THE SCHOOL FOR EXCELLENCE (TSFX)

VCE MATHEMATICAL METHODS UNITS 3 & 4

WRITTEN EXAMINATION 2 – 2018

Reading Time: 15 minutes
Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Student Number: ☐☐ ☐☐☐ ☐☐☐ **Letter** ☐

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
1	20	20	20
2	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory **DOES NOT** need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 22 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

$f : D \rightarrow R,\ f(x) = (x-2)^2 - 1$ has a range of $[3, \infty)$. The domain D could be

A. $[0, \infty)$

B. $(0, \infty)$

C. $(-\infty, 0)$

D. $(-\infty, 0]$

E. $[-1, \infty)$

QUESTION 2

$f(x) = (x-1)(x+3)P(x) + m(x-1) + n$, where $P(x)$ is a polynomial and m and n are real numbers.

When $f(x)$ is divided by $(x-1)$ the remainder is – 10.
When $f(x)$ is divided by $(x+3)$ the remainder is 2.

The values of m and n are (respectively)

A. 2 and –10

B. 3 and –10

C. –3 and –10

D. – 10 and 3

E. 10 and 3

QUESTION 3
The equations $(a+2)x+y=3$ and $ax+2y=b$ will have a unique solution if

A. $a=-4$ and $b=3$

B. $a\in R\backslash\{-4\}$ and $b=3$ only.

C. $a=-4$ and $b\in R$

D. $a\in R\backslash\{2\}$ and $b\in R$

E. $a\in R\backslash\{-4\}$ and $b\in R$

QUESTION 4
Let $g:[-a,a]\rightarrow R, g(x)=3\sin\left(4\left(x+\frac{\pi}{6}\right)\right)$. If the inverse function with rule $g^{-1}(x)$ exists then the maximum possible value of a is

A. $-\frac{7\pi}{24}$

B. $-\frac{\pi}{24}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{6}$

E. $\frac{5\pi}{24}$

QUESTION 5
The graph of the function f is obtained from the graph of the function g with rule $g(x)=x^3-4x$ by a dilation of factor of 2 from the *y*-axis, a reflection in the *y*-axis and then a horizontal translation of 2 units in the negative direction of the *x*-axis. The rule of the function f is given by

A. $f(x)=4+2x-\frac{(x+2)^3}{8}$

B. $f(x)=2x-4-\frac{(x-2)^3}{8}$

C. $f(x)=4-2x-\frac{(x-2)^3}{8}$

D. $f(x)=8x-16-8(x-2)^3$

E. $f(x)=8x+16-8(x+2)^3$

QUESTION 6

The equation $(x+2)^2(x-3)^3 - p = 0$ has only one solution for x when the constant p satisfies the following inequalities

A. $-108 \le p \le 0$

B. $-108 \le p < 0$

C. $p > 1$ and $p < -109$

D. $p < 0$ and $p > -108$

E. $p > 0$ and $p < -108$

QUESTION 7

A particular function satisfies the functional equation $f(x+y) = f(x)f(y)$ where $x, y \in R^+$.
A possible rule for the function is

A. $f(x) = \log_e(x)$

B. $f(x) = e^{(x-1)}$

C. $f(x) = e^{-x}$

D. $f(x) = x^2 - 1$

E. $f(x) = \cos(-x)$

QUESTION 8

If $f(x) = x^3 g(x) - 3x$ and $g'(3) = 2$ and $g(3) = -1$ then $f'(3)$ equals

A. -81
B. -84
C. -87
D. 24
E. 27

QUESTION 9

The average rate of change of the function f with rule $f(x) = x^3 - \sqrt[3]{(28x - 48)}$ between $x = 2$ and $x = 4$, is closest to

A. 11

B. 24.4

C. 25.9

D. 27

E. 29.1

QUESTION 10

The function f is differentiable for all $x \in [0, 8)$ and satisfies the following conditions:

$f'(x) > 0$ for $x > 6$
$f'(x) = 0$ for $x = 6$
$f'(x) < 0$ for $1 < x < 6$
$f'(x) = 0$ for $x = 1$
$f'(x) < 0$ for $x < 1$

Which one of the following statements is necessarily true?

A. The graph of $y = f(x)$ is strictly decreasing for $0 < x < 6$.

B. The graph of $y = f(x)$ has a stationary point of inflection at $x = 6$.

C. The graph of $y = f(x)$ has a local maximum at $x = 6$.

D. The graph of $y = f(x)$ has a local minimum at $x = 1$.

E. The graph of $y = f(x)$ is strictly increasing for $6 \le x < 8$.

QUESTION 11

If $\int_1^k \left(\frac{3}{2x-1}\right) dx = \frac{3\log_e 5}{2}$, $k > 1$ then k equals

A. -2
B. 3
C. $-2, 3$
D. -3
E. $2, -3$

QUESTION 12

Given that $\int_{3}^{-1} 2f(x)\,dx = -5$ then $\int_{-1}^{3} \left(4f(x) - 3x\right)dx$ is

A. -32
B. -12
C. -5
D. -2
E. 8

QUESTION 13

If $\frac{d}{dx}(x\log_e x) = 1 + \log_e x$ then $\int(\log_e x)\,dx$ is

A. $x\log_e x + c$

B. $x\log_e x - x + c$

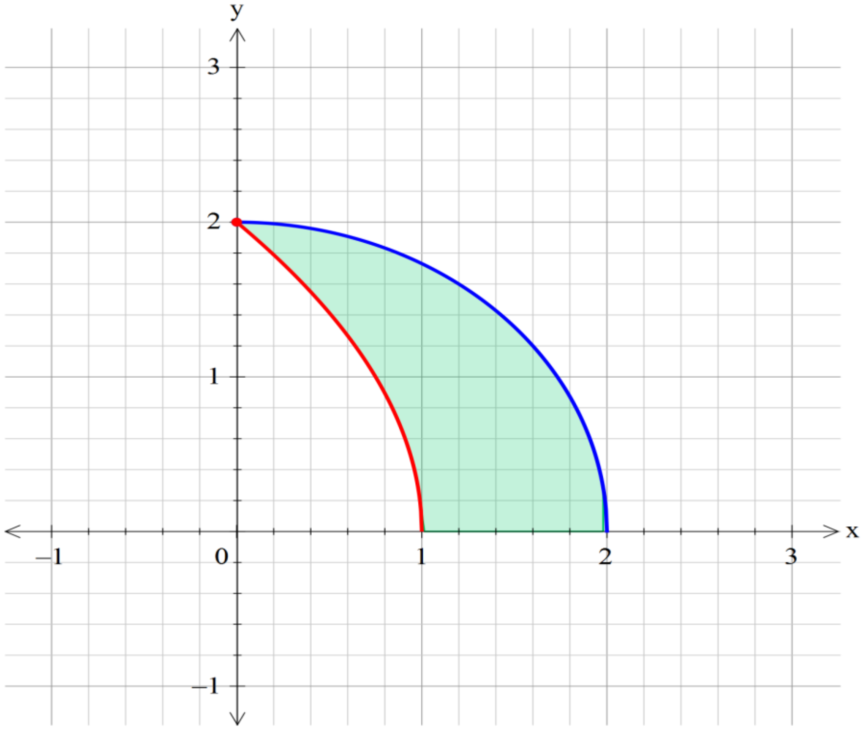
C. $x\log_e x + x + c$

D. $x\log_e x - 1 + c$

E. $x\log_e x + 1 + c$

QUESTION 14

The area between the x-axis and the two curves with equations $y = \sqrt{(4 - x^2)}$ and $y = 2\sqrt{(1 - x)}$ has been shaded below.



This area (in square units) has a value of

A. $\dfrac{3\pi - 4}{3}$

B. $\dfrac{10\pi - 13}{10}$

C. $\dfrac{3\pi - 2}{3}$

D. 1.808

E. $\dfrac{2\pi - 1}{2}$

QUESTION 15
A bag contains five blue socks and three red socks. Three socks are selected from the bag, without replacement.

The probability that they are **NOT** all the same colour is

A. $\frac{45}{56}$

B. $\frac{11}{56}$

C. $\frac{41}{42}$

D. $\frac{1}{42}$

E. $\frac{33}{256}$

QUESTION 16
Consider the discrete probability distribution with random variable X shown in the table below.

x	0	1	2	3	5
$\Pr(X = x)$	m	m	$2m$	n	0.1

Given that $E(X) = 2.5$, the values of m and the variance are respectively

A. 0.1 and 0.5

B. 0.55 and 1.65

C. 0.1 and 1.7

D. 0.1 and 1.65

E. 0.5 and 56.16

QUESTION 17
Westfold Stadium claims that 7 out of every 10 customers gain entry through the turnstile within 6 minutes of queuing at the entrance. Ten random customers have their entry times recorded. Given that at least eight customers gain entry within the six minutes, the probability that there are exactly nine of them who enter within the six minutes, correct to four decimal places, is

A. 0.1961

B. 0.1211

C. 0.3163

D. 0.3164

E. 0.3828

QUESTION 18
The random variable, X, has a normal distribution with mean 50 and variance 4. If the random variable, Z, has the standard normal distribution, then the probability that X lies **outside** the interval [42, 54] is

A. 0.0288

B. $\Pr(-4 < Z < 2)$

C. $\Pr(Z > -4) + \Pr(Z < 2)$

D. $\Pr(Z < -4) + \Pr(Z > 2)$

E. $\Pr(Z < -2) + \Pr(Z > 1)$

QUESTION 19
15% of oranges in an orchard are 5 cm or smaller in diameter and 30% are 8 cm or more in diameter. If the diameter of oranges is normally distributed, the mean and standard deviation, correct to 4 decimal places are

A. $\sigma = 1.9221$ and $\mu = 6.9921$

B. $\sigma = 8.3289$ and $\mu = 3.6323$

C. $\sigma = 5.8590$ and $\mu = 4.9275$

D. $\sigma = 3.6323$ and $\mu = 8.3289$

E. $\sigma = 2.0000$ and $\mu = 6.0000$

QUESTION 20

A company is conducting a survey to determine the popularity of the Liberal Party. They find that 55% of people support the opposing Party. If the company wishes to have a 95% level of confidence in their result with a margin of error of 1%, the number of people that should be surveyed is

A. 2233
B. 2234
C. 9507
D. 9508
E. 9604

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1 (18 marks)
There are three athletes exercising along a path called ***The Sharewalk Path***. This path is shared at any time between runners, walkers and riders.

Sarah, for these purposes called S_1, is a runner whose running plan follows a Normal distribution where her mean and variance of the time of running her usual distance is 60 minutes and 2 minutes respectively.

a. Find the probability that Sarah will run her usual distance in less than 58 minutes. Give your answer correct to four decimal places. 1 mark

Sharon, S_2, is a walker whose exercise plan for a certain number of walks, n, for the probability of finishing that walk, p, follows the Binomial distribution where her mean and variance is 30 and 5 respectively.

b. **(i)** Find the values of n and p for Sharon's Distribution. 2 marks

(ii) Find the probability that Sharon will finish between 30 and 36 walks, correct to four decimal places. 1 mark

(iii) Find the maximum number of walks that Sharon will complete if the probability of completing at least two walks is at most 99%. 3 marks

Susan, S_3, is a bike rider whose planned ride on ***The Sharewalk Path*** is frustrated by the number of times she has to slow down for others sharing the same path. The time for Susan's ride is described by the probability density function

$$S_3(t)=\begin{cases}\frac{1}{10}, & 0\le t<3\\ a\left(t-3\right)^2+\frac{1}{10}, & 3\le t\le 9\\ 0, & elsewhere\end{cases}$$

where t is the time in minutes.

c. **(i)** Show that $a=\frac{1}{720}$. 2 marks

(ii) Determine the expected time, in minutes, for Susan's ride on ***The Sharewalk Path.*** 1 mark

The records kept by a very large number of runners, walkers and riders of ***The Sharewalk Path*** indicate that 55% of runners run for more than 60 minutes per day. Sarah is a director of an athletics company and randomly samples 100 such runners.

d. **(i)** Find the standard deviation of the sample proportion, $\hat{P}$. Give your answer correct to four decimal places. 2 marks

(ii) Find the probability that more than 60% of the sample of 100 runners run for more than 60 minutes per day, correct to four decimal places. Do not use the Normal approximation. 2 marks

(iii) Using the Normal approximation, find the probability that more than 60% of the sample of 100 runners run for more than 60 minutes per day. Give your answer correct to three decimal places. 2 marks

Similar records kept by a very large number of walkers of ***The Sharewalk Path*** indicate that 20% of walkers complete their walk. Sharon is a director of the same athletics company as Sarah and randomly samples 200 walkers.

e. Determine a 95% confidence interval for the population proportion from this sample, correct to two decimal places. 2 marks

QUESTION 2 (18 marks)

Consider the hybrid function $h(t) = \begin{cases} (t-\pi)^2 + 3, 0 \le t < \pi \\ 3\sin(2t) + 3, \pi \le t \le 6 \end{cases}$

Find:

a. **(i)** $h(0)$ 1 mark

(ii) $h(\pi)$ 1 mark

(iii) $\{t : h(t) = 0\}$ 1 mark

b. **(i)** Is the graph $h(t)$ continuous at all points in its domain? Why/Why not? 2 marks

(ii) Write down the domain for which $h(t)$ is differentiable. 1 mark

c. Sketch the graph the graph of $h(t)$, labelling all significant features on the grid below.

3 marks

d. **(i)** On your graph in **part c.** shade the area that would find $\int_0^3 h(t)dt$. 1 mark

(ii) Show that the area under the graph of $h(t)$ from $t=0$ to $t=3$ is $\frac{(3-\pi)^3}{3}+\frac{\pi^3}{3}+9$.

2 marks

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(iii) Find the **left endpoint approximation** for the area under the graph of $h(t)$ from $t=0$ to $t=3$ using six equal width strips and explain whether this answer under or over estimates the area found in **part (ii)**. Give your answer correct to 3 decimal places. 2 marks

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The graph of $h(t)$ models the sales of cars (in $100,000s) in a particular car sales yard where t is measured in months.

e. (i) Find the average car sales over the six month period. Give your answer to the nearest $1,000. 2 marks

(ii) The manager of the car yard is in danger of losing his job if car sales fall below $300,000. For what proportion of the six months is the manager in danger of losing his job? Give your answer as a percentage to the nearest %. 2 marks

QUESTION 3 (11 marks)
Geoff is a truck driver who travels regularly between two different towns. Geoff takes exactly one hour to drive from a certain point, *A*, in one town to a certain point, *B*, in the second town. He knows that there are fixed speed cameras at points *A* and *B* that measure his average speed. The speed limit for trucks on Victorian roads is $100\,km/hr$. If the average speed recorded is more than $100\,km/hr$ then it is known that Geoff is speeding.

Geoff's velocity can be modelled by the function

$$v(t) = a\sin\left(\frac{5\pi}{3}(t+\beta)\right) + c$$

with $v(t)$ in km/hr, t in hours and a, β, c are real constants.

Geoff's maximum velocity is $120\,km/hr$ and he begins his journey at rest.

a. Show that $c + a = 0$ and $a = -60$. 2 marks

b. Show that the value of β is 0.3. 1 mark

c. Use calculus to find the equation describing the displacement of $x(t)$. 3 marks

d. How far has Geoff travelled halfway through his one hour trip? Give your answer in km, correct to 1 decimal place. 3 marks

e. For how many minutes during Geoff's trip is he speeding? Give your answer to the nearest minute. 2 marks

QUESTION 4 (13 marks)

Let $g:[-14,1] \to R, g(x) = -3\log_e(2-x)+2$.

a. **(i)** Using calculus, show there are no stationary point(s) on the graph of g. 2 marks

(ii) Hence find the value of x for which g has its maximum value. 1 mark

b. **(i)** Sketch the graph of g on the axes below, labelling the endpoints with their coordinates. 2 marks

(ii) Find the equation of the straight line that joins the endpoints of the graph of g .

1 mark

c. Find the area enclosed by the graph of $g(x)$ and the straight line that joins the endpoints of the graph of g . Give your answer correct to 1 decimal place.

2 marks

The mean value theorem states that between any two points on a graph there will exist at least one point where the instantaneous rate of change will equal the average rate of change.

Let point M be where $x = -14$ and point N be where $x = 1$.

d. **(i)** Find the coordinate(s) on the $g(x)$ graph that illustrate(s) the mean value theorem.
3 marks

(ii) Hence find the equation of, and sketch the tangent to, the graph, g, at the point found in **part d (i)**. Give your answer(s) correct to 3 decimal places. 2 marks

END OF QUESTION AND ANSWER BOOK