

Student Name……………………………………

### MATHEMATICAL METHODS UNITS 3 & 4

### TRIAL EXAMINATION 1

**2019**

#### Reading Time: 15 minutes

Writing time: 1 hour

###### Instructions to students

This exam consists of 10 questions.

All questions should be answered in the spaces provided.

There is a total of 40 marks available.

The marks allocated to each of the questions are indicated throughout.

Students may **not** bring any calculators or notes into the exam.

Where a numerical answer is required, an exact value must be given unless otherwise directed.

Where more than one mark is allocated to a question, appropriate working must be shown.

Diagrams in this trial exam are not drawn to scale.

A formula sheet can be found on pages 14 and 15 of this exam.

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**Question 1** (3 marks)

1. If . 1 mark
2. Let .

Evaluate . 2 marks

**Question 2** (3 marks)

Given that . 3 marks

**Question 3** (2 marks)

Find all the possible solutions for . 2 marks

**Question 4** (2 marks)

In a large population of school leavers, the proportion who play organized sport is 0.2.

The random variable  represents the sample proportion of school leavers who play organized sport, for samples of *n* school leavers drawn from the population.

Find the largest sample size that can be drawn from the population such that the standard deviation of  will have a minimum of 0.04. 2 marks

**Question 5** (3 marks)

The random variable *X* is normally distributed. It has a mean of 20, a variance of 9 and .

The random variable *Z* is the standard normal distribution.

1. Find in terms of *q*. 1 mark
2. Find in terms of *q*. 2 marks

**Question 6** (6 marks)

Let .

1. Sketch the graph of *f* on the axes below. Label the axis intercepts with their coordinates and label any asymptotes with their equation. 3 marks



1. State the rule and domain of . 2 marks
2. Hence find the values of *x* for which . 1 mark

**Question 7** (5 marks)

The discrete random variable *X* has the probability mass function



1. Show that . 2 marks
2. Given that . 2 marks
3. Evaluate . 1 mark

**Question 8** (5 marks)

Let . The graph of *f* is shown below.



The point *P* lies on the graph of *f*.

The point  lies on the *x*-axis.

1. Find the minimum distance from *P* to *Q.* Express your answer in the form where *a* and *b* are positive integers. 3 marks
2. The transformation *T* maps the graph of  onto the graph of  where

.

Show that the point  lies on the graph of *h*. 2 marks

**Question 9** (5 marks)

Let .

1. Show that . 1 mark
2. Hence evaluate , where *n* is a positive, even integer. 2 marks

Part of the graph of *f* is shown below. One of the *x*-intercepts of this graph occurs at . The region enclosed by the graph of *f*, the *x*-axis and the line  has been shaded.



1. Given that  and using your result from part **b.** or otherwise, find the area of the shaded region. 2 marks

**Question 10** (6 marks)

Let .

1. Show that the solution to the equation . 2 marks

Let *A* be the area bounded by the curves  and the lines . This area *A* is shaded in the graph below.



The corner points of area *A* are *P*, *Q*, *R* and *S*.

1. Write down, but do **not** attempt to evaluate, a definite integral, that gives the value of *A.* 1 mark
2. The area of the trapezium *PQRS* is used to approximate *A*.

The area of the trapezium *PQRS* can be found using the result from part **a.** and can be expressed in the form  where *c* and *d* are simplified expressions in terms of *e*.

Find *c* and *d.* 3 marks

**Mathematical Methods formulas**

## Mensuration

|  |  |  |  |
| --- | --- | --- | --- |
| area of a trapezium |  | volume of a pyramid |  |
| curved surface area of a cylinder |  | volume of a sphere |  |
| volume of a cylinder |  | area of a triangle |  |
| volume of a cone |  |  | |

## Calculus

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
| product rule |  | quotient rule |  |
| chain rule |  |  | |

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## Probability

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
| mean |  | variance |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Probability distribution** | | **Mean** | **Variance** |
| discrete | = |  |  |
| continuous |  |  |  |

## Sample proportions

|  |  |  |  |
| --- | --- | --- | --- |
|  | | mean |  |
| standard  deviation |  | approximate  confidence  interval |  |