



Trial Examination 2018

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (4 marks)

$$\begin{aligned} \text{a. } \frac{d}{dx}[x^3 \log_e(x+2)] &= 3x^2 \times \log_e(x+2) + x^3 \times \frac{1}{x+2} \\ &= 3x^2 \log_e(x+2) + \frac{x^3}{x+2} \end{aligned}$$

A1

$$\text{b. } f'(x) = \frac{-2x \sin(2x) - \cos(2x)}{x^2}$$

A1

$$f'\left(\frac{\pi}{4}\right) = \frac{-2 \times \frac{\pi}{4} \times \sin\left(2 \times \frac{\pi}{4}\right) - \cos\left(2 \times \frac{\pi}{4}\right)}{\left(\frac{\pi}{4}\right)^2}$$

M1

$$= -\frac{8}{\pi}$$

A1

Question 2 (3 marks)

$$g(x) = \int (4-x)^2 dx$$

$$= -\frac{1}{3}(4-x)^3 + c$$

A1

$$g(1) = 0$$

$$\Rightarrow 0 = -\frac{1}{3}(4-x)^3 + c$$

M1

$$c = 9$$

$$\therefore g(x) = -\frac{1}{3}(4-x)^3 + 9$$

A1

Question 3 (4 marks)

$$\text{a. } (3^2)^{1-3x} = 3^{-3}$$

$$2(1-3x) = -3$$

M1

$$x = \frac{5}{6}$$

A1

$$\text{b. } \log_2\left(\frac{3x(x+4)}{15}\right) = 0$$

$$\frac{3x(x+4)}{15} = 1$$

M1

$$3x(x+4) = 15$$

$$3x^2 + 12x - 15 = 0$$

$$3(x+5)(x-1) = 0$$

$$\therefore x = 1 \text{ as } x > 0$$

A1

Question 4 (4 marks)

$$\text{a. } f(g(x)) = \frac{1}{\sqrt{1 - \frac{x^2}{4}}}$$

M1

$$= \frac{1}{\sqrt{\frac{4-x^2}{4}}}$$

$$= \frac{1}{\frac{1}{2}\sqrt{4-x^2}}$$

$$= \frac{2}{\sqrt{4-x^2}}$$

A1

$$\text{b. } \text{range } g(x) \subseteq \text{domain } f(x)$$

$$\text{range } g(x) = (-\infty, 1]$$

$$\text{domain } f(x) = (0, \infty)$$

$$\Rightarrow 1 - \frac{x^2}{4} > 0$$

A1

$$\therefore x \in (-2, 2)$$

A1

Question 5 (7 marks)

$$\text{a. } \text{range } f = [0, 6]$$

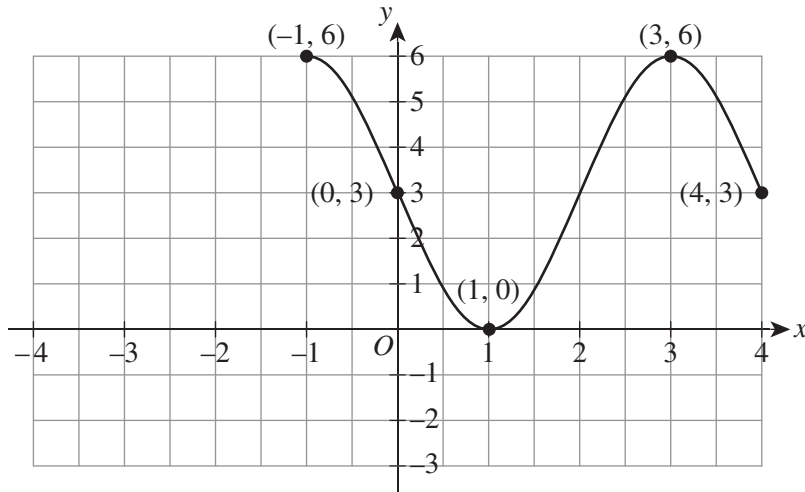
A1

$$\text{period} = \frac{2\pi}{\frac{\pi}{2}}$$

$$= 4$$

A1

b.



correct shape (must not be linear between turning points) A1
 correct intercepts labelled as coordinates A1
 correct turning points labelled as coordinates A1

c. $-3 \sin\left(\frac{\pi x}{2}\right) + 3 = \frac{3}{2}$

$$\sin\left(\frac{\pi x}{2}\right) = \frac{1}{2}$$

A1

$$\frac{\pi x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{1}{3} \text{ or } x = \frac{5}{3}$$

A1

Question 6 (9 marks)

a. $\Pr(\text{JJJ}) = \left(\frac{1}{4}\right)^3$
 $= \frac{1}{64}$

A1

b. $\Pr(\text{JJJ} | \text{at least 1 J}) = \frac{\Pr(\text{JJJ} \cap \text{at least 1 J})}{\Pr(\text{at least 1 J})}$
 $= \frac{\Pr(\text{JJJ} \cap \text{at least 1 J})}{1 - \Pr(\text{no J})}$
 $= \frac{\frac{1}{64}}{1 - \left(\frac{3}{4}\right)^3}$
 $= \frac{\frac{1}{64}}{\frac{37}{64}}$
 $= \frac{1}{37}$

M1

A1

- c.** $\Pr(J) = p$
 $\Pr(J') = 1 - p$
 $\Pr(\text{exactly 1 J}) = \Pr(JJ') + \Pr(J'J)$
 $\Rightarrow p(1 - p) + (1 - p)p = \frac{8}{25}$ M1
 $2p(1 - p) = \frac{8}{25}$
 $25p^2 - 25p + 4 = 0$
 $(5p - 1)(5p - 4) = 0$
 $p = \frac{1}{5}$ or $p = \frac{4}{5}$ M1
- However, $\Pr(J) < \frac{1}{4}$.
 $\therefore p = \frac{1}{5}$ only A1
- d.** $X \sim N(20, 36)$
 $Z \sim N(0, 1)$
 $\Pr(X < 14) = \Pr(Z < -1)$
 $= \Pr(Z > 1)$
 $= 0.16$ A1
- e.** $\Pr(14 < X < 20 | X > 14) = \frac{\Pr(14 < X < 20)}{\Pr(X > 14)}$
 $= \frac{\Pr(-1 < Z < 0)}{\Pr(Z > -1)}$
 $= \frac{0.34}{0.84}$
 $= \frac{17}{42}$ A1
- f.** $\Pr(X < 10) = \Pr(X > 30) = k$
 $\Pr(X < 26) = \Pr(Z < 1) = 0.84$
 $\Pr(10 < X < 26) = 0.84 - k$ A1

Question 7 (9 marks)

a. $f'(x) = -\frac{2k}{x^3}$

$$f'(k) = -\frac{2}{k^2}$$

$$f(k) = \frac{1}{k}$$

The tangent passes through $\left(k, \frac{1}{k}\right)$ with gradient $-\frac{2}{k^2}$. A1

$$y - \frac{1}{k} = -\frac{2}{k^2}(x - k)$$

$$y = -\frac{2}{k^2}x + \frac{3}{k}$$
A1

b. point A (y-intercept of tangent line): $\left(0, \frac{3}{k}\right)$ A1

point B: $\left(0, -\frac{3}{k}\right)$ A1

Let $y = 0$ (for point C).

$$\Rightarrow -\frac{2}{k^2}x + \frac{3}{k} = 0$$

$$\therefore x = \frac{3k}{2}$$

point C: $\left(\frac{3k}{2}, 0\right)$ A1

c. area = $\frac{1}{2}bh$

$$= \frac{1}{2} \times 2 \times \frac{3}{k} \times \frac{3k}{2}$$
M1

$$= \frac{9}{2}$$
A1

\therefore The area is constant and independent of k .

d. $\hat{ABC} = \frac{\pi}{3}$

$$\Rightarrow \tan\left(\frac{\pi}{6}\right) = m_{BC}$$

$$\frac{\sqrt{3}}{3} = \frac{2}{k^2}$$
M1

$$k^2 = 2\sqrt{3}$$

$$= \sqrt{12}$$

$$\therefore k = 12^{\frac{1}{4}} \text{ as } k > 0$$
A1