Neap:

Trial Examination 2018

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

a. $\frac{d}{dx}[x^3\log_e(x+2)] = 3x^2 \times \log_e(x+2) + x^3 \times \frac{1}{x+2}$

$= 3x^2\log_e(x+2) + \frac{x^3}{x+2}$ A1

b. $f'(x) = \frac{-2x\sin(2x) - \cos(2x)}{x^2}$ A1

$f'\left(\frac{\pi}{4}\right) = \frac{-2 \times \frac{\pi}{4} \times \sin\left(2 \times \frac{\pi}{4}\right) - \cos\left(2 \times \frac{\pi}{4}\right)}{\left(\frac{\pi}{4}\right)^2}$ M1

$= -\frac{8}{\pi}$ A1

Question 2 (3 marks)

$g(x) = \int (4-x)^2 dx$

$= -\frac{1}{3}(4-x)^3 + c$ A1

$g(1) = 0$

$\Rightarrow 0 = -\frac{1}{3}(4-x)^3 + c$ M1

$c = 9$

$\therefore g(x) = -\frac{1}{3}(4-x)^3 + 9$ A1

Question 3 (4 marks)

a. $(3^2)^{1-3x} = 3^{-3}$

$2(1-3x) = -3$ M1

$x = \frac{5}{6}$ A1

b. $\log_2\left(\dfrac{3x(x+4)}{15}\right) = 0$

$\dfrac{3x(x+4)}{15} = 1$ M1

$3x(x+4) = 15$

$3x^2 + 12x - 15 = 0$

$3(x+5)(x-1) = 0$

$\therefore x = 1$ as $x > 0$ A1

Question 4 (4 marks)

a. $f(g(x)) = \dfrac{1}{\sqrt{1 - \dfrac{x^2}{4}}}$ M1

$= \dfrac{1}{\sqrt{\dfrac{4 - x^2}{4}}}$

$= \dfrac{1}{\dfrac{1}{2}\sqrt{4 - x^2}}$

$= \dfrac{2}{\sqrt{4 - x^2}}$ A1

b. range $g(x) \subseteq$ domain $f(x)$

range $g(x) = (-\infty, 1]$

domain $f(x) = (0, \infty)$

$\Rightarrow 1 - \dfrac{x^2}{4} > 0$ A1

$\therefore x \in (-2, 2)$ A1

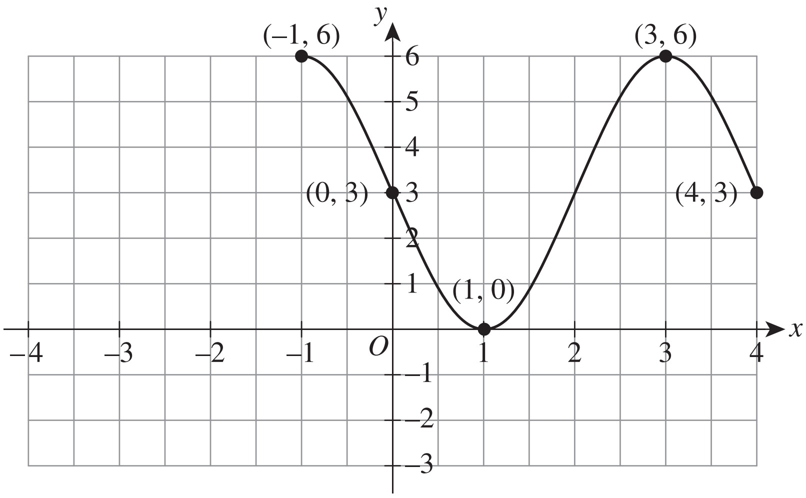
Question 5 (7 marks)

a. range $f = [0, 6]$ A1

period $= \dfrac{2\pi}{\dfrac{\pi}{2}}$

$= 4$ A1

b.



correct shape (must not be linear between turning points) A1
correct intercepts labelled as coordinates A1
correct turning points labelled as coordinates A1

c. $-3\sin\left(\frac{\pi x}{2}\right)+3=\frac{3}{2}$

$\sin\left(\frac{\pi x}{2}\right)=\frac{1}{2}$ A1

$\frac{\pi x}{2}=\frac{\pi}{6},\frac{5\pi}{6}$

$x=\frac{1}{3}$ or $x=\frac{5}{3}$ A1

Question 6 (9 marks)

a. $\Pr(\text{JJJ})=\left(\frac{1}{4}\right)^3$

$=\frac{1}{64}$ A1

b. $\Pr(\text{JJJ}\,|\,\text{at least 1 J})=\frac{\Pr(\text{JJJ}\cap\text{at least 1 J})}{\Pr(\textit{at least 1 J})}$ M1

$=\frac{\Pr(\text{JJJ}\cap\text{at least 1 J})}{1-\Pr(\text{no J})}$

$=\frac{\frac{1}{64}}{1-\left(\frac{3}{4}\right)^3}$

$=\frac{\frac{1}{64}}{\frac{37}{64}}$

$=\frac{1}{37}$ A1

c. $\Pr(\mathrm{J}) = p$

$\Pr(\mathrm{J}') = 1 - p$

$\Pr(\text{exactly 1 J}) = \Pr(\mathrm{JJ}') + \Pr(\mathrm{J}'\mathrm{J})$

$\Rightarrow p(1-p) + (1-p)p = \frac{8}{25}$ M1

$2p(1-p) = \frac{8}{25}$

$25p^2 - 25p + 4 = 0$

$(5p-1)(5p-4) = 0$

$p = \frac{1}{5}$ or $p = \frac{4}{5}$ M1

However, $\Pr(\mathrm{J}) < \frac{1}{4}$.

$\therefore p = \frac{1}{5}$ only A1

d. $X \sim N(20, 36)$

$Z \sim N(0, 1)$

$\Pr(X < 14) = \Pr(Z < -1)$

$= \Pr(Z > 1)$

$= 0.16$ A1

e. $\Pr(14 < X < 20 | X > 14) = \frac{\Pr(14 < X < 20)}{\Pr(X > 14)}$

$= \frac{\Pr(-1 < Z < 0)}{\Pr(Z > -1)}$

$= \frac{0.34}{0.84}$

$= \frac{17}{42}$ A1

f. $\Pr(X < 10) = \Pr(X > 30) = k$

$\Pr(X < 26) = \Pr(Z < 1) = 0.84$

$\Pr(10 < X < 26) = 0.84 - k$ A1

Question 7 (9 marks)

a. $f'(x) = -\frac{2k}{x^3}$

$f'(k) = -\frac{2}{k^2}$

$f(k) = \frac{1}{k}$

The tangent passes through $\left(k, \frac{1}{k}\right)$ with gradient $-\frac{2}{k^2}$. A1

$y - \frac{1}{k} = -\frac{2}{k^2}(x - k)$

$y = -\frac{2}{k^2}x + \frac{3}{k}$ A1

b. point A (y-intercept of tangent line): $\left(0, \frac{3}{k}\right)$ A1

point B: $\left(0, -\frac{3}{k}\right)$ A1

Let $y = 0$ (for point C).

$\Rightarrow -\frac{2}{k^2}x + \frac{3}{k} = 0$

$\therefore x = \frac{3k}{2}$

point C: $\left(\frac{3k}{2}, 0\right)$ A1

c. area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 2 \times \frac{3}{k} \times \frac{3k}{2}$ M1

$= \frac{9}{2}$ A1

$\therefore$ The area is constant and independent of k.

d. $A\hat{B}C = \frac{\pi}{3}$

$\Rightarrow \tan\left(\frac{\pi}{6}\right) = m_{BC}$

$\frac{\sqrt{3}}{3} = \frac{2}{k^2}$ M1

$k^2 = 2\sqrt{3}$

$= \sqrt{12}$

$\therefore k = 12^{\frac{1}{4}}$ as $k > 0$ A1

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