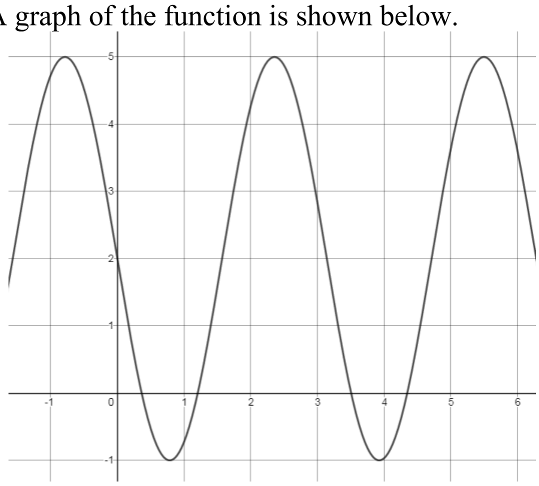
MATHS METHODS

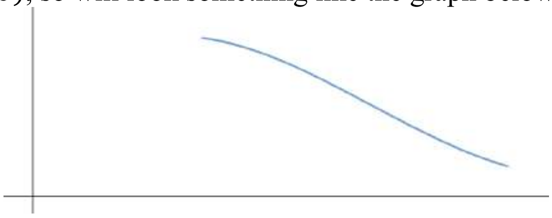
PRACTICE EXAM 2
SOLUTIONS
2019

Section A – Multiple-choice questions

Question	Answer	Notes
1	A	The period of a sine function of the form $y = a\sin(nx) + k$ is given by $$\text{period} = \frac{2\pi}{n}$$ For this function, $n = 2$ so the period will be $$\text{period} = \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$$ The amplitude of a sine function of the form $y = a\sin(nx) + k$ is a if $a > 0$ or $-a$ is $a < 0$. So the amplitude of this function is 3. The $+2$ add the end of the function f will translate the function up 2 units. This means that the range of the function will be $[2-3, 2+3] = [-1, 5]$. One nice way to do this question is just to graph the function on your calculator. A graph of the function is shown below.  By graphing the function, you can read off the range and also check if your value for the period matches with the graph.

2	E	We are told that the remainder is 5 when f is divided by $(x+1)$. Whenever we see the word 'remainder', it's likely that we'll be using the remainder theorem, and this is the case here. By the remainder theorem,
		$$f(-1) = 5$$
		Evaluating $f(-1)$ and setting it equal to 5 gives
		$$f(-1) = 2(-1)^3 - 3(-1)^2 - a(-1) + b = -2 - 3 + a + b$$
		$$= -5 + a + b = 5$$
		$$a + b = 10$$
		Thus the sum of a and b is 10.
		Most of this question can be done quite simply in your calculator, as follows:
		`define f(x)=2x^3-3x^2-a*x+b`
		`done`
		`solve(f(-1)=5,a`
		`{a=-b+10}`
		You would then just need to swap the b onto the other side of the equation.

3	E	Whenever you see the word 'midpoint' on an exam, it's likely that you'll be using the midpoint formula: $$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ $$(1, 1) = \left(\frac{4 + a}{2}, \frac{5 + b}{2}\right)$$ We can then solve the x-coordinate and y-coordinate separately to find the point P: $$1 = \frac{4 + a}{2}$$ $$2 = 4 + a$$ $$a = -2$$ $$1 = \frac{5 + b}{2}$$ $$2 = 5 + b$$ $$b = -3$$ So the coordinates of the point P are $(-2, -3)$.
4	E	There are probably a few good ways to do this question (trial and error would work, for instance), so here's one way. We can note that g is a 'positive' parabola, so the minimum value of -14 will occur at g's local minimum. We can differentiate g and set the derivative equal to zero to find this stationary point: $$g'(x) = 2x + b$$ $$g'(x) = 2x + b = 0$$ $$x = -\frac{b}{2}$$ So the x-coordinate of the local maximum is $-\frac{b}{2}$. We also know that the y-coordinate of this local maximum will be -14. So we can substitute in these values and then solve for b: $$-14 = \left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) + 2$$ $$-16 = \frac{b^2}{4} - \frac{b^2}{2} = -\frac{b^2}{4}$$ $$b^2 = 64$$ $$b = \pm 8$$ Since we are told in the question that $b > 0$, we can rule out the $b = -8$ solution. So $b = 8$. For the sake of time, this question should be exclusively done in your calculator. All the calculations can be done in 3 or 4 lines, as follows: `define f(x)=x^2+b*x+2` → done `solve(diff(f(x))` → $\left\{x=\frac{-b}{2}\right\}$ `solve(f(`$\frac{-b}{2}$`)=-14,b` → {b=-8,b=8} Note that the calculator used above is a Classpad. If you're using a TI-Inspire, you won't be able to use some of the shortcuts used above (like not specifying the variable for differentiation or when solving).

5	E	First, let's see why E is the correct answer. We are told that the function f is strictly decreasing over the interval (a, b). This means that f will have a negative gradient over the interval (a, b), so will look something like the graph below.  This means that $f(a)$, the y-coordinate when $x = a$, will be greater than $f(b)$, the y-coordinate when $x = b$. So: $$f(a) > f(b)$$ $$f(a) - f(b) > 0$$ so E is correct. So now let's debunk the other answers. We know no information about $f(b - a)$, so A and E are not necessarily true. We are not told any information about the points $\left(a, f(a)\right)$ and $\left(b, f(b)\right)$, as the interval (a, b) does not include those points. Thus, B and C are not necessarily true, as it is possible that the gradient is zero at both of those points.
6	E	The formula for a confidence interval is $$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$ This means that $\hat{p}$ will be halfway between the two values of the confidence interval. So: $$\hat{p} = \frac{0.85 + 0.93}{2} = 0.89$$ For a 95% confidence interval, $z = 1.96$. Using these values, we can solve for n: $$0.85 = \hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.89 - 1.96\sqrt{\frac{0.89(1-0.89)}{n}}$$ This equation can then be solved on the calculator, as follows: $\text{solve}\left(0.85=0.89-1.96\sqrt{\frac{0.89(1-0.89)}{n}},n\right)$ $\{n=235.0579\}$ Rounding this value gives $n = 235$.
7	D	Using the formula for average value: $$\frac{1}{b-a}\int_a^b f(x)\,dx = k$$ Rearranging to make the integral the subject yields $$\int_a^b f(x)\,dx = k(b-a)$$ Now we can evaluate the integral we're asked to solve in the question $$\int_a^b \left(2 - f(x)\right)dx = \int_a^b 2\,dx - \int_a^b f(x)\,dx = [2x]_a^b - k(b-a)$$ $$= 2b - 2a - kb + ka = (a-b)(k-2)$$

8	A	The formula for average rate of change is $$\frac{f(b)-f(a)}{b-a}$$ which is essentially just 'rise over run'.
		Substituting in the values from the question yields $$-3=\frac{f(b)-f(-2)}{b-(-2)}=\frac{(b^2-1)^2-((-2)^2-1)^2}{b-(-2)}$$ This can be solved on the calculator as follows:
		`define f(x)=(x^2-1)^2`
		`done`
		`solve((f(b)-f(-2))/(b-(-2))=-3,b`
		`{b=1}`
		Note that defining the function allows a much quicker use of the function than typing it out each time. It's less vital for these quick multiple choice questions, but is a great way to save time on longer multi-part extended response questions.

9	A	For g to exist, the range on the inside function ($\log_e(kx)$) needs to be a subset of the domain of the outside function ($\sqrt{x}$). The maximal domain of g will be the largest possible domain such that this condition is true. The domain of the inside function is $[0,\infty)$, as the number inside a square root cannot be negative. Thus, we want the range of $\log_e(kx)$ to be $[0,\infty)$. The domain which will give this range will start at the x-value of the x-intercept and go up to infinity. Finding this x-value: $$0=\log_e(kx)$$ $$e^0=kx$$ $$x=\frac{e^0}{k}=\frac{1}{k}$$ So the maximal domain of g will be $\left[\frac{1}{k},\infty\right)$.
10	D	(This explanation uses the 'dash method' for working out transformations. Throughout, x and y refer to the x and y pre-transformation, while x' and x' refer to the x and y post-transformation.) We can use the 'dash method' to see how the transformation T_1 will map the graph of $y=f(x)$. Simplifying the matrices of T_1 yields $$T_1\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}\frac{1}{2}x\\y\end{bmatrix}$$ so we can use the 'dash method' as follows: $$x'=\frac{1}{2}x,\qquad y'=y$$ Then we can follow the normal process with the 'dash method': rearrange these equations to make x and y the subject and then substitute these values into the function: $$x=2x',\qquad y=y'$$ $$y=\sqrt{x^3}$$ $$y'=\sqrt{(2x')^3}$$ This is our transformed function, g, so $g=\sqrt{2x^3}$. Expanding out these brackets gives: $$g(x)=\sqrt{8x^3}=2\sqrt{2}\sqrt{x^3}$$

		We are asked to find T_2, an equivalent transformation to T_1. All possible solutions for T_2 are of the form $$T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ ay \end{bmatrix}$$ where a is a real constant. So we just need to work out which value of a is correct. We know the starting point of the transformation ($y = \sqrt{x^3}$) and we know the ending point ($y = 2\sqrt{2}\sqrt{x^3}$). Basically what we're going to do is the backwards process to the one before: we have the 'before' and 'after' functions, we just need to work out what the transformation itself is. Rearranging these equations slightly, we can see that $$y = \sqrt{x^3}$$ should transform into $$\frac{y}{2\sqrt{2}} = \sqrt{x^3}$$ This means that $$x = x', \qquad y = \frac{y'}{2\sqrt{2}}$$ Rearranging these to make x' and y' the subject yields: $$x' = x\,, \qquad y' = 2\sqrt{2}y$$ Now we can compare these equations to the transformation T_2. Reading the equivalent equation out of the matrix equation for T_2 yields: $$x' = x\,, \qquad y' = ay$$ Thus, $a = 2\sqrt{2}$. Using the 'dash method' here is honestly a very long-winded way of doing this question: it's just here for the sake of rigour. A quicker way to do this question is by recognise and awareness of how the matrix form of a transformation will affect a graph. Of course, doing this is going to be a little more prone to errors, but will save you time.
11	E	We can use the formula $$Z = \frac{X - \mu}{\sigma}$$ to convert the Z values into their corresponding values on the X curve. For $Z = -1$: $$-1 = \frac{X-1}{2}$$ $$X = -2 + 1 = -1$$ For $Z = 2$: $$2 = \frac{X-1}{2}$$ $$X = 4 + 1 = 5$$ Thus, $\Pr(-1 < Z < 2) = \Pr(-1 < X < 5)$. However, this is not listed as a solution. To find an alternate solution, we can recognise that a normal distribution is symmetrical about the mean. This means that $\Pr(-1 < Z < 2)$ will be equal to $\Pr(-2 < Z < 1)$. Repeating the solving for X values as above yields: $$\Pr(-1 < Z < 2) = \Pr(-2 < Z < 1) = \Pr(-3 < X < 3)$$ This is best visualised by drawing quick sketches of each probability on a bell curve.

12	B	Two people are random chosen. Let's given names to these people to make the explanation a little clearer. Let's say the first person is Anna and the second is Briana. In what ways could Anna and Briana own a total of 2 pets? Well, there are three different ways: • Anna owns 2 pets and Briana owns none. • Anna and Briana own 1 pet each. • Briana owns 2 pets and Anna owns none. We can work out the probability for each of these scenarios individually. Since the number of pets owned by a person is independent of the number of pets that anyone else owns, we can use the independent events formula $$\Pr(A \cap B) = \Pr(A)\Pr(B)$$ So then: $\Pr(\text{Anna has 2 pets} \cap \text{Briana has no pets})$ $= \Pr(Anna\ has\ 2\ pets)\Pr(Briana\ has\ no\ pets)$ $= 0.24 \times 0.2 = 0.048$ $\Pr(\text{Anna has 1 pet} \cap \text{Briana has 1 pet})$ $= \Pr(Anna\ has\ 1\ pet)\Pr(Briana\ has\ 1\ pet)$ $= 0.34 \times 0.34 = 0.1156$ $\Pr(\text{Anna has no pets} \cap \text{Briana has 2 pets})$ $= \Pr(Anna\ has\ \text{no}\ pets)\Pr(Briana\ has\ 2\ pets)$ $= 0.2 \times 0.24 = 0.048$ The probability that they own two pets in total will be the sum of each of these probabilities: $$\Pr(2\ pets\ in\ total) = 0.048 + 0.1156 + 0.048 = 0.2116$$ You don't necessarily have to think about using the independent events formula here: a nice way to think about this question is with a big tree diagram where we're only interested in 3 particular paths (those which will give a total of 2 pets).
13	E	Noting the properties (like symmetry) of the standard normal curve, we can make the following observations: $$\Pr(Z < a) = 1 - \Pr(Z > a) = 1 - p$$ $$\Pr(Z < 0) = 0.5$$ $$\Pr(Z < -a) = p$$ $$\Pr(-a < Z < 0) = \Pr(Z < 0) - \Pr(Z < -a) = 0.5 - p$$ So we can evaluate the required probability: $\Pr(-a < Z < 0 \mid Z < a) = \dfrac{\Pr(-a < Z < 0 \cap Z < a)}{\Pr(Z < a)}$ $= \dfrac{\Pr(-a < Z < 0)}{\Pr(Z < a)} = \dfrac{0.5 - p}{1 - p} = \dfrac{-2}{-2}\left(\dfrac{0.5 - p}{1 - p}\right)$ $= \dfrac{2p - 1}{2p - 2}$

<table>
<tr>
<td>14</td>
<td>A</td>
<td>The player takes two throws. Let's say that X is the number of points that the player scores. X will be distributed binomially, with $n = 2$. So we can work out the probability that the player will score 0, 1 or 2 points:
$$\Pr(X = 0) = {}^2C_0 p^0 (1-p)^{2-0} = (1-p)^2$$
$$\Pr(X = 1) = {}^2C_1 p^1 (1-p)^{2-1} = 2p(1-p)$$
$$\Pr(X = 2) = {}^2C_2 p^2 (1-p)^{2-2} = p^2$$
Now that we have the probability of each possibility of X, the number of points scored, we can think of it as a discrete probability distribution.
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>$\Pr(X = x)$</td><td>$(1-p)^2$</td><td>$2p(1-p)$</td><td>p^2</td></tr>
</table>
Now we can use the formula for expected value to find the expected number of points:
$$\mu = \sum x\, p(x) = 0 \times (1-p)^2 + 1 \times 2p(1-p) + 2 \times p^2$$
$$= 2p - 2p^2 + 2p^2 = 2p$$
Instead of use the binomial formula, you can easily work out the different probabilities with a tree diagram.</td>
</tr>
<tr>
<td>15</td>
<td>D</td>
<td>For $f\big(g(x)\big)$ to exist, the range of g needs to be a subset of the domain of f. Since $g'(x) > 0$, we know that the range of g will be $[g(c), g(d)]$. This interval must be a subset of the domain of f, which is $[a, b]$. Thus, $a \le g(c) < g(d) \le b$.</td>
</tr>
<tr>
<td>16</td>
<td>D</td>
<td>A tree diagram is a nice way to represent this information. We can work out the probability as follows:
$$\Pr(\text{same colour}) = \Pr(\text{both are red}) + \Pr(\text{both are green})$$
$$= \left(\frac{3}{7} \times \frac{2}{6}\right) + \left(\frac{4}{7} \times \frac{3}{6}\right) = \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$$</td>
</tr>
<tr>
<td>17</td>
<td>D</td>
<td>$y = 3ax^2$ is a quadratic with a turning point at the origin. $y = mx + c$, where $m \neq 0$, is a straight line. This question is best done by drawings diagrams to visualise the graphs. For A and B, it's easy to draw a straight line graph, with either a positive or negative gradient, which won't intersect a quadratic. For C, $a > 0$ means we will have a 'positive quadratic', while the $c < 0$ means the y-intercept of the straight line will be below the x-axis. A quick sketch of this shows that it's easy for the two graphs to intersect. For D, $a < 0$ means we will have a 'negative quadratic', while the $c < 0$ means the y-intercept of the straight line will be below the x-axis. A quick sketch of this shows that the two graphs must intersect twice. The answer E is derived from the discriminant. Intersections between the graphs will be solutions to the equation $3ax^2 = mx + c$, so the discriminant of this equation will tell us how many solutions (which are intersections) there will be. For 2 intersections, the discriminant needs to be greater than zero. Thus:
$$\Delta = (-m)^2 - 4(3a)(-c) > 0$$
$$m^2 + 12ac > 0$$
$$m^2 > -12ac$$
Solving this for m yields $m > \sqrt{-12ac}$ and $m < -\sqrt{-12ac}$. If either of these conditions were true, there would be 2 intersections. However, neither is equivalent to answer E.</td>
</tr>
</table>

<table>
<tr><td>18</td><td>D</td><td>The first step is to find the stationary point of f. To find this, we can find the derivative of f and set that equal to zero, solving for x:
$$f'(x) = axe^{ax} + e^{ax} = e^{ax}(ax+1)$$
$$e^{ax}(ax+1) = 0$$
Using the null factor law, $e^{ax} = 0$ or $ax + 1 = 0$. The only solution for x is $x = -\frac{1}{a}$. This is the x-coordinate of the stationary point of f.
Now we want to set up the distance between two points formula:
$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{\left(-\frac{1}{a} - 0\right)^2 - \left(f\left(-\frac{1}{a}\right) - 0\right)^2}$$
We want to find when this distance is a minimum. So we need to differentiate this equation, set it equal to zero and solve for a. The whole question, including these last steps, can be solved in your calculator as follows:
define $f(x)=x*e^{a*x}+a$
done
diff(f(x),x
$a\cdot x\cdot e^{a\cdot x}+e^{a\cdot x}$
solve(ans=0,x
$\left\{x=\frac{-1}{a}\right\}$
define $g(a)=\sqrt{(\frac{-1}{a})^2+f(\frac{-1}{a})^2}$
done
diff(g(a),a
$\frac{-\sqrt{(a^4+1)\cdot e^2-2\cdot a^2\cdot e+1}\cdot \text{signum}(a)\cdot e^{-1}}{a^2}$ ▸
solve(ans=0,a
{a=-1.032240831,a=1.032240831}
Since $a \geq 0$, $a = 1.03$.</td></tr>
<tr><td>19</td><td>D</td><td>f is a probability density function, so the area under graph must be equal to one. Setting up this information in an equation:
$$\int_0^b \frac{1}{m}\sin\left(\frac{x}{m}\right)\,dx = 1$$
This can be solved in a calculator:
solve($\int_0^b \frac{1}{m}\sin(\frac{x}{m})dx=1$,b
$\left\{b=m\cdot\pi\cdot\text{constn}(1)-\frac{m\cdot\pi}{2}\right\}$
This constn(1) is a whole number and is there to encompass all the infinitely many possible solutions for b. Only the solution $b = \frac{\pi m}{2}$ makes sense for our probability density function, as it is the only value for b for which the function will always be above the x-axis.</td></tr>
<tr><td>20</td><td>D</td><td>Using the formula $\hat{P} = \frac{X}{n}$ yields
$$\Pr(\hat{P} > 0.8 \mid \hat{P} > 0.5) = \Pr(X > 8 \mid X > 5) = \frac{\Pr(X > 8 \cap X > 5)}{\Pr(X > 5)}$$
$$= \frac{\Pr(X > 8)}{\Pr(X > 5)}$$
X is a binomially distributed variable with $n = 10$ and $p = 0.6$. We can use the calculator's binomialCDf function to evaluate this:
$\frac{\text{binomialCDf}(9,10,10,0.6)}{\text{binomialCDf}(6,10,10,0.6)}$
0.07322249734
Note that the lower bounds are 9 and 6 respectively. This is because, for example, $\Pr(X > 8)$ does not include the event when $X = 8$.</td></tr>
</table>

Section B – Short-answer questions

Question 1 a.

Answer:

$$f'(x) = 2x\cos(x) + 2\sin(x)$$

Notes:

Given we have a few questions involving the same function, it's a good idea to define it in your calculator in the first question. We can solve this question in our calculator.

Question 1 b.

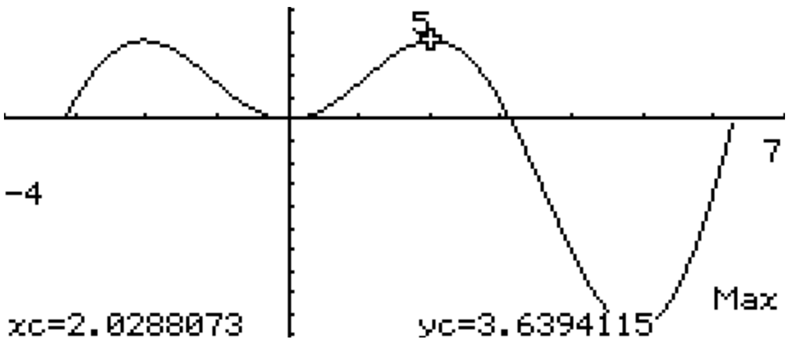
Answer:

$$[-9.63, 3.64]$$

Notes:

Your answers must be rounded to two decimal places, and square brackets (not round brackets) must be used.

We can plot $y = f(x)$ in our calculator, and use the fMax and fMin functions to find the highest and lowest y-values respectively. For example, when the fMax function is used, the calculator will find a point with the maximum y-value:



Alternatively, we could use the derivative found in the previous question. Setting this derivative equal to zero and solving for x finds the stationary points of the function. Substituting these x-values into the function f gives the corresponding y-values. Note that we also have to evaluate the y-values at the endpoints of the domain. Since the absolute maximum and minimum will occur at either a stationary point or an endpoint, the largest and smallest y-values found here will be the boundaries of the range.

```
define f(x)=2x*sin(x)
                                          done
diff(f(x),x
                          2·x·cos(x)+2·sin(x)
solve(ans=0,x)|-π≤x≤2π
◂7838,x=0,x=2.028757838,x=4.913180439}
f(-2.028757838)
                                   3.639411482
f(0)
                                             0
f(2.028757838)
                                   3.639411482
f(4.913180439)
                                  -9.628939779
f(-π)
                                             0
f(2π)
                                             0
```

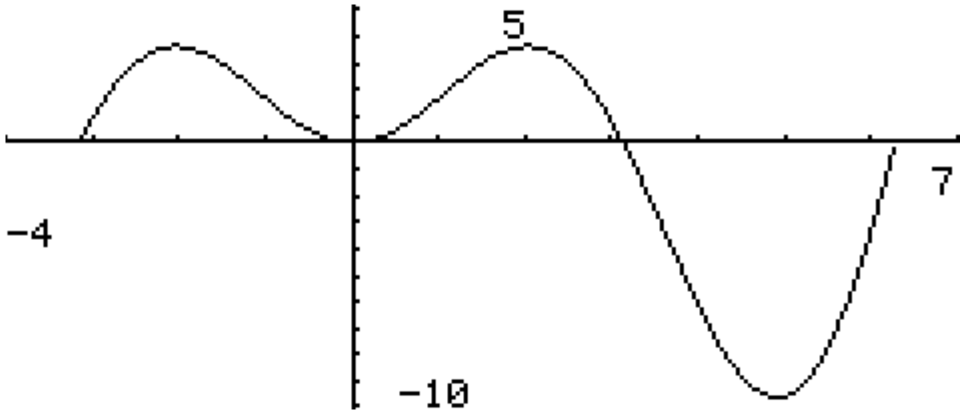
Question 1 c.

Answer:

10π

Notes:

We first need to look at the graph of $y = f(x)$:



We can see that there are three different regions bound by the graph and the x-axis. We will need to evaluate this area in two different integrals, as there is some area above and below the x-axis. Putting a negative sign in front of the integral evaluating the area below the x-axis will mean that the resulting number will be positive. So:

$$\text{Area} = \int_{-\pi}^{\pi} f(x)\,dx - \int_{\pi}^{\pi} f(x)\,dx$$

This can be evaluated in the calculator.

Question 1 d.

Answer:

$$c = -\frac{\pi}{2}$$

$$d = 2$$

Notes:

We need to work out how one vertical translation and one horizontal translation can map the graph of

$$y = 2x\sin(x)$$

onto the graph of

$$y = (2x + \pi)\cos(x) + 2$$

The trick here is to note that a graph of cos is equivalent to a graph of sin with a horizontal translation:

$$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$$

This allows us to rewrite the rule $y = (2x + \pi)\cos(x) + 2$ as:

$$y - 2 = 2\left(x + \frac{\pi}{2}\right)\sin\left(x + \frac{\pi}{2}\right)$$

Comparing our graphs pre- and post-transformation, we can observe that

$$x = x' + \frac{\pi}{2}$$

$$y = y' - 2$$

Rearranging these to make x and y the subject gives:

$$x' = x - \frac{\pi}{2}$$

$$y' = y + 2$$

We can now compare this result to the matrix form of T:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - \frac{\pi}{2} \\ y + 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{2} \\ 2 \end{bmatrix}$$

So $c = -\frac{\pi}{2}$ and $d = 2$.

Question 1 e.

Answer:

2.88

Notes:

g^{-1} will exist when g is a one-to-one function. g won't be a one-to-one function if the graph of g includes any turning points.

This means that the biggest different between a and b will mean the largest domain such that g contains no turning points. To find this, we can find the x-coordinate of all the potential turning points of g and find the biggest difference between these. Note that we also need to check the different between the endpoints and their nearest turning point, as one of these may give the solution.

```
solve(ans=0,x)|-π≤x≤2π
{x=-2.028757838,x=0,x=2.028757838,x=4▸
2π-4.913180439
                                   1.370004868
4.913180439-2.028757838
                                   2.884422601
2.028757838-0
                                   2.028757838
0-(-2.028757838)
                                   2.028757838
-2.028757838-(-π)
                                   1.112834816
```

The largest of these differences is 2.88.

Question 1 f.

Answer:

$$-\frac{8}{3}=\frac{1}{2\pi-(-\pi)}\int_{-\pi}^{2\pi}(2x\sin(x)+c)\,dx$$

$$c=-2$$

Notes:

The formula for average value is

$$\frac{1}{b-a}\int_{a}^{b}f(x)\,dx$$

Using the relevant values and formula:

$$-\frac{8}{3}=\frac{1}{2\pi-(-\pi)}\int_{-\pi}^{2\pi}(2x\sin(x)+c)\,dx$$

This can be solved on the calculator.

Remember that the formula for average value is not on the formula sheet, so you need to remember it (especially for Exam 1).

Question 1 g.

Answer:

Equation of the tangent: $y=-2x+c,\quad c=-13$

Notes:

The tangent line will be of the form $y=mx+k$. (I've changed the normal c to a k to avoid confusion with the c in the equation for h).

The tangent to the graph at $x=-\frac{\pi}{2}$ will have the same gradient as the graph of h at $x=-\frac{\pi}{2}$. To find the gradient of h, we need to differentiate it:

$$h'(x)=2x\cos(x)+2\sin(x)$$

$$h'\left(-\frac{\pi}{2}\right)=2\left(-\frac{\pi}{2}\right)\cos\left(-\frac{\pi}{2}\right)+2\sin\left(-\frac{\pi}{2}\right)=-\pi(0)+2(-1)=-2$$

So $m=-2$. Now we need to find the value of k, which we can do by substituting the point $\left(-\frac{\pi}{2},h\left(-\frac{\pi}{2}\right)\right)$ into the equation for the tangent.

$$y=mx+k$$

$$2\left(-\frac{\pi}{2}\right)\sin\left(-\frac{\pi}{2}\right)+c=-2\left(-\frac{\pi}{2}\right)+k$$

$$\pi+c=\pi+k$$

$$k=c$$

So the equation for the tangent is $y = -2x + c$.

We are told that this tangent passes through the point $(4, -21)$. Substituting in these values will mean we can solve for c:

$$y = -2x + c$$

$$-21 = -2(4) + c$$

$$c = -21 + 8 = -13$$

This question can be done in two lines or so in your calculator:

$$\text{tanLine}\left(f(x)+C, x, -\frac{\pi}{2}\right) \qquad -2\cdot x+C$$

$$\text{solve}(-21=-2\cdot(4)+C, C) \qquad \{C=-13\}$$

Even if you do this completely in your calculator, make sure that you include some working, as the question is worth more than one mark.

Question 1 h.

Answer:

$$-200\pi = \int_{-\pi}^{2\pi} (2x\sin(x) + c)\, dx$$

$$c = -66$$

Notes:

Here, it's important to note that $c \in R^-$, as specified earlier. This means that c is a negative number, so c will shift the graph of h down as c becomes a 'larger' negative number. This means that the area specific in the question will be wholly below the x-axis, so when the relevant integral is evaluated, a negative number will result.

$$\text{solve}\left(-200\pi=\int_{-\pi}^{2\pi} f(x)+c\,dx, c\right) \qquad \{c=-66\}$$

Notes:

If the above expression is solved with the value of 200π, a different value of c will be found. However, this c value is not a negative number, so is not a suitable solution.

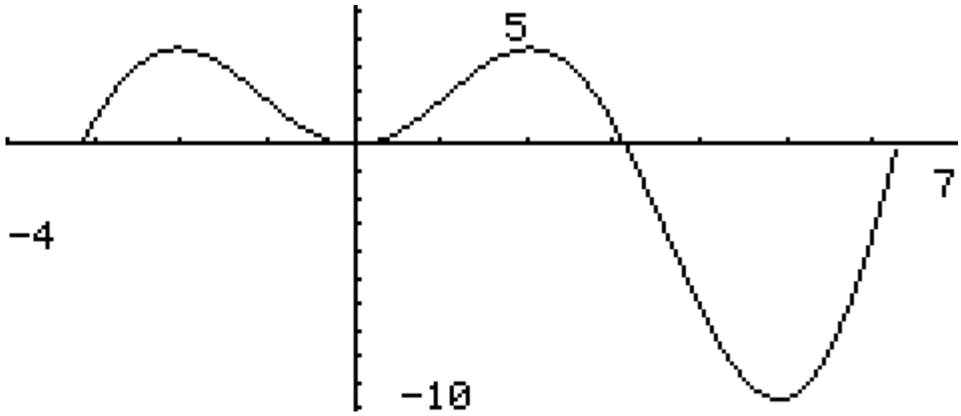
Question 1 i.

Answer:

$$c = -3.64$$

Notes:

A good way to think about this one is by looking at the graph of $y = h(x)$. The graph of $y = h(x)$ when $c = 0$ is shown below:



Since c is a negative number, it will shift the graph of h down. If you imagine shifting the above graph down, there's only one particular shift at which there are only two x-intercepts (when both of the local maxima are sitting on the x-axis).

This shift down will occur when c is equal in size to the y-coordinates of the maxima in the above graph. Thus, $c = -3.64$.

Question 2 a.

Answer:

$$t = \frac{100}{k}$$

Notes:

The maximum volume of bubbles is a stationary point of the graph $y = V(t)$. So we can differentiate $V(t)$, set it equal to zero and solve for t. This is best done in the calculator, as follows:

define $f(x)=k^2*x*e^{-\frac{k}{100}x}$

done

diff(f(x),x

$$\frac{-(k^3\cdot x-100\cdot k^2)\cdot e^{\frac{-k\cdot x}{100}}}{100}$$

solve(ans=0,x

$$\left\{x=\frac{100}{k}\right\}$$

Question 2 b.

Answer:

$$\frac{500}{e}$$

Notes:

To find the maximum amount of bubbles when $k = 5$, we can substitute the value of t from **part a.** into the equation for bubble volume. This can be done on the calculator. Since $k = 5$,

$$V = 500e^{-1} = \frac{500}{e}$$

The straight vertical line | on your calculator allows you to substitute in variables:

$$f\left(\frac{100}{k}\right)|k=5$$

$$500 \cdot e^{-1}$$

Question 2 c.

Answer:

$$[2.8, 13.2]$$

Notes:

We want the volume of bubbles remaining after 30 minutes to be greater than 100 litres. This means we're looking for solutions to the inequality

$$f(30) \geq 100$$

We could ideally solve this in our calculator:

$$\text{solve}(f(30)\geq 100, k$$

$$\left\{-\left(100 \cdot e^{\frac{3 \cdot k}{10}} - 30 \cdot k^2\right) \cdot e^{\frac{-3 \cdot k}{10}} \geq 0\right\}$$

but it isn't able to solve it. Instead, we can solve for when $f(30) = 100$:

$$\text{solve}(f(30)=100, k$$

$$\{k=-1.465457857, k=2.763563297, k=13.17\ldots$$

We have three different solutions for k. One is a negative value, which doesn't make sense as a percentage. So we can then look at the two positive values of k. We need to work out which values of k will satisfy the desired result. The interval of suitable values will involve $k = 2.8$ and $l = 13.2$, but we need to work out which values of k will work (e.g. do we need $k \geq 2.8$ or $k \leq 2.8$). To work this out, we can substitute in different values for k and test the water (for bubble volume):

$$f(30)|k=1 \qquad 22.22454662$$

$$f(30)|k=10 \qquad 149.3612051$$

$$f(30)|k=15 \qquad 74.98572663$$

We can see that the values of k outside $[2.8, 13.2]$ give a volume less than 100 litres, so suitable values of k will be in the region $[2.8, 13.2]$.

Question 2 d.

Answer:

$$\frac{a}{2} = ae^{-bt}$$

$$t = \frac{1}{b}\log_e(2)$$

Notes:

The initial volume can be found by evaluating $D(0)$:

$$D(0) = ae^{-b(0)} = ae^0 = a$$

which can also be found on the calculator:

```
define f(x)=a*e^(-b*x)
                                  done
f(0)
                                     a
```

Since the initial volume is a, half of the initial volume will be $\frac{a}{2}$. Setting this equal to $D(t)$, we can solve for t.

$$\frac{a}{2} = ae^{-bt}$$

$$\frac{1}{2} = e^{-bt}$$

$$\log_e\left(\frac{1}{2}\right) = -bt$$

$$t = -\frac{1}{b}\log_e\left(\frac{1}{2}\right) = \frac{1}{b}\log_e\left(\frac{1}{2}\right)$$

This can also be solved on your calculator:

```
solve(a/2=f(x),x
                              {x=ln(2)/b}
```

Question 2 e.

Answer:

To invert, swap y and t.

$$D^{-1}(t) = -\frac{1}{b}\log_e\left(\frac{t}{a}\right)$$

Notes:

Note that this step of writing 'to invert, swap y and t' is an essential step. If you don't write a statement like this one, your working does not show a logical flow and VCAA will take off a mark.

We then need to make y the subject:

$$\frac{t}{a} = e^{-by}$$

$$-by = \log_e\left(\frac{t}{a}\right)$$

$$y = -\frac{1}{b}\log_e\left(\frac{t}{a}\right)$$

Hence, $D^{-1}(t) = -\frac{1}{b}\log_e\left(\frac{t}{a}\right)$.

Question 2 f.

Answer:

$$1 : \frac{1}{e^2}$$

Notes:

We can work out the volume in each sink at $t = 2$.

In Sink 1 with $b = 1$:

$$D(2) = ae^{-(1)(2)} = ae^{-2}$$

In Sink 2 with $b = 2$:

$$D(2) = ae^{-(2)(2)} = ae^{-4}$$

The ratio will be given by:

$$ae^{-2} : ae^{-4}$$

$$1 : \frac{ae^{-4}}{ae^{-2}}$$

$$1 : \frac{1}{e^2}$$

The ratio can also be found in one line in your calculator:

$$\frac{f(2)|b=2}{f(2)|b=1} \qquad e^{-2}$$

Question 2 g.

Answer:

$$\frac{W^2}{a}$$

Notes:

We are told that, at a certain time, the volume of foam in Sink 1 is W. By setting $D(t)$ equal to W, we can find the time, in terms of a and W. Hence, when the volume in sink 1 is W, $t = \log_e\left(\frac{a}{W}\right)$.

We can substitute this value of t into the rule for the volume of foam in sink 2.

Question 3 h.

Answer:

$$t = \log_e(4)$$

Notes:

We are asked to find when the value of $\frac{dD}{dt}$ for Sink 1 is double the value of $\frac{dD}{dt}$ for Sink 2. We can differentiate the formulas in our calculator.

For Sink 1:

```
diff(f(x),x)|b=1
                                  -a·e^-x
```

For Sink 2:

```
diff(f(x),x)|b=2
                                  -2·a·e^-2·x
```

We want to find when

$$\frac{dD}{dt}(\text{for Sink 1}) = 2\frac{dD}{dt}(\text{for Sink 2})$$

which can be solved in our calculator:

```
solve(-a·e^-x=2*-2·a·e^-2·x,x
                                  {x=2·ln(2)}
```

We can use log laws to get this value in the desired form:

$$t = 2\log_e(2) = \log_e(2^2) = \log_e(4)$$

Question 3 a.

Answer:

$$\mu = \int_{15}^{30} t\,f(t)\,dt = 21.52$$

Notes:

The formula for mean is

$$\mu = \int_{\infty}^{\infty} x\, f(x)\, dx$$

Using the values and formula for this question:

$$\mu = \int_{15}^{30} t\, f(t)\, dt$$

This can be evaluated in the calculator:

$$\int_{15}^{30} t*f(t)dt$$

21.52173913

Question 3 b.

Answer:

$$\int_{15}^{m} f(t)\, dt = 0.5$$

$$m = 19.58$$

Notes:

The median, m, is the value which has 50% of the probability on either side of it. This means that the median will satisfy this equation:

$$\int_{15}^{m} f(t)\, dt = 0.5$$

We can solve this on the calculator:

$$\text{solve}(\int_{15}^{m} f(t)dt = 0.5, m$$

{m=9.44868599,m=19.5761843}

The value of $m = 9.45$ can be ruled out as an answer as $\int_{-\infty}^{9.45} f(t)\, dt = 0$.

We could alternatively solve this equation:

$$\text{solve}(\int_{m}^{30} f(t)dt = 0.5, m$$

{m=9.44868599,m=19.5761843}

Question 3 c.

Answer:

$$\Pr(T \geq 25) = \int_{25}^{30} f(t)\, dt = \frac{7}{23}$$

Notes:

The probability will be given by the area under the graph of $y = f(t)$ between $t = 25$ and $t = 30$. This can be evaluated on the calculator, but since the question is worth more than one mark, you need to write some working, even if you just do it all in the calculator.

Question 3 d.

Answer:

$$\Pr(T \leq 20 \mid T \leq 25) = \frac{\int_{15}^{20} f(t)\, dt}{\int_{15}^{25} f(t)\, dt} = \frac{37}{48}$$

Notes:

We can use the conditional probability formula to evaluate this probability:

$$\Pr(T \leq 20 \mid T \leq 25) = \frac{\Pr(T \leq 20 \cap T \leq 25)}{\Pr(T \leq 25)} = \frac{\Pr(T \leq 20)}{\Pr(T \leq 25)} = \frac{\int_{15}^{20} f(t)\, dt}{\int_{15}^{25} f(t)\, dt}$$

This can be evaluated on the calculator.

Question 3 e.

Answer:

$$\text{binomialCdf}\left(3, 5, 5, \frac{16}{23}\right) = 0.831$$

Notes:

As worked out in **part c.**, the probability that Eliza will be late for school is $16/23$.

The number of days, X, which Eliza is late for school is a binomially distributed random variable with $n = 5$ and $p = 16/23$. To evaluate the required probability, we can use the binomialCDf function in the calculator (CDf because there is a range of possible X values, not just the one).

$$\text{binomialCDf}\left(3, 5, 5, \frac{16}{23}\right)$$

0.8311204049

Question 3 f.

Answer:

$$Z = -0.8416\ldots$$

$$-0.8416\ldots = \frac{7-8}{\sigma}$$

$$\sigma = 1.19$$

Notes:

You would get 1 mark for either $Z = -0.8416\ldots$ or $-0.8416\ldots = \frac{7-8}{\sigma}$, or an equivalent equation, and 1 mark for $\sigma = 1.19$. Your answer must be rounded to 2 decimal places.

We know that 80% of the time Eliza's walk from the bus stop to school will take more than 7 minutes. Since the time taken for the walk is normally distributed, we can find the Z value that corresponds to this X value of 7. We can use the invNormCDf function to do this.

```
invNormCDf("R",0.8,1,0)
                         -0.8416212336
```

(Note that if you have a TI-Inspire calculator, you can't specify whether you want the area to be on the left or right: your calculator will automatically presume the area to be on the left. So when you have an area p which has its tail on the right, take $1-p$ to get the area on the left.)

Now we know that the X value of 7 corresponds to the Z value of $-0.8416\ldots$. So we can use the formula that converts between X and Z values, with standard deviation being the only unknown:

$$Z = \frac{X-\mu}{\sigma}$$

$$-0.8416\ldots = \frac{7-8}{\sigma}$$

This can be solved on the calculator:

$$\text{solve}\left(-0.8416212336=\frac{7-8}{s},s\right)$$

$$\{s=1.18818295\}$$

Question 3 g.

Answer:

$$^{10}C_3(0.4)^3(1-0.4)^{10-3} = 0.215$$

Notes:

The number of songs, X, released before 2000 is a binomially distributed random variable with $n = 10$ and $p = 0.4$. The binomial formula can then be used to evaluate the probability of $X = 3$:

$$\Pr(X=3) = {}^{10}C_3(0.4)^3(1-0.4)^{10-3}$$

This can be evaluated in the calculator using the binomialPDf function.

Question 3 h.

Answer:

$$\frac{\text{binomialCdf}(5, 10, 10, 0.4)}{\text{binomialCDf}(3, 10, 10, 0.4)} = 0.441$$

Notes:

We can use the formula

$$\hat{P} = \frac{X}{n}$$

to convert the $\hat{P}$ values into X values.

Then we can use the conditional probability formula to evaluate the probability:

$$\Pr\left(\hat{P} \geq 0.5 \mid \hat{P} \geq 0.3\right) = \Pr(X \geq 5 \mid X \geq 3) = \frac{\Pr(X \geq 5 \cap X \geq 3)}{\Pr(X \geq 3)} = \frac{\Pr(X \geq 5)}{\Pr(X \geq 3)}$$

Since X is binomially distributed, we can use the binomialCDf function to evaluate this:

$$\frac{\text{binomialCDf}(5,10,10,0.4)}{\text{binomialCDf}(3,10,10,0.4)} \quad 0.4406055335$$

Question 3 i.

Answer:

$$n = 36$$

Notes:

The formula for standard deviation is

$$\text{sd}\left(\hat{P}\right) = \sqrt{\frac{p(1-p)}{n}}$$

Substituting in our known values gives:

$$\frac{\sqrt{6}}{30} = \sqrt{\frac{0.4(1-0.4)}{n}}$$

This can be solved on the calculator,

Question 4 a.

Answer:

$$L = \sqrt{(2-(k-4)^3)^2 + k^2}$$

Notes:

The distance between two points is given by

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Substituting in the values for this question yields:

$$L = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(-(k-4)^3 + 2 - 0)^2 + (k-0)^2} = \sqrt{(2-(k-4)^3)^2 + k^2}$$

Question 4 b.

Answer:

$$L = 5, m = 1.56 \text{ and } m = 0.34$$

Notes

We can set the equation for L found in **part a.** equal to 5 and then solve for k.

$$L = \sqrt{(2-(k-4)^3)^2 + k^2} = 5$$

We then need to use these values for k to find values for m. We can do this with the formula for the gradient of a straight line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(k) - 0}{k - 0} = \frac{f(k)}{k}$$

Thus we can solve the question in the calculator as follows:

$\text{define } f(x) = -(x-4)^3 + 2$

done

$\text{define } L(k) = \sqrt{(f(k))^2 + k^2}$

done

solve(L(k)=5,k

{k=2.697448616,k=4.73743217}

$\frac{f(k)}{k} | k=2.697448616$

1.560719598

$\frac{f(k)}{k} | k=4.73743217$

0.3375203597

Question 4 c.

Answer:

$$\frac{dL}{dk} = 0, k = 3.31, m = 0.70$$

Notes:

We can take the equation for distance from **part a** and differentiate it with respect to k. Solving the resulting equation will find the value of k which gives the minimum distance. Again, we can then use the formula

$$m = \frac{f(k)}{k}$$

to find the value of m.

All these steps can be done in the calculator as follows:

```
diff(L(k),k
       3·k^5-60·k^4+480·k^3-1926·k^2+3889·
  ──────────────────────────────────────────▸
  (k^6-24·k^5+240·k^4-1284·k^3+3889·k^2-633
solve(ans=0,k
                          {k=3.311248739}
f(k)
──── |ans
 k
                             0.7026740769
```

Question 4 d.

Answer:

$$\int_0^k \frac{f(k)}{k} x \, dx + \int_k^{2^{\frac{1}{3}}+4} f(x) \, dx = 18$$

$$k = -3.97 \text{ and } k = 2.12$$

$$m = 4.11$$

Notes:

You would get 2 marks for a correctly set up integral equation for the area, but only 1 mark if there are small errors like incorrect bounds on the integrals. Note that the integral equation may use either k or m as the variable. Then, you would get 1 mark for the correct value of m.

The area described in the question will be given by two different integrals: the integral of the straight line $y = mx$ from 0 to k, and then the integral of the curve $y = -(x-4)^3 + 2$ from k until $2^{\frac{1}{3}} + 4$ (which is where the curve intersects the x-axis). Thus, we will need to set up two separate integrals.

The first integral will be

$$\int_0^k mx \, dx = \int_0^k \frac{f(k)}{k} x \, dx$$

while the second integral will be

$$\int_k^{2^{\frac{1}{3}}+4} (-(x-4)^3 + 2) \, dx$$

The sum of these two integrals must be equal to 18. We can solve this in the calculator and find the value of k, which we can then use to find m.

Question 4 e.

Answer:

$$\theta = 26.57$$

Notes:

Normally in an angle question, the angle asked for will be the angle from the positive direction of the x-axis to the line, measured in the anticlockwise direction. However, the angle required in this question is slightly different.

We can start off by finding the angle β from the positive direction of the x-axis to the straight road $y = 2x$, measured in the anticlockwise direction.

$$m = \tan(\beta)$$

$$2 = \tan(\beta)$$

This can be solved in our calculator, noting that the question is wanting an angle in degrees:

```
solve(2=tan(β),β
                {β=180·constn(1)+63.43494882}
```

To make this solution for β a little clearer, we could specify a possible domain for β. A quick sketch of the road shows that the angle is clearly going to be between 0° and 90°, so we can specify this:

```
solve(2=tan(β),β)|0<β<90
                              {β=63.43494882}
```

This angle is the angle between the straight road and the line $y = 0$. However, the question wants the acute angle between the straight road and the line $x = 0$. A quick sketch of the situation reveals that these two angles are complementary: they will sum to 90°. Hence:

$$\theta = 90 - \beta = 90 - 63.43 = 26.57$$

Question 4 f.

Answer:

$$\sqrt{(2a-3)^2 + (a-3)^2}$$

Notes:

Once again, we can use the distance between two points formula:

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Substituting in the point $(a, 2a)$ and $(3,3)$ yields:

$$\sqrt{(2a-3)^2 + (a-3)^2} = \sqrt{5a^2 - 18a + 18}$$

Question 4 g.

Answer:

$$T=\frac{\sqrt{(3a-0)^2+(a-0)^2}}{6}+\frac{\sqrt{(2a-3)^2+(a-3)^2}}{3}=\frac{\sqrt{5}a}{6}+\frac{\sqrt{5a^2-18a+18}}{3}$$
$$=\frac{2\sqrt{5a^2-18a+18}+\sqrt{5}a}{6}$$

Notes:

You would receive 2 marks if the working clearly showing an understanding of how T is derived. No mathematical leaps of logic should be present in the working.

The total time taken will be the sum of the time taken to travel along the straight road to $(a, 2a)$ and the time taken to travel from $(a, 2a)$ to $(3,3)$. Let's let these times be T_1 and T_2 respectively.

Let's first think about T_1. The formula for time taken is

$$T=\frac{d}{s}$$

where d is the distance and s is the speed. The distance for T_1 will be the distance from the origin to the point $(a, 2a)$. Hence:

$$T_1=\frac{d_1}{s_1}=\frac{\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}}{6}=\frac{\sqrt{(3a-0)^2+(a-0)^2}}{6}=\frac{\sqrt{5}a}{6}$$

Now for T_2, the distance will be that found in **part f.** of the question. Hence:

$$T_2=\frac{d_2}{s_2}=\frac{\sqrt{(2a-3)^2+(a-3)^2}}{3}$$

The total time will be the sum of T_1 and T_2:

$$T_{total}=T_1+T_2=\frac{\sqrt{5}a}{6}+\frac{\sqrt{(2a-3)^2+(a-3)^2}}{3}=\frac{2\sqrt{5a^2-18a+18}+\sqrt{5}a}{6}$$

Question 4 h.

Answer:

$$\frac{dT}{da}=\frac{10a+\sqrt{5(5a^2-18a+18)}-18}{6\sqrt{5a^2-18a+18}}=0$$
$$a=\frac{1}{5}\left(9-\sqrt{3}\right)$$

Notes:

The minimum time will occur when $\frac{dT}{da} = 0$. We need to derive T with respect to a, set it equal to zero and then solve for a. This is best done on the calculator:

```
define f(a)=(√5 a+2√(5·a²−18·a+18))/6
                                        done
diff(f(a),a
        (10·a+√(5·(5·a²−18·a+18)) −18)/(6·√(5·a²−18·a+18))
solve(ans=0,a
                                {a=−√3/5+9/5}
```

Since this question is worth more than one mark, it's necessary to include some working, even if you did all your working on your calculator.

Question 4i.

Answer:

For the straight road section:

$$D(x) = \sqrt{3}x$$

$$\text{average distance from origin} = \frac{1}{2.54 - 0}\int_0^{2.54} \sqrt{3}x\, dx = 2.20$$

For the curved road section:

$$D(x) = \sqrt{x^2 + (2 - (x - 4)^3)^2}$$

$$\text{average distance from origin} = \frac{1}{3 - 2.54}\int_{2.54}^{3} \sqrt{x^2 + (2 - (x - 4)^3)^2}\, dx = 2.61$$

Overall:

$$\frac{2.20 + 2.61}{2} = 2.41$$

Notes:

Here, there is 1 mark for $D(x) = \sqrt{3}x$, or an equivalent expression, 1 mark for 2.20 (Kaylee's average distance from the origin over the straight road section), 1 mark for $D(x) = \sqrt{x^2 + (2-(x-4)^3)^2}$, or an equivalent expression, 1 mark for 2.61 (Kaylee's average distance from the origin over the curved road section), and 1 mark for 2.41.

The question asks for Kaylee's average distance from the origin over her journey's duration. We need to look at the two sections of the journey individually.

For the journey on the straight road:

Kaylee will journey from the origin $(0,0)$ to the intersection of the two roads. We can find this intersection by solving the equation

$$2x = -(x-4)^3 + 2$$

which we can do in the calculator as follows:

```
solve(2x=-(x-4)^3+2,x
                          {x=2.543835754}
-(x-4)^3+2|ans
                              5.087671508
```

So Kaylee will travel from $(0, 0)$ to $(2.54, 5.09)$. To find average distance from the origin, we first need an equation which will tell us Kaylee's distance from the origin. So let's set up this function:

$$D(x) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(2x - 0)^2 + (x - 0)^2} = \sqrt{3}x$$

where D is the distance from Kaylee's position to the origin and x is the x-coordinate of Kaylee's position.

Then to find the average distance, we can use the average value formula:

$$\text{average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Substituting in the values and formula for this section of Kaylee's journey yields:

$$\text{average distance from origin} = \frac{1}{2.54 - 0}\int_0^{2.54} \sqrt{3}x\,dx$$

This can be solved in the calculator:

$$\frac{1}{2.543835754-0}\int_0^{2.543835754} \sqrt{3}\,x\,dx$$

$$2.203026386$$

Now we need to repeat the process looking at the second section of Kaylee's journey: along the curved road from $(2.54, 5.08)$ to her destination $(3,3)$.

For this section:

$$D(x) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(-(x-4)^3 + 2 - 0)^2 + (x-0)^2}$$
$$= \sqrt{x^2 + (2 - (x-4)^3)^2}$$

We can use the average value formula to find the average distance from the origin over this section of Kaylee's journey:

$$\text{average distance from the origin} = \frac{1}{3 - 2.54}\int_{2.54}^{3} \sqrt{x^2 + (2 - (x-4)^3)^2}\ dx$$

Again this can be solved in the calculator:

```
define f(x)=-(x-4)^3+2
                                    done
define D(x)=√(f(x)^2-x^2)
                                    done
     1              3
─────────────       ∫       D(x)dx
3-2.543835754  2.543835754
                               2.610564908
```

So now we know that over the first section of her journey Kaylee's average distance from the origin is 2.20 and that over the second section her average distance from the origin is 2.61. Since we are told that the amount of time that she will spend on the straight road will be the same as the length of time she'll spend on the curved road, the overall average distance from the origin will be the average of these two values:

$$\text{average distance from the origin} = \frac{2.20 + 2.61}{2} = 2.41$$

which we should find in our calculator to make sure the decimals will all round correctly:

```
2.203026386+2.610564908
───────────────────────
           2
                               2.406795647
```