)^n\right)=na\left(ax+b\right)^{n-1}$		$\int\left(ax+b\right)^n dx=\frac{1}{a\left(n+1\right)}\left(ax+b\right)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$		$\int\frac{1}{x}dx=\log_e\left(x\right)+c,\ x>0$	
$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$		$\int\sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$	
$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$		$\int\cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$	
$\frac{d}{dx}\left(\tan\left(ax\right)\right)=\frac{a}{\cos^2\left(ax\right)}=a\sec^2\left(ax\right)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\ ,\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET