

Year 2018

VCE

Mathematical Methods

Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

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**Victorian Certificate of Education
2018**

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 17 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
 In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
 In questions where more than one mark is available, appropriate working **must** be shown.
 Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{\cos(2x)}{2x}$, find $\frac{dy}{dx}$.

2 marks

b. Let $g(x) = e^{\sqrt{x}}$. Evaluate $g'(4)$.

2 marks

Question 2 (3 marks)

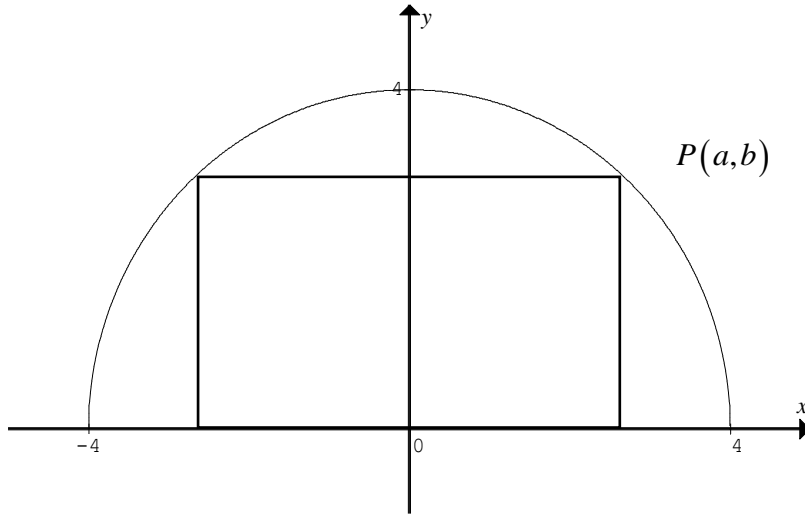
A binomial distribution of the random variable X , with four independent trials, is such that $\Pr(X = 1) = 2\Pr(X = 2)$. If p is the probability of a success on any trial, find the value of p .

Question 3 (6 marks)

a. The tangent to the curve $y = \sqrt{16 - x^2}$ at the point where $x = p$ makes an angle of 60° measured with the positive direction of the x -axis. Find the value of p .

3 mark

- b. A rectangle has two vertices on the graph of $y = \sqrt{16 - x^2}$, one at the point $P(a, b)$ where $a > 0$ and two on the x -axis as shown in the diagram below.



- i. Let A be the area of the rectangle, show that $A = 2a\sqrt{16 - a^2}$

1 mark

- ii. Hence find the value of a , for which the area of the rectangle is a maximum.

2 marks

Question 4 (4 marks)

a. Solve the equation $\log_8(x+5) + \log_8(3x-1) = 2$ for x .

2 marks

b. Solve the equation $32 \times 64^x - 12 \times 8^x + 1 = 0$ for x .

2 marks

Question 5 (4 marks)

Packets of smarties contain red smarties and other smarties of other colours.

a. A small packet of ten smarties contains three red smarties.

Let \hat{P} represent the sample proportion of red smarties in a packet.

i. What values can \hat{P} take.

1 mark

ii. Find $\Pr\left(\hat{P} = \frac{1}{5}\right)$

1 mark

b. A large jar of smarties contains over 600 smarties. One in four of these are red smarties. A random sample of 300 smarties is selected.

Find an approximate 95% confidence interval for \hat{P} , the sample proportion of red smarties. Use an integer multiple of the standard deviation in your calculations.

2 marks

Question 6 (5 marks)

Consider the functions with the rules $f(x) = \log_e(x+2)$ and $g(x) = 3+4x-x^2$, defined on their maximal domains.

i. Show that $f(g(x))$ does not exist.

2 marks

ii. If $g : D \rightarrow R$, $g(x) = 3+4x-x^2$, find the largest subset D of R , such that $f(g(x))$ is defined and determine the function $f(g(x))$.

3 marks

Question 7 (3 marks)

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$2\cos^2(k)$	$\cos(k)$

a. Find the possible values of k , given $0 \leq k \leq 2\pi$.

2 marks

b. Find $\text{var}(X)$.

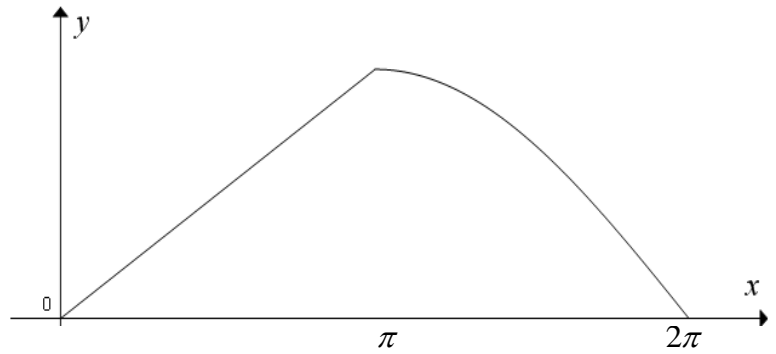
1 mark

Question 8 (5 marks)

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kx & 0 \leq x \leq \pi \\ a \sin(nx) & \pi \leq x \leq 2\pi \end{cases}$$

The graph of $f(x)$ is shown.



- i.** Explain why $n = \frac{1}{2}$.

1 mark

- ii.** Find the values of a and k .

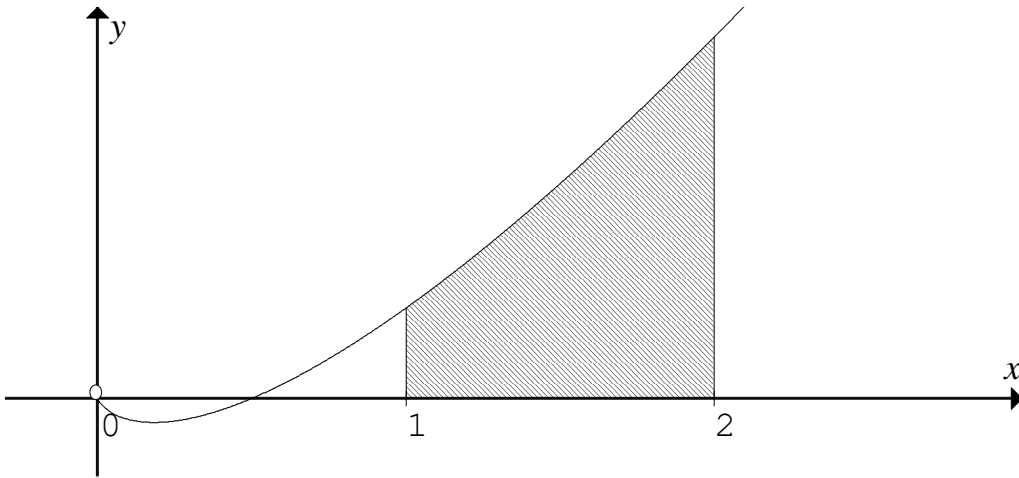
4 marks

Question 9 (6 marks)

a. Differentiate $x^2 \log_e(2x)$ with respect to x .

1 mark

b. Part of the graph of the function $f : \mathbb{R}^+ \rightarrow \mathbb{R}$, $f(x) = x \log_e(2x)$ is shown below.



i. Find the coordinates of the minimum turning point on the graph of f .

2 marks

- ii. Find area of the shaded region.
Give your answer in the form $a \log_e(2) + b$, where $a, b \in \mathbb{R}$.

3 marks

END OF QUESTION AND ANSWER BOOKLET
END OF EXAMINATION

EXTRA WORKING PAGE

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax+b)^n) = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{P} = \frac{X}{n}$	mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval $\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

END OF FORMULA SHEET