2019 PRACTICE EXAM

MATHS METHODS UNITS 3&4

Exam 1 Question Booklet
Exam 2 Question Booklet
Worked Solution Booklet

ATARNotes

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ATARNotes

STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Written examination 1

2019

Reading time: 9:00 a.m. to 9:15 a.m. (15 minutes)
Writing time: 9:15 a.m. to 10:15 a.m. (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 10 pages
- Formula sheet
- Working space is provided throughout the book

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $f: R\backslash\{-2\} \to R, f(x) = \frac{\sin(x)}{x+2}$.
Differentiate f with respect to x.

1 mark

b. Let $g(x) = (3-x)\log_e(x^2)$.
Evaluate $g'(1)$.

2 marks

Question 2 (3 marks)

Let $f: R \to R$, $f(x) = \frac{1}{2}\tan\left(x + \frac{\pi}{4}\right)$.

a. Find $f'(x)$.

1 mark

Let θ be the angle from the positive direction of the x-axis to the tangent of the graph of f, measured in the anticlockwise direction.

b. Find the smallest positive value for x for which $\theta = 45°$.

2 marks

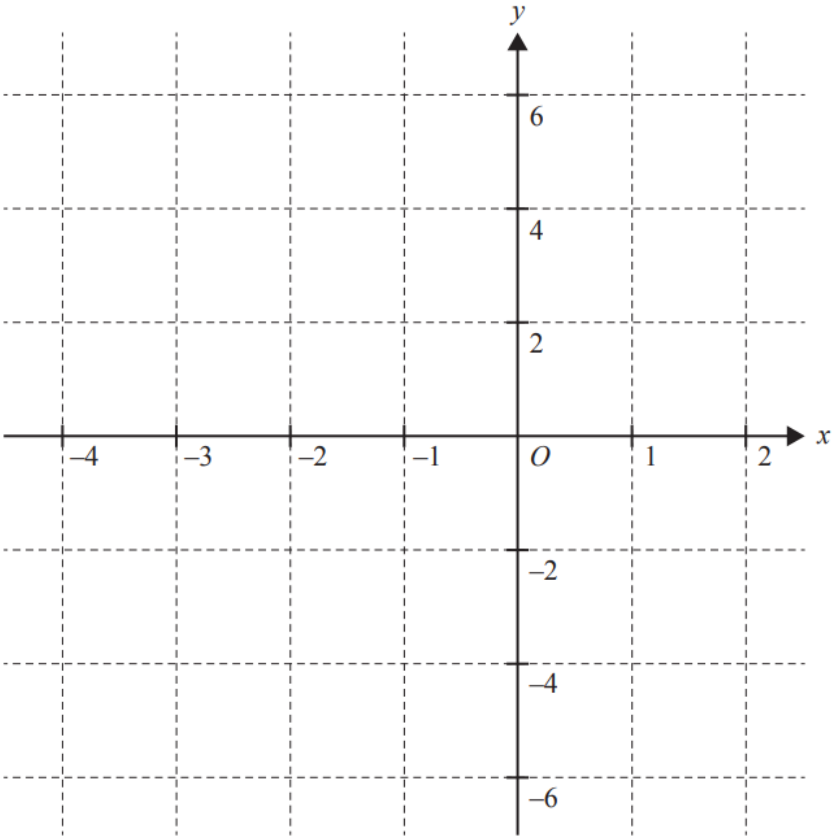
TURN OVER

Question 3 (5 marks)

Let $f: [-4,0) \to R, f(x) = \frac{2}{x^2} + 1$.

a. Sketch the graph of f. Label any endpoints and axis intercepts with their coordinates and label any asymptotes with the appropriate equation.

3 marks



b. Find the area enclosed by the graph of f, the lines $x = -3$ and $x = -1$, and the x-axis.

2 marks

Question 4 (6 marks)

In a large forest, one quarter of all trees are infected with cinnamon fungus. For a sample of size n taken from the population of trees, $\hat{P}$ is the random variable that represents the proportion of trees infected with cinnamon fungus.

Let $n = 3$.

a. Find $\mathrm{E}(\hat{P})$.

1 mark

b. Find $\Pr\left(\hat{P} \geq \frac{1}{3}\right)$.

2 marks

Let $n = 27$. Assume $\hat{P}$ is normally distributed.

c. Find $\mathrm{sd}(\hat{P})$. Express your answer in the form $\frac{1}{a}$, where a is a positive integer.

1 mark

d. Find $\Pr\left(\hat{P} \geq \frac{1}{3}\right)$.

2 marks

TURN OVER

Question 5 (5 marks)

Let $f: R \to R, f(x) = x^2$ and $g: [0, a) \to R, g(x) = 2\cos(x)$, where a is a real constant.

a. Find the rule of h, where $h(x) = f\big(g(x)\big)$.

1 mark

b.

i. State the value of a which gives the maximal domain of h.

1 mark

ii. For the value of a found in **part b. i.**, state the range of $h(x)$.

1 mark

c. Find the largest value of a such that g^{-1}, the inverse function of g, exists.

2 marks

Question 6 (2 marks)

Two events, A and B, from a given event space, are such that $\Pr(A) = \frac{2}{3}$ and $\Pr(B) = p$.

a. If A and B are independent events, find $\Pr(B|A')$ in terms of p.

1 mark

b. If A and B are mutually exclusive events, find $\Pr(B|A')$ in terms of p.

1 mark

TURN OVER

Question 7 (4 marks)

A toaster manufacturer puts their products through three independent tests for quality. The three tests occur in sequence, and toasters that do not pass a test are immediately rejected. The probability that a toaster will be rejected at the first test is $\frac{1}{2}$, at the second test is $\frac{2}{5}$ and at the third test is $\frac{1}{6}$.

a. Calculate the probability that a toaster will be rejected. Express your answer in the form $\frac{a}{b}$, where a and b are positive integers.

2 marks

b. Calculate the probability that, given a particular toaster is not rejected by the first test, the toaster will not be rejected. Express your answer in the form $\frac{c}{d}$, where c and d are positive integers.

2 marks

Question 8 (4 marks)

a. Solve $4\sin^2(x) + 1 = 4\sin(x)$ for x, where $0 \le x \le \pi$.

2 marks

b. Solve $2\log_3(3 - t) - \log_3(t + 7) = 2$ for t, where $-7 < t < 3$.

2 marks

Question 9 (8 marks)

Let $y = x\cos(2\pi x + k)$, where k is a real constant.

a. Find $\frac{dy}{dx}$.

1 mark

A probability density function f is given by

$$f(x) = \begin{cases} \sin(2\pi x + k) + 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

where $0 \le k < 2\pi$.

b. Using your result from **part a.**, or otherwise, show that the mean, μ, of this function is $\frac{1}{2\pi}(\pi - \cos(k))$.

4 marks

c. Find the maximum value of μ and the value for k at which this maximum occurs.

3 marks

END OF QUESTION AND ANSWER BOOK