Year 2018

VCE

Mathematical Methods

Trial Examination 2

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Victorian Certificate of Education
2018

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 33 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Consider the function $f(x)=2x-a$ where $a>0$. If the range is $(-a,a]$ then the domain is

A. $(0,a)$

B. $[0,a]$

C. $(0,a]$

D. $[0,a)$

E. $[-a,a)$

Question 2

Let $P(x)=(x+2)(x-4)Q(x)+a(x+2)+b$, where $Q(x)$ is a polynomial. When $P(x)$ is divided by $x+2$ the remainder is 6, when $P(x)$ is divided by $x-4$ the remainder is 12. When $P(x)$ is divided by $(x+2)(x-4)$ the remainder is

A. 6

B. 12

C. $Q(4)$

D. $Q(-2)$

E. $x+8$

Question 3

Which of the following is **not** a continuous function?

A. $f(x)=\begin{cases} x^2-3 & \text{for } x\ge 2 \\ 1 & \text{for } x<2 \end{cases}$

B. $f(x)=\begin{cases} \sqrt{3x-5} & \text{for } x\ge 2 \\ 1 & \text{for } x<2 \end{cases}$

C. $f(x)=\begin{cases} \dfrac{1}{(3x-5)^2} & \text{for } x<2 \\ 1 & \text{for } x\ge 2 \end{cases}$

D. $f(x)=\begin{cases} \log_2(3x-5) & \text{for } x>2 \\ 1 & \text{for } x\ge 2 \end{cases}$

E. $f(x)=\begin{cases} \tan\left(\dfrac{\pi x}{8}\right) & \text{for } x\le 2 \\ 1 & \text{for } x>2 \end{cases}$

Question 4

Several students were investigating the system of linear simultaneous equations

$3x-(k+2)y=2$ and $-2kx+10y=k-7$ where k is a real constant.

Alan stated that there is no solution when $k=3$ and an infinite number of solutions when $k=-5$.

Ben stated that there is no solution when $k=-5$ and an infinite number of solutions when $k=3$.

Colin stated that there is a unique solution when $k\in R\setminus\{-5,3\}$

David stated that there is a unique solution when $k=-5$ or $k=3$.

Edward stated that there is no solution when $k=-3$ and an infinite number of solutions when $k=5$, and a unique solution when $k\in R\setminus\{5,-3\}$.

Then

A. Alan and Colin are correct.

B. Ben and Colin are correct.

C. Alan and David are correct.

D. Ben and David are correct.

E. Edward is correct.

Question 5

If f and g are two continuous and differentiable functions with the following properties

$f(1)=4$, $f(2)=3$, $f'(1)=5$, $f'(2)=-4$
$g(1)=2$, $g(2)=2$, $g'(1)=-2$, $g'(2)=5$

If $h(x)=f(g(x))$ then

A. $h(1)=8$ and $h'(1)=-16$

B. $h(1)=8$ and $h'(1)=-10$

C. $h(1)=3$ and $h'(1)=-4$

D. $h(1)=3$ and $h'(1)=-10$

E. $h(1)=3$ and $h'(1)=8$

Question 6

The same students where investigating the function
$f:[0,b]\to R$, $f(x)=x(2b-x)$ where $b>0$.

Alan stated the range is R.

Ben stated the average value of the function is $\frac{2b^2}{3}$ and the average rate of change of the function is b.

Colin stated the average value of the function is b and the average rate of change of the function is $\frac{2b^2}{3}$.

David stated that the function is always increasing and that the gradient at $x=b$ does not exist.

Edward stated that the function is always increasing and that $f'(b)=0$.

Then

A. Alan and Colin are correct.

B. Colin and David are correct.

C. Alan and David are correct.

D. Ben and David are correct.

E. Ben and Edward are correct.

Question 7

A box contains six black and five red marbles. Three marbles are drawn from the box without replacement. The probability that there are at least one of each colour is

A. $\frac{9}{11}$

B. $\frac{3}{11}$

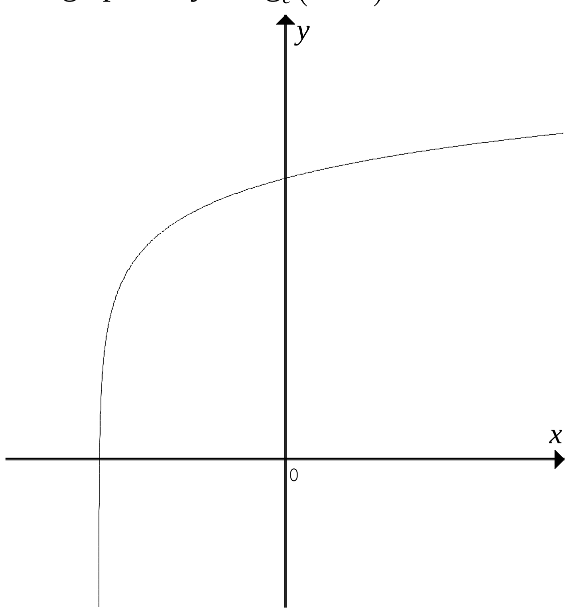
C. $\frac{17}{99}$

D. $\frac{90}{121}$

E. $\frac{33}{110}$

Question 8

The graph of $y = \log_e(x+a)+b$ is shown below, then



A. $a > 0$ and $a > e^{-b}$

B. $a > 0$ and $a < e^{-b}$

C. $a < 0$ and $a > e^{b}$

D. $a < 0$ and $a < e^{-b}$

E. $a > 0$ and $a > e^{b}$

Question 9

If $\int_{-1}^{2}\left(3x^2-2f(x)+g(x)\right)dx=18$ and $\int_{2}^{-1}g(x)\,dx=3$ then $\int_{2}^{-1}f(x)\,dx$ is equal to

A. 7

B. 6

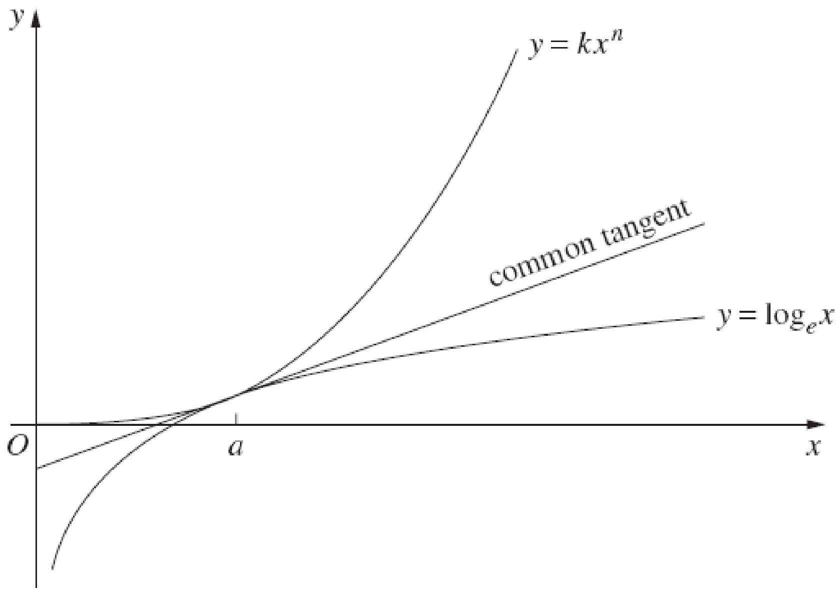
C. 4

D. -3

E. -7

Question 10

The graphs of the functions $y=kx^n$ and $y=\log_e(x)$ have a common tangent at $x=a$, as shown.



Then which of the following is **false**?

A. $k=\dfrac{\log_e(a)}{a^n}$

B. $k=\dfrac{1}{na^n}$

C. $k=\dfrac{1}{en}$

D. $a=ne^k$

E. $n=\log_a(e)$

Question 11

The graphs of $y = kx + 1$ and $y = x^2 - kx + 2$ do not intersect when

A. $k < 1$

B. $k < -1$

C. $-1 < k < 1$

D. $k > 1$

E. $k > -1$

Question 12

Which of the following is the inverse function of $f : (-\infty, a) \to R$, $f(x) = 2ax - x^2$

where a is a positive real number.

A. $f^{-1} : R \to R$, $f^{-1}(x) = \dfrac{1}{2ax - x^2}$

B. $f^{-1} : R \to R$, $f^{-1}(x) = a + \sqrt{a^2 - x}$

C. $f^{-1} : (-\infty, a) \to R$, $f^{-1}(x) = a + \sqrt{a^2 - x}$

D. $f^{-1} : (-\infty, a) \to R$, $f^{-1}(x) = a - \sqrt{a^2 - x}$

E. $f^{-1} : (-\infty, a^2) \to R$, $f^{-1}(x) = a - \sqrt{a^2 - x}$

Question 13

If $f(x)$ is a non-zero even function, and $g(x)$ is a non-zero odd function, then if $a > 0$

$\int_{-a}^{a} \left(f(x) + g(x) \right) dx$ is equal to

A. $2\int_{0}^{a} \left(f(x) + g(x) \right) dx$

B. $2\int_{0}^{a} f(x)\, dx$

C. $2\int_{0}^{a} g(x)\, dx$

D. $\int_{0}^{a} f(x)\, dx$

E. $\int_{0}^{a} g(x)\, dx$

Question 14

A certain curve has its gradient given by $1-4\sin\left(\frac{x}{2}\right)$. If the curve crosses the *x*-axis at $x=2\pi$ then the equation of the curve is

A. $y=x+2\cos\left(\frac{x}{2}\right)+2-2\pi$

B. $y=x+8\cos\left(\frac{x}{2}\right)+8-2\pi$

C. $y=x+2\cos\left(\frac{x}{2}\right)-2-2\pi$

D. $y=x+8\cos\left(\frac{x}{2}\right)-8-2\pi$

E. $y=x+8\cos\left(\frac{x}{2}\right)$

Question 15

Let $f(x)=\frac{1}{1+x^2}$. Which of the following statements about f is **not** true?

A. $f\left(\frac{1}{x}\right)=x^2 f(x)$ for $x\neq 0$

B. $f\left(\frac{1}{x}\right)+f(x)=1$ for $x\neq 0$

C. $f\left(\frac{1}{x}\right)f(x)=x^2\left(f(x)\right)^2$ for $x\neq 0$

D. $f\left(x^2\right)+f\left(-x^2\right)=2f\left(x^2\right)$

E. $\left(f\left(\sqrt{x}\right)\right)^2=f(x)$ for $x>0$

Question 16

The graph of $y=\cos(x)$ is transformed into the graph of $y=-3\sin(x)$ by

A. A dilation by a scale factor of 3 parallel to the *y*-axis, a reflection in the *x*-axis, and a translation of $\frac{\pi}{2}$ to the right parallel to the *x*-axis.

B. A dilation by a scale factor of 3 away from the *y*-axis, a reflection in the *x*-axis, and a translation of $\frac{\pi}{2}$ to the left parallel to the *x*-axis.

C. A dilation by a scale factor of -3 parallel to the *y*-axis, a reflection in the *y*-axis, and a translation of $\frac{\pi}{2}$ to the right parallel to the *y*-axis.

D. A dilation by a scale factor of 3 away from the *y*-axis, a reflection in the *y*-axis, and a translation of $\frac{\pi}{2}$ to the left parallel to the *x*-axis.

E. A dilation by a scale factor of 3 away from the *x*-axis, a reflection in the *x*-axis, and a translation of $\frac{\pi}{2}$ to the left parallel to the *y*-axis.

Question 17

If $\Pr(A)=\sqrt{p}$, $\Pr(B)=10p$, $\Pr(A|B)=\frac{1}{4}$ and $\Pr(A'\cap B')=\frac{1}{2}$, then

A. $p=0.25$

B. $p=0.4$

C. $p=0.04$

D. $p=0.09$

E. $p=0.01$

Question 18

It is known that 55% of the population have brown eyes. For a random sample of 20 students, $\hat{P}$ is the random variable of the proportion of students who have brown eyes. Then $\Pr(\hat{P}>0.5)$, correct to four decimal places is

A. 0.1771

B. 0.1593

C. 0.2493

D. 0.5914

E. 0.7507

Question 19

The random variable Z has the standard normal distribution, with $\Pr(Z > a) = A$, and $\Pr(a < Z < b) = P$, where $b > a$, then $\Pr(Z > b \mid Z > a)$ is equal to

A. $\dfrac{1+A-P}{A}$

B. $\dfrac{1-P}{A}$

C. $1-\dfrac{P}{A}$

D. $\dfrac{1+A-P}{1-A}$

E. $\dfrac{P}{A}-1$

Question 20

A discrete random variable X has the following probability distribution.

X	0	1	2
$\Pr(X = x)$	a	b	c

Given that $E(X) = \text{var}(X) = \dfrac{1}{2}$ then

A. $a = \dfrac{5}{8}$, $b = \dfrac{1}{4}$, $c = \dfrac{1}{8}$

B. $a = \dfrac{1}{2}$, $b = 0$, $c = \dfrac{1}{2}$

C. $a = \dfrac{1}{2}$, $b = \dfrac{1}{2}$, $c = 0$

D. $a = 0$, $b = \dfrac{7}{8}$, $c = \dfrac{1}{8}$

E. $a = \dfrac{11}{16}$, $b = \dfrac{1}{4}$, $c = \dfrac{1}{16}$

END OF SECTION A

SECTION B

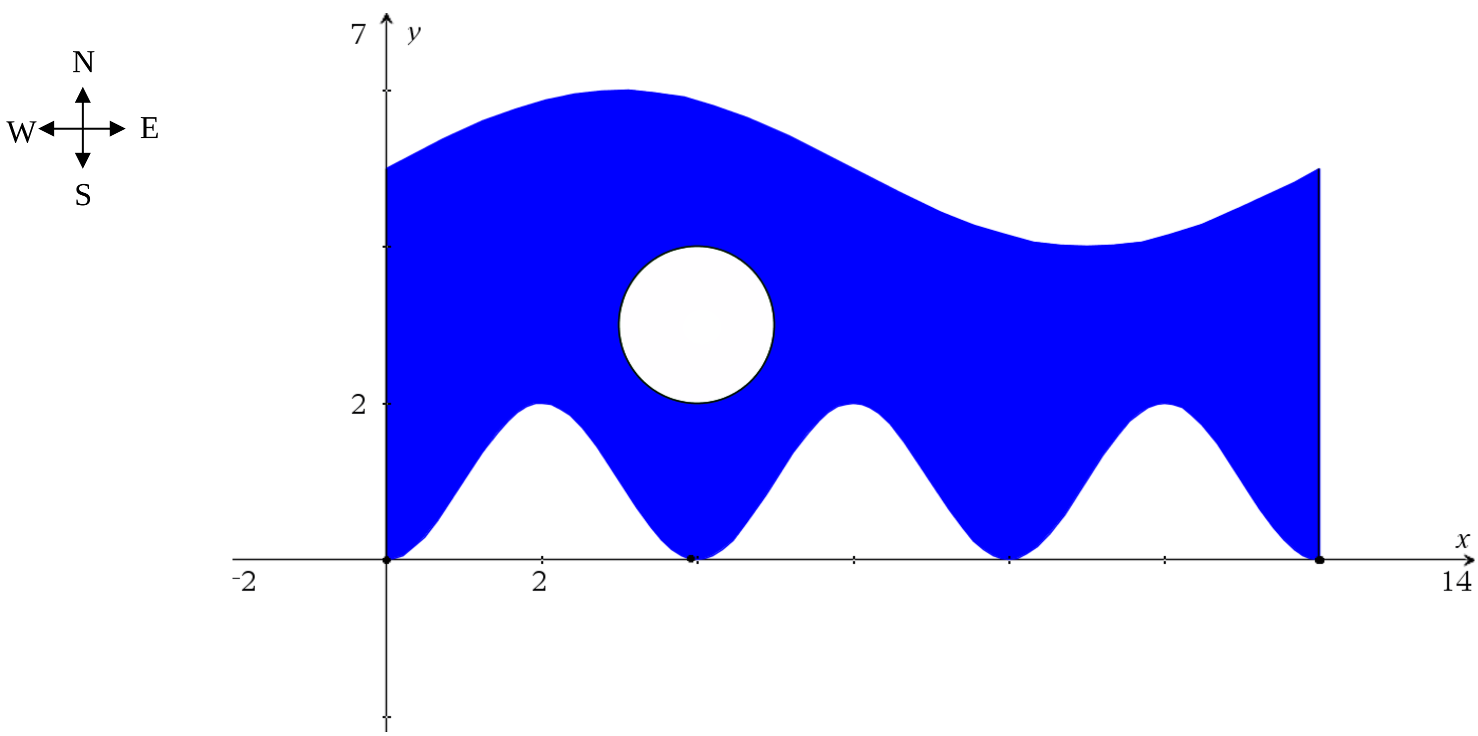
Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (12 marks)

The diagram shows a plane cross-sectional view of a swimming pool at a hotel resort.

The x and y axes are shown, and are in the directions of east-west and north-south respectively. Distance are measured in metres and the length of the pool is 12 metres.



The pool is bounded by the northern most curve which is the function

$f:[0,12] \to R\,,\; f(x) = 5 + \sin\left(\frac{\pi x}{6}\right)$

and the southern most curve which is the function

$g:[0,12] \to R\,,\; g(x) = 2\sin^2\left(\frac{\pi x}{4}\right)$.

The pool also contains a circular island, which has the equation $(x-4)^2 + (y-3)^2 = 1$.

To get to the island, a bridge is to be built from the centre of the circular island to the northern most point on the graph of f.

a. Find the length of this bridge in metres.

2 marks

Another bridge is to be built as a tangent line, from the southern curve at the point where $x = 5$.

b.i. Find the equation of this tangent line.

2 marks

ii. Find the coordinates where this bridge intersects the northern curve.

Give your answers correct to three decimal places.

2 marks

iii. Determine the length of this bridge, in metres, giving your answer correct to three decimal places.

1 mark

c.i. Write down a definite integral which gives the total cross-sectional area in square metres of the water in the swimming pool.

1 mark

ii. The pool is filled to a constant depth of 1.5 metres. Find the volume in cubic metres of water in the swimming pool.

1 mark

d. Find the maximum and minimum widths, in metres measured north-south of the swimming pool. Give your answers correct to three decimal places.

3 marks

Question 2 (14 marks)

Consider the function $f: R \to R$, $f(x) = x^3 - 6x^2 + cx + d$, where $c, d \in R$.

For different values of c and d, the graph has different properties.

a.i. Find a range of values of c, for which the graph of the function f has two stationary points.

1 mark

ii. For the case when $c = 9$, find a range of values of d, when the graph of the function f, crosses the x-axis at three distinct points.

3 marks

iii. If the graph of the function f has one stationary point at $(1, 9)$, find the values of c and d, in this particular case and determine the coordinates of the other stationary point.

1 mark

b.i. Find the value of c, for which the graph of the function f has exactly one stationary point.

1 mark

ii. If the graph of the function f is translated p units to the left away from the y-axis, it becomes the graph of $y = x^3$. Find the values of c, d and p in this particular case.

2 marks

c. If the graph of the function f crosses the y-axis at -4, and the tangent to the function f at $x = a$, is $y = -3x + 4$, find the values of a, c, and d in this particular case.

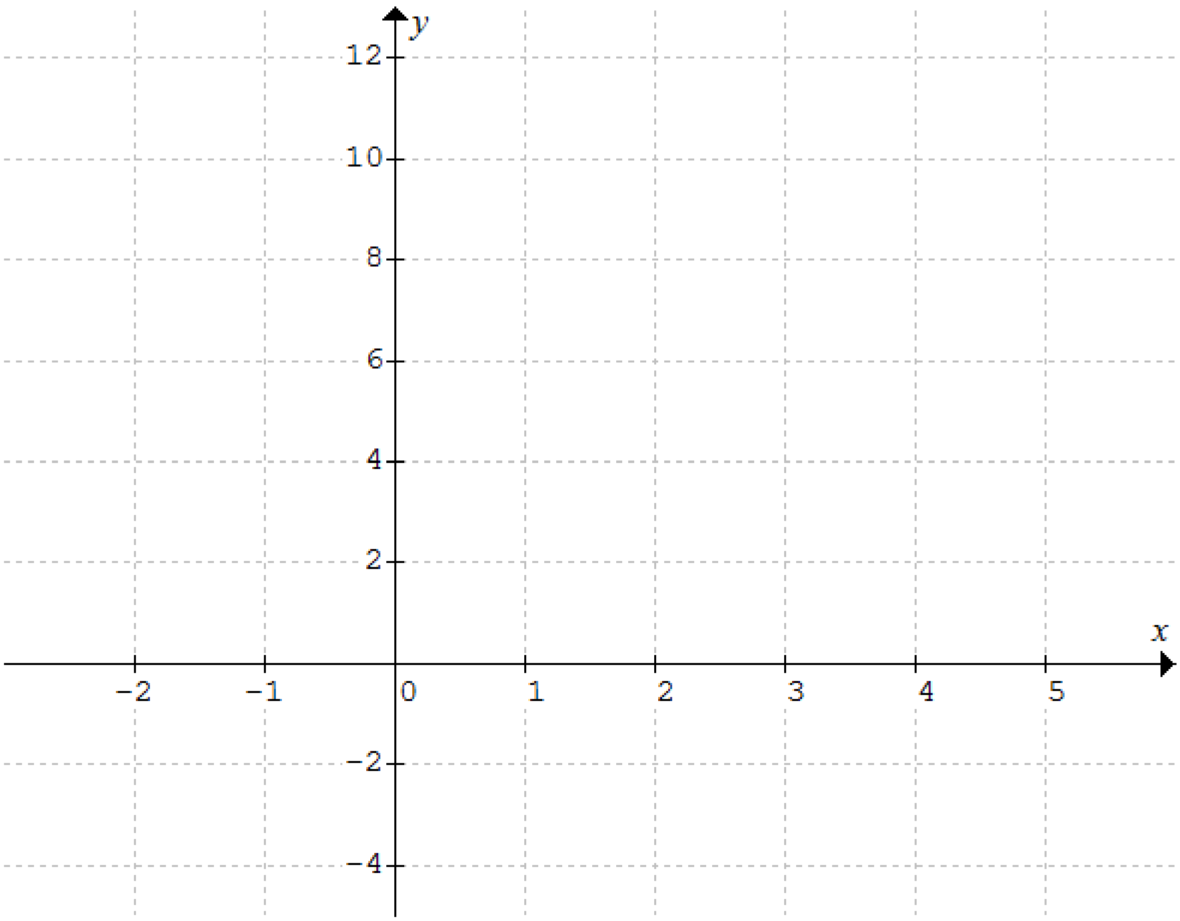
2 marks

d.i. Let A be the area bounded by the graph of the function f, the coordinate axes, and the line $x = 2$, and that $f(x) > 0$ for $0 \le x \le 2$. If this area is approximated by four equally spaced left rectangles, the area is $15\frac{1}{4}$. If this area is approximated by four equally spaced right rectangles, the area is $16\frac{1}{4}$, find the values of c and d in this particular case, and the true area A.

3 marks

ii. Sketch the graph of $f(x)$ on the axes below, for this case, shading the required area.

1 mark



Question 3 (16 marks)

a. The length of time for which a certain type of candle burns before going out is normally distributed with a mean of one hour and a standard deviation of ten minutes.

i. Find the probability that a randomly selected candle of this type burns for between 45 and 55 minutes, giving your answer correct to four decimal places.

1 mark

ii. Find the probability that a candle of this type will burn for less than 45 minutes, giving your answer correct to four decimal places.

1 mark

iii. A birthday cake has 10 such candles, assuming that all candles burn independently, find the probability that at least one candle burns for less than 45 minutes, giving your answer correct to four decimal places.

1 mark

b. The manufacturers of the candles decide to take a random sample of 80 of these types of candles. For samples of size 80 from the population which burn for a mean of one hour and a standard deviation of ten minutes, let $\hat{P}$ be the random variable of the distribution of sample proportions of these types of candles which burn for less than 45 minutes.

i. Find the probability $\Pr\left(\hat{P} \geq 0.1 \mid \hat{P} \geq 0.05\right)$. Give your answer to four decimal places.

Do not use a normal approximation.

2 marks

ii. The manufacturers find that in a particular sample of 80 of these types of candles, 16 of them burn for less than 45 minutes. Determine a 90% confidence interval for the manufacturers estimate of the proportion of these candles which burn for less than 45 minutes. Give your values correct to three decimal places.

1 mark

c. For another type of long life candles, the times that these candles burn for is found to be normally distributed. It is found that 25% of these candles burn for more than 252 minutes, while 9% of these candles burn for less than 246 minutes. Find the mean and standard deviation of the times that these candles burn for. Give your answers to the nearest minute.

3 marks

d. Every year on her birthday Lilly has either a chocolate cake or an ice-cream cake. If she has an ice-cream cake one year, the probability that she has an ice-cream cake the following year is 0.65, while if she has a chocolate cake one year, the probability she has a chocolate cake the following year is 0.55. Suppose that on her 13^{th} birthday she had an ice-cream cake, determine the probability that from her 13^{th} to 16^{th} birthday inclusive she has exactly three ice-cream cakes. Give your answer to four decimal places.

2 marks

e. When Lilly has a party, the time t, in hours that the guests stay, for has found to satisfy a continuous probability density function defined by

$$T(t)=\begin{cases}\dfrac{9}{16}te^{-\frac{3t}{4}} & \text{for } t\ge 0\\ 0 & \text{elsewhere}\end{cases}$$

i. Find the probability that guests stay for more than two hours, if it is known that they stayed for at least one hour.

2 marks

ii. Find the mean and variance of the times in hours that guests stay at the party.

2 marks

iii. Find the median time in hours that guests stay at the party.
Give your answer correct to three decimal places.

1 mark

Question 4 (18 marks)

Consider the function $f : B \to R$, $f(x) = 2\sqrt{x+2}$

a. The function f can be obtained from the graph of $y = \sqrt{x}$ under a transformation,

$$T : R^2 \to R^2 \ , \ T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$$

Find the values of a, b, h and k.

2 marks

b. State the maximal domain B of the function f.

1 mark

c. Find f^{-1}, the inverse function f.

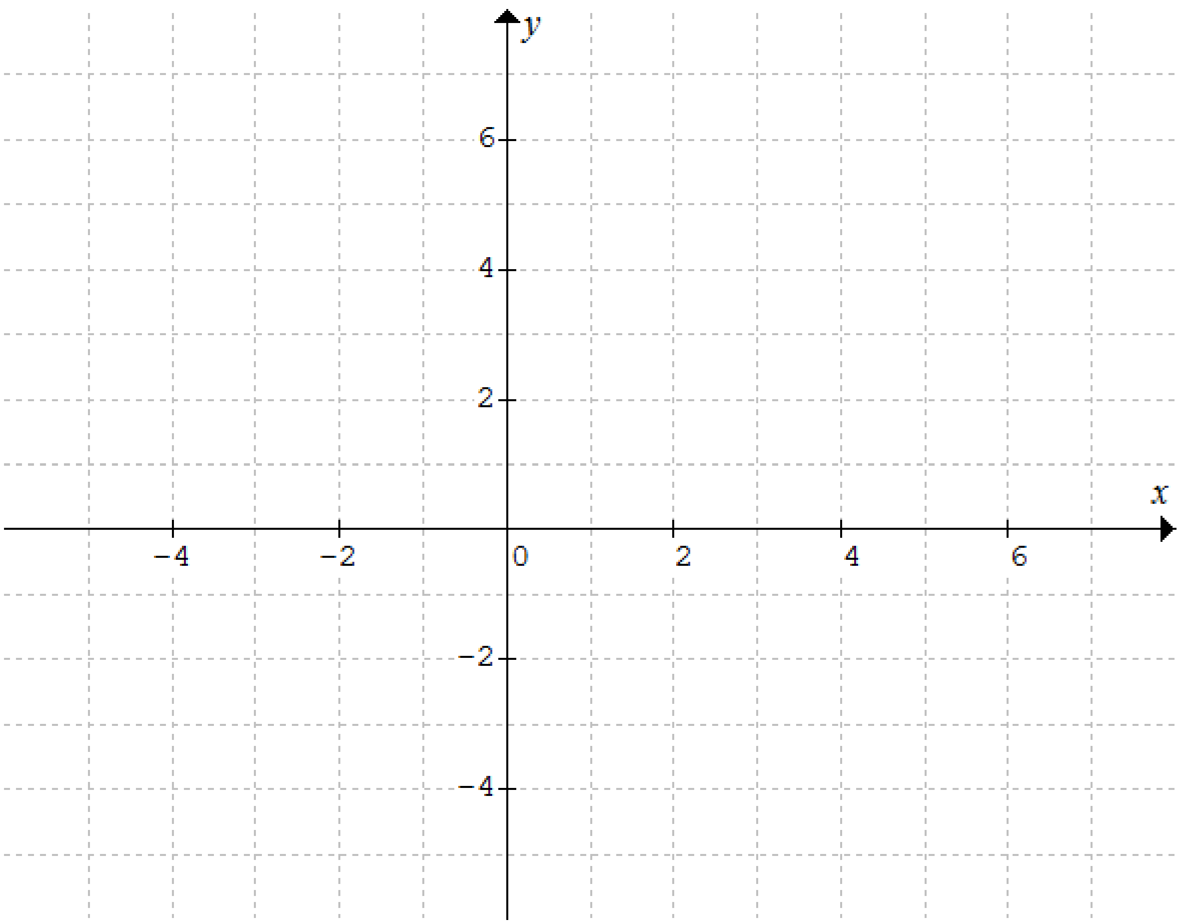
2 marks

d. Find all values of u, such that $f(u) = f^{-1}(u)$.

1 mark

e. Sketch the graphs of $y = f(x)$, $y = f^{-1}(x)$ and the graph of $y = x$ on the diagram below.

2 marks



f.i. Find the area enclosed between the graphs of $y = f(x)$ and $y = x$.

1 mark

ii. Find the area enclosed between the graphs of $y = x$ and $y = f^{-1}(x)$.

1 mark

g. Find the angle between the tangents to both curves $y = f(x)$ and $y = f^{-1}(x)$ at $x = u$.
Give your answer in degrees correct to one decimal place.

2 marks

Consider now the function $g: D \to R$, $g(x) = k\sqrt{x+k}$ where $k \neq 0$

h. Describe the transformations that map the graph of $y = \sqrt{x}$ onto the graph of g.

1 mark

i. Find g^{-1} the inverse function g, stating the maximal domain D of g.

1 mark

j. The point with co-ordinates $Q(c,c)$ lies on the graphs of $y = g(x)$ and $y = g^{-1}(x)$
Express c in terms of k.

1 mark

k. Find values of *k*, for when the graphs of $y = g(x)$ and $y = g^{-1}(x)$ do not intersect.

1 mark

l. Determine the value of *k*, when the tangents to both curves $y = g(x)$ and $y = g^{-1}(x)$ intersect at $x = c$ at 45^0.

2 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A)=1-\Pr(A')$		$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$	
$\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$			
mean	$\mu=E(X)$	variance	$\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$

Sample proportions

$\hat{P}=\dfrac{X}{n}$		mean	$E(\hat{P})=p$
standard deviation	$\operatorname{sd}(\hat{P})=\sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\,,\ \hat{p}+z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Words									

SIGNATURE ______________________________________

SECTION A

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**