MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

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2018 Trial Examination

SOLUTIONS

SECTION A

Question 1
Answer: B

Explanation:
$\text{period} = \frac{2\pi}{n}$ and $\text{range} = [1-2, 1+2]$

Question 2
Answer: A

Explanation:

Define $f(x)=x^2-1$	*Done*
Define $g(x)=\frac{3-x}{4}$	*Done*
$g(f(2))$	0

Question 3
Answer: C

Explanation:
$RB + BR + RR = \left(\frac{5}{9}\times\frac{4}{8}\right) + \left(\frac{4}{9}\times\frac{5}{8}\right) + \left(\frac{5}{9}\times\frac{4}{8}\right)$

Question 4
Answer: E

Explanation:
The interval where the tangents to the graph have negative gradients.

Question 5
Answer: D

Explanation:
$y+5=b^{a-\frac{x}{2}} \rightarrow \log_b(y+5)=a-\frac{x}{2} \rightarrow \frac{x}{2}=a-\log_b(y+5) \rightarrow x=2\left(a-\log_b(y+5)\right)$

Question 6
Answer: A

Explanation:

Define $f(x)=5\cdot x+x^3$ *Done*

solve$\left(\dfrac{f(a)-f(0)}{a-0}=8,a\right)$ $a=-\sqrt{3}$ or $a=\sqrt{3}$

Question 7
Answer: E

Explanation:
$x'=-x,\ y'=\frac{1}{4}y \rightarrow x=-x',\ y=4y'$
$4y'=\sin\left(3\left(-x'-\frac{\pi}{2}\right)\right) \rightarrow y=-\frac{1}{4}\sin\left(3x+\frac{\pi}{2}\right)$

Question 8
Answer: B

Explanation:

$\text{solve}\left(\cos(2\cdot x)=\frac{1}{2},x\right)|-\pi\le x\le\pi$

$x=\frac{-5\cdot\pi}{6}$ or $x=\frac{-\pi}{6}$ or $x=\frac{\pi}{6}$ or $x=\frac{5\cdot\pi}{6}$

$\frac{-5\cdot\pi}{6}+\frac{-\pi}{6}+\frac{\pi}{6}+\frac{5\cdot\pi}{6}$ 0

Question 9
Answer: C

Explanation:

Define $h(x)=\frac{1}{(x-1)^2}$ *Done*

$h(x)\cdot h(-x)=h(x^2)$ true

Question 10
Answer: A

Explanation:
$Var(X) = E(X^2) - \left(E(X)\right)^2 = 4 - 5p - (2 - 3p)^2$

Question 11
Answer: E

Explanation:
Solve $20p = 2\sqrt{20p(1-p)}$ on CAS

Question 12
Answer: D

Explanation:
The area is below the x-axis between -2 and $\frac{1}{3}$, hence the negative sign.

Question 13
Answer: A

Explanation:

$$\text{solve}\left(\int_0^{\pi}(k\cdot\sin(x))\,dx=1,k\right)|0<k<2 \qquad k=\frac{1}{2}$$

Question 14
Answer: B

Explanation:
Solve $f'(x)=0$ for each option

Question 15
Answer: E

Explanation:

$$\text{solve}\left((1-p)^4=\frac{1}{625},p\right) \qquad p=\frac{4}{5} \text{ or } p=\frac{6}{5}$$

$$\text{binomCdf}\left(4,\frac{4}{5},3,4\right) \qquad 0.8192$$

Question 16
Answer: C

Explanation:

$\text{solve}(4+4\cdot(2\cdot p-1)\cdot(1-p)\geq 0,p)$ $\quad 0\leq p\leq \frac{3}{2}$

Question 17
Answer: C

Explanation:

$\text{invNorm}(0.1488,16,2)$ $\quad 13.9168122047$

Question 18
Answer: B

Explanation:
Since 6 is not included in the domain it cannot yield the maximum value.

Question 19
Answer: E

Explanation:

$\text{solve}\left(\frac{m}{2}\neq\frac{4}{m-2},m\right)$ $\quad m\neq -2 \text{ and } m\neq 4$

Question 20
Answer: A

Explanation:

$\frac{1}{4-0}\cdot\int_0^4 (x\cdot\sqrt{x})\,dx$ $\quad \frac{16}{5}$

SECTION B

Question 1

a. $\left(\frac{-\sqrt{6}}{6}, \frac{-\sqrt{6}}{9}\right)$ *and* $\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{9}\right)$

Define $f(x)=x-2\cdot x^3$	*Done*
$\text{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$	$x=\frac{-\sqrt{6}}{6}$ or $x=\frac{\sqrt{6}}{6}$
$f\left(\frac{-\sqrt{6}}{6}\right)$	$\frac{-\sqrt{6}}{9}$
$f\left(\frac{\sqrt{6}}{6}\right)$	$\frac{\sqrt{6}}{9}$

3 marks

b.

i. $(-2, 14)$ *and* $(2, -14)$
$y = -7x + k \rightarrow 14 = 14 + k \rightarrow k = 0$
Equation : $7x + y = 0$

2 marks

ii. $20\sqrt{2}$ *units*

$\sqrt{28^2+4^2}$	$20\cdot\sqrt{2}$

1 mark

c.

i. The 2 points will be $C(-2, 8k-2)$ and $D(2, 2-8k)$
$y = (1-4k)x + c \rightarrow 8k - 2 = -2(1-4k) + c \rightarrow c = 0$
Equation : $y = (1-4k)x$

2 marks

ii. $-1 = 5(1-4k) \rightarrow k = \frac{3}{10}$

1 mark

d.

i. $-a = a - ka^3$

$-2 = -ka^2$

$a = \sqrt{\frac{2}{k}}$

2 marks

ii.

$$\int_{-\sqrt{\frac{2}{k}}}^{0} \left(-x - x + k \cdot x^3\right) dx + \int_{0}^{\sqrt{\frac{2}{k}}} \left(x - k \cdot x^3 + x\right) dx$$

$$\frac{2}{k}$$

2 marks

Question 2

a. $\int_0^2 \frac{ct}{2}\, dt + \int_2^6 \frac{c}{4}(10 - t)\, dt = 1$

$\left[\frac{ct^2}{4}\right]_0^2 + \left[\frac{c}{4}\left(10t - \frac{t^2}{2}\right)\right]_2^6 = 1$

$c + \frac{c}{4}(60 - 18 - 20 + 2) = 1 \rightarrow c = \frac{1}{7}$

3 marks

b. $\int_1^2 \frac{t}{14}\, dt + \int_2^5 \frac{(10-t)}{28}\, dt$

$= \left[\frac{t^2}{28}\right]_1^2 + \left[\frac{20t - t^2}{56}\right]_2^5$

$= \frac{3}{28} + \frac{39}{56} = \frac{45}{56}$

2 marks

c. This is a conditional probability.

$$\frac{\Pr(T \le 1 \cap T \le 5)}{\Pr(T \le 5)} = \frac{\Pr(T \le 1)}{\Pr(T \le 5)} = \frac{\Pr(T \le 1)}{\Pr(T \le 1) + \Pr(1 \le T \le 5)} = \frac{\frac{1}{28}}{\frac{1}{28} + \frac{45}{56}} = \frac{2}{47}$$

2 marks

d. $\Pr(0 \le T \le 2) = \frac{1}{7}$

$\Pr(2 \le T \le a) = \frac{1}{7}$

$$\text{solve}\left(\int_2^a \left(\frac{\frac{1}{7}}{4} \cdot (10-t)\right) dt = \frac{1}{7}, a\right)$$

$$a=2.51668522645 \text{ or } a=17.4833147735$$

$a = 2.5167$

3 marks

e.

i. $Bi\left(7, \frac{3}{7}\right)$

$\Pr(T \ge 4) = \text{binomcd}f\left(7, \frac{3}{7}, 4\right) = 0.8734$

2 marks

ii. $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{27}{343}$

1 mark

f. $\Pr(T \ge 13) = 0.2312 \rightarrow \Pr\left(Z \ge \frac{2}{b}\right) = 0.2312$

$\frac{2}{\text{b}} = \text{invorm}(1 - 0.2312, 0, 1) \rightarrow \frac{2}{b} = 0.73490 \rightarrow b = 2.72$ days

3 marks

Question 3

a. $a = 3,\ b = -1,\ c = -3$

Dialtion by a factor of 3 from the x-axis, horizontal translation 1 unit to the left and vertical translation of 3 units down.

1 mark for each

b. $x = 3 \times 2^{y+1} - 3 \rightarrow \frac{x+3}{3} = 2^{y+1} \rightarrow y + 1 = \log_2\left(\frac{x+3}{3}\right) \rightarrow y = \log_2\left(\frac{x+3}{3}\right) - 1$

$f^{-1}(x) = \log_2\left(\frac{x+3}{3}\right) - 1$, Domain: $(-3, \infty)$

2 marks

c. 2.60 square units

$$\int_{-1}^{0}\left(3\cdot 2^{x+1}-3-\log_{2}\left(\frac{x+3}{3}\right)+1\right)dx$$

$$2.60085516211$$

2 marks

d.

i. Range of $f:(-3,\infty)$, Domain of $g: R\backslash\{0\}$
R_f is not part of D_g, hence $g(f(x))$ is not defined

2 marks

ii. $f(g(x)) = f\left(\frac{1}{x}\right) = 3 \times 2^{\frac{1}{x}+1} - 3$
Domain of $f(g(x))$: $R\backslash\{0\}$

2 marks

e.

i. $\text{tangentline}(f(x), x, -3)$
$y = 0.52x - 0.69$

2 marks

ii. $\tan(\theta) = 0.52 \rightarrow \theta = 27.5°$

1 mark

iii. $k = 1.837$

$$\text{tangentLine}\left(3\cdot 2^{x+1}-3, x, -3\right)$$
$$0.5199\cdot x - 0.6904$$

$$\text{tangentLine}\left(3\cdot 2^{x+1}-3, x, k\right)$$
$$4.159\cdot(2.)^{k}\cdot x - 4.159\cdot\left((k-1.443)\cdot(2.)^{k}+0.7\right.$$ ▸

$$\text{solve}\left(-0.39=4.1588830833597\cdot(2.)^{k}\cdot 0.5\right.$$ ▸
$$k=-2.997 \text{ or } k=1.837$$

Substitute (0.57, -0.39) into the tangent line at $x = k$ obtained on CAS

3 marks

Question 4

a. $\text{Max} = 45 + 30 = 75 \text{ m}, \quad \text{Min} = 45 - 30 = 15 \text{ m}$

1 mark for each

b.

i. $\text{Period}, T = \frac{2\pi}{\frac{\pi}{3}} = 6 \text{ minutes}$

Ride finishes in 12 minutes

2 marks

ii. After 3 minutes and 9 minutes

$\text{Define } h(t) = 45 - 30 \cdot \cos\left(\frac{\pi \cdot t}{3}\right)$ — *Done*

$\text{solve}(h(t) = 75, t) | 0 \le t \le 12$ — $t = 3 \text{ or } t = 9$

2 marks

c.

i.

$\frac{d}{dt}(h(t))$ — $10 \cdot \pi \cdot \sin\left(\frac{\pi \cdot t}{3}\right)$

1 mark

ii. $t = 3, 6, 9$

$\text{solve}\left(\frac{d}{dt}(h(t)) = 0, t\right) | 0 \le t \le 12$

$t = 0 \text{ or } t = 3 \text{ or } t = 6 \text{ or } t = 9 \text{ or } t = 12$

2 marks

d. $\tan^{-1}\left(\frac{75}{350}\right) = 12.09°$

1 mark

e. $CF = \frac{75}{\tan\alpha}$

1 mark

f. $CF = \frac{75}{\tan(26.09)} = 153.16 \text{ m}$

2 marks