# 2019 VCE Mathematical Methods Trial Examination 1

# Detailed Answers


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**Question 1**

**a.** Let $y=\dfrac{\sin(2x)}{x^2}$ using the quotient rule

$u=\sin(2x) \qquad v=x^2$

$\dfrac{du}{dx}=2\cos(2x) \qquad \dfrac{dv}{dx}=2x$ M1

$\dfrac{dy}{dx}=\dfrac{2x^2\cos(2x)-2x\sin(2x)}{x^4}=\dfrac{2x\left(x\cos(2x)-\sin(2x)\right)}{x^4}$

$\dfrac{dy}{dx}=\dfrac{2\left(x\cos(2x)-\sin(2x)\right)}{x^3}=\dfrac{2f(x)}{x^3}$

$f(x)=x\cos(2x)-\sin(2x)$ A1

**b.** Let $y=g(x)=\tan\left(\dfrac{2}{x}\right)=\tan(u) \qquad u=\dfrac{2}{x}=2x^{-1}$ chain rule

$\dfrac{dy}{du}=\dfrac{1}{\cos^2(u)} \qquad \dfrac{du}{dx}=-2x^{-2}=-\dfrac{2}{x^2}$

$g'(x)=-\dfrac{2}{x^2\cos^2\left(\dfrac{2}{x}\right)}$ M1

$g'\left(\dfrac{6}{\pi}\right)=-\dfrac{2}{\left(\dfrac{6}{\pi}\right)^2\cos^2\left(\dfrac{\pi}{3}\right)}=-2\times\dfrac{\pi^2}{36}\times\dfrac{1}{\left(\dfrac{1}{2}\right)^2}$

$g'\left(\dfrac{6}{\pi}\right)=-\dfrac{2\pi^2}{9}$ A1

**Question 2**

$X\overset{d}{=}Bi(n=7,\ p=?)$

$M=\Pr(X=3)=\dbinom{7}{3}p^3(1-p)^4=35p^3(1-p)^4$ differentiating using the product rule A1

$\dfrac{dM}{dp}=35\left[\left(\dfrac{d}{dp}\left(p^3\right)\right)\times(1-p)^4+p^3\times\dfrac{d}{dp}\left((1-p)^4\right)\right]$

$\dfrac{dM}{dp}=35\left[3p^2(1-p)^4-4p^3(1-p)^3\right]$

$\dfrac{dM}{dp}=35p^2(1-p)^3\left[3(1-p)-4p\right]$ M1

$\dfrac{dM}{dp}=35p^2(1-p)^3(3-7p)=0$ since $M$ has the largest probability

since $0<p<1$ , $p=\dfrac{3}{7}$ A1

**Question 3**

**a.** $2-4\cos\left(\frac{\pi x}{3}\right)=0$

$4\cos\left(\frac{\pi x}{3}\right)=2$

$\cos\left(\frac{\pi x}{3}\right)=\frac{1}{2}$

$\frac{\pi x}{3}=2n\pi\pm\cos^{-1}\left(\frac{1}{2}\right)=2n\pi\pm\frac{\pi}{3}=\frac{\pi}{3}(6n\pm1)$

$x=6n\pm1\ ,\ n\in Z$ A1

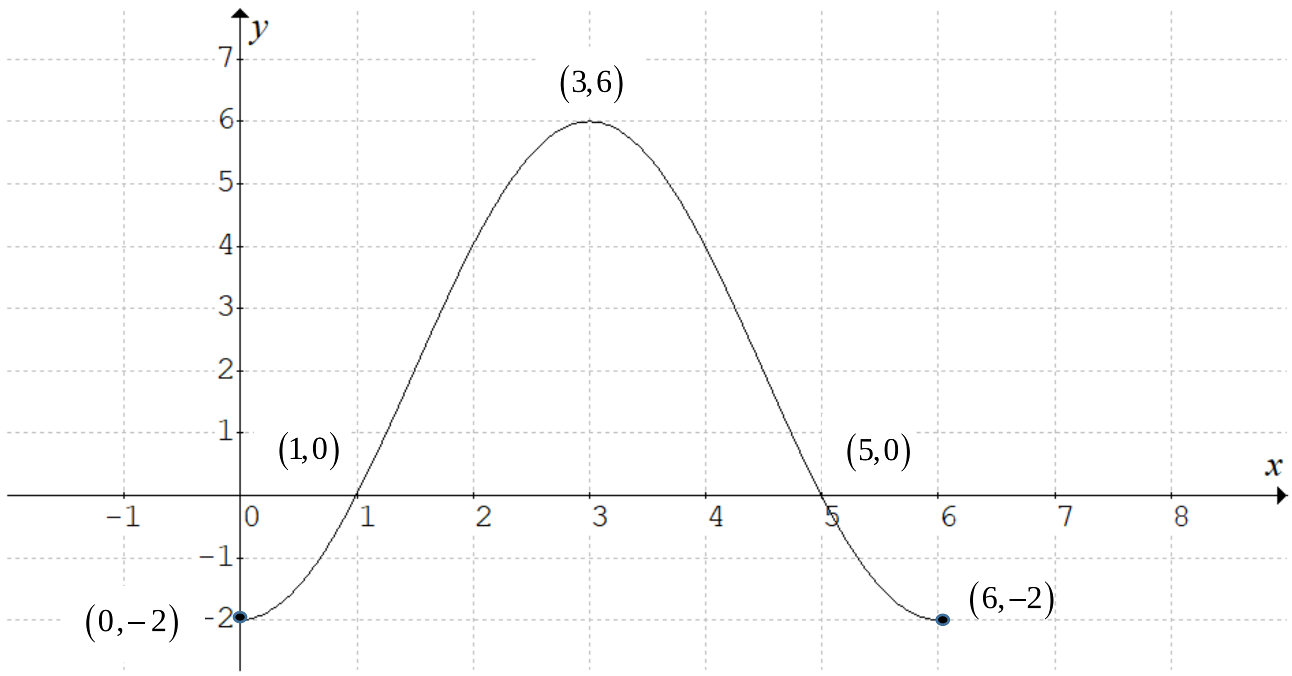
**b.** endpoints $f(0)=2-4\cos(0)=-2\ ,\ f(6)=2-4\cos(2\pi)=-2$

maximum at $y=6,$ when $\cos\left(\frac{\pi x}{3}\right)=-1\ ,\ \frac{\pi x}{3}=\pi\ ,\ x=3$

crosses $x$-axis when $2-4\cos\left(\frac{\pi x}{3}\right)=0$ from **a.** $x=1,5$

endpoints $(0,-2)\ ,\ (6,-2)$ maximum $(3,6)$, $x$-intercepts $(1,0)\ ,(5,0)$ A1

G1



**Question 4**

$y=\sqrt{5+cx}=(5+cx)^{\frac{1}{2}}$ differentiating using the chain rule

$\frac{dy}{dx}=\frac{c}{2\sqrt{5+cx}}$

at the point $x=2$, the gradient is $\left.\frac{dy}{dx}\right|_{x=2}=\frac{c}{2\sqrt{5+2c}}$ M1

this makes an angle of $135^0$, so the gradient $m=\tan\left(135^0\right)=-1$

so $\frac{c}{2\sqrt{5+2c}}=-1$ and therefore $c<0$ A1

$c=-2\sqrt{5+2c}$ squaring both sides

$c^2=4(5+2c)=20+8c$

$c^2-8c-20=0$ M1

$(c-10)(c+2)=0$

$c=10\ ,\ -2$ but $c<0$

$c=-2$ only A1

**Question 5**

**a.** $\frac{d}{dx}\left[\log_e\left(x^2+4\right)\right]=\frac{2x}{x^2+4}$ A1

**b.** the average value $\bar{f}=\frac{1}{b-0}\int_0^b\frac{x}{x^2+4}\,dx=\log_e\left(\sqrt[b]{2}\right)=\log_e\left(2^{\frac{1}{b}}\right)$

from **a.** $\frac{1}{b}\int_0^b\frac{x}{x^2+4}\,dx=\left[\frac{1}{2b}\log_e\left(x^2+4\right)\right]_0^b=\frac{1}{b}\log_e(2)$ A1

$=\frac{1}{2}\left[\log_e\left(b^2+4\right)-\log_e(4)\right]=\frac{1}{2}\log_e(4)$

$=\frac{1}{2}\log_e\left(\frac{b^2+4}{4}\right)=\frac{1}{2}\log_e(4)$

$\frac{b^2+4}{4}=4$ A1

$b^2+4=16$

$b^2=12$

$b=\pm\sqrt{12}$ but $b>0$

$b=2\sqrt{3}$ only A1

**Question 6**

**a.** $x$ is the number of broken biscuits, in a total of 12, so $\hat{P} = \dfrac{x}{12}$

$\Pr\left(\hat{P} = \dfrac{1}{12}\right) = \Pr(\text{one broken}) = \Pr(X = 1)$, $X \overset{d}{=} Bi\left(n = 12, p = \dfrac{1}{6}\right)$ M1

$\Pr(X = 1) = \binom{12}{1}\left(\dfrac{1}{6}\right)\left(\dfrac{5}{6}\right)^{11} = 12 \times \dfrac{1}{6} \times \left(\dfrac{5}{6}\right)^{11}$

$\Pr(X = 1) = 2\left(\dfrac{5}{6}\right)^{11}$ , $c = 2$ , $a = 5$ , $b = 6$ , $n = 11$ A1

**b.** $n = 144$ , $p = \dfrac{1}{6}$

$\text{sd}\left(\hat{P}\right) = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{\frac{1}{6} \times \frac{5}{6}}{144}} = \dfrac{\sqrt{5}}{72}$ A1

**c.** Packet 1, contains 2 broken and 10 not broken, total of 12.

Let packet 2, contain $12 - b$ broken and $b$ not broken, total of 12.

$\Pr(\text{both not broken}) = \dfrac{1}{2} \times \dfrac{10}{12} \times \dfrac{9}{11} + \dfrac{1}{2} \times \dfrac{b}{12} \times \dfrac{b-1}{11} = \dfrac{23}{66}$

$90 + b(b-1) = \dfrac{23}{66} \times 2 \times 132 = 92$ M1

$b^2 - b - 2 = 0$

$(b-2)(b+1) = 0$

$b = 2$ since $b > 0$

there are 10 broken biscuits in the packet that was dropped onto the floor. A1

**d.** Let $W$ be the weights of a packet, $W \overset{d}{=} N\left(\mu = 250, \sigma^2 = 8^2\right)$

$Z$ is the standard normal, $Z \overset{d}{=} N\left(\mu = 0, \sigma^2 = 1\right)$

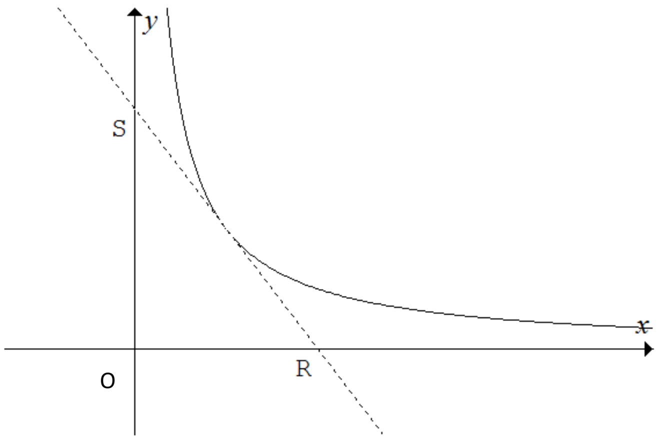
$\Pr(244 \le W \le 256)$

$= \Pr\left(\dfrac{244 - 250}{8} \le Z \le \dfrac{256 - 250}{8}\right)$

$= \Pr\left(-\dfrac{3}{4} \le Z \le \dfrac{3}{4}\right) = 2\Pr\left(0 \le Z \le \dfrac{3}{4}\right) = 2\Pr\left(-\dfrac{3}{4} \le Z \le 0\right)$

$w = -\dfrac{3}{4}$ A1

**Question 7**




**a.** Let $y = f(x) = \dfrac{1}{x} = x^{-1}$

$\dfrac{dy}{dx} = f'(x) = -x^{-2} = -\dfrac{1}{x^2}$

$m_T = f'(a) = -\dfrac{1}{a^2}$ M1

$T: \quad y - \dfrac{1}{a} = -\dfrac{1}{a^2}(x - a)$

$T: \quad y = -\dfrac{x}{a^2} + \dfrac{2}{a}$ , $m = -\dfrac{1}{a^2}$ , $c = \dfrac{2}{a}$ A1

**b.** at S $x = 0 \Rightarrow y = \dfrac{2}{a}$ , $\mathrm{S}\left(0, \dfrac{2}{a}\right)$.

at $R$ $y = 0 \Rightarrow \dfrac{x}{a^2} = \dfrac{2}{a}$ , $x = 2a$ , $\mathrm{R}(2a, 0)$.

$\Delta\mathrm{ORS} = \dfrac{1}{2} \times \dfrac{2}{a} \times 2a = 2$ A1

**Question 8**

$A = \dfrac{1}{2} \times 2u \times f(u) = uf(u) = ue^{-u^2}$

using the product rule

$\dfrac{dA}{du} = \dfrac{d}{du}(u)e^{-u^2} + u\dfrac{d}{du}\left(e^{-u^2}\right)$ , now $\dfrac{d}{du}\left(e^{-u^2}\right) = -2ue^{-u^2}$ by the chain rule A1

$\dfrac{dA}{du} = e^{-u^2} - 2u^2 e^{-u^2} = e^{-u^2}\left(1 - 2u^2\right) = 0$ for maximum M1

$2u^2 = 1$

$u^2 = \dfrac{1}{2}$

$u = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ since $u > 0$ A1

**Question 9**

**a.** completing the square

$y = 11 + 2x - x^2 = -\left(x^2 - 2x + 1\right) + 1 + 11 = 12 - \left(x-1\right)^2$

$y = x^2 \quad y' = 12 - \left(x'-1\right)^2 \quad , \quad \dfrac{y'-12}{-1} = \left(x'-1\right)^2$

$x = x' - 1 \quad , \quad y = \dfrac{y'-12}{-1} \quad \Rightarrow \quad x' = x + 1 \quad , \quad y' = -y + 12$

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 12 \end{bmatrix} \quad , \; a = 1 \,,\; b = -1 \,,\; h = 1 \,,\; k = 12$ A1

**b.**

Complete a function domain table

| | $f(x)$ | $g(x)$ |
|---|---|---|
| domain | $(-4,\infty)$ | $R$ |
| Range | $R^+ = (0,\infty)$ | $(-\infty,12]$ |

Note that since range $g = (-\infty,12] \not\subset$ domain $f = (-4,\infty)$

so that $f\left(g\left(x\right)\right)$ does not exist. A1

**c.** solving $g\left(x\right) = 11 + 2x - x^2 = -4$

$\Rightarrow \quad x^2 - 2x - 15 = 0$

$\left(x-5\right)\left(x+3\right) = 0$ M1

$\Rightarrow \quad x = -3 \;,\; 5$

So we can restrict the domain of $g$, and domain $f\left(g\left(x\right)\right) =$ domain $g\left(x\right) = D = \left(-3,5\right)$ A1

$g : \left(-3,5\right) \to R,\; g\left(x\right) = 11 + 2x - x^2$

now domain $g = \left(-3,5\right)$ , range $g = \left(-4,12\right]$

range $g \subset$ domain $f$ , so that $f\left(g\left(x\right)\right)$ exists

$f\left(g\left(x\right)\right) = f\left(11 + 2x - x^2\right)$

$= \dfrac{1}{\sqrt{11 + 2x - x^2 + 4}}$

$= \dfrac{1}{\sqrt{15 + 2x - x^2}}$

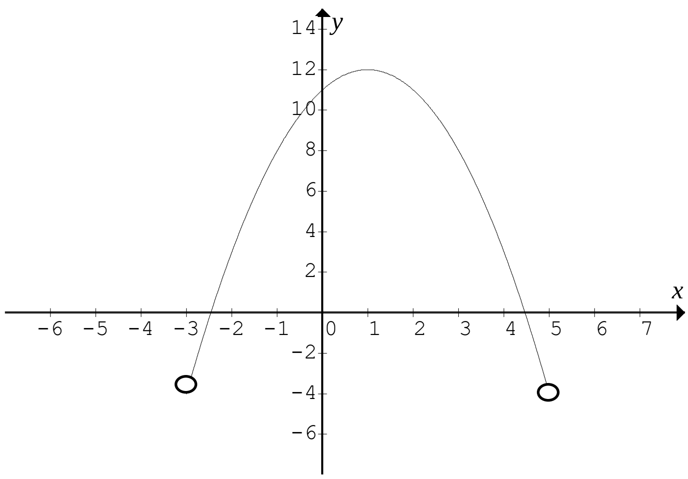
alternatively $f\left(g\left(x\right)\right)$ now exists when

$15 + 2x - x^2 > 0$

$= -\left(x^2 - 2x - 15\right) > 0$

$= -\left(x-5\right)\left(x+3\right) > 0 \;\; \Rightarrow \;\; D = \left(-3,5\right)$

Graph of restricted domain $g\left(x\right)$



**Question 10**

**a.** $f(x) = e^{-2x}\sin(2x)$ using the product rule

$f'(x) = e^{-2x}\frac{d}{dx}\left[\sin(2x)\right] + \sin(2x)\frac{d}{dx}\left(e^{-2x}\right)$

$= 2e^{-2x}\cos(2x) - 2e^{-2x}\sin(2x)$ M1

$= 2e^{-2x}\left(\cos(2x) - \sin(2x)\right)$

for turning points $f'(x) = 0 \quad \Rightarrow \quad 2e^{-2x}\left(\cos(2x) - \sin(2x)\right) = 0$

$\cos(2x) - \sin(2x) = 0$ , $\cos(2x) = \sin(2x)$ , $\tan(2x) = 1$

$2x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{\pi}{8}, \frac{5\pi}{8}$ since $0 \le x \le \pi$ A1

**b.** Let $y = e^{-2x}\left(\cos(2x) + \sin(2x)\right)$ using the product rule

$\frac{dy}{dx} = e^{-2x}\frac{d}{dx}\left[\cos(2x) + \sin(2x)\right] + \left(\cos(2x) + \sin(2x)\right)\frac{d}{dx}\left(e^{-2x}\right)$

$= e^{-2x} \times \left(-2\sin(2x) + 2\cos(2x)\right) - 2e^{-2x}\left(\cos(2x) + \sin(2x)\right)$

$= -4e^{-2x}\sin(2x)$ A1

**c.** the graph of $f(x) = e^{-2x}\sin(2x)$ crosses the $x$-axis when $e^{-2x}\sin(2x) = 0$

that is when $\sin(2x) = 0$ , $2x = 0, \pi, 2\pi$ at $x = 0, \frac{\pi}{2}, \pi$

required area $A = \int_0^{\frac{\pi}{2}} e^{-2x}\sin(2x)\,dx$ A1

from **b.** $\frac{d}{dx}\left[e^{-2x}\left(\cos(2x) + \sin(2x)\right)\right] = -4e^{-2x}\sin(2x)$

$A = -\frac{1}{4}\left[e^{-2x}\left(\cos(2x) + \sin(2x)\right)\right]_0^{\frac{\pi}{2}}$

A1

$A = -\frac{1}{4}\left(e^{-\pi}\left(\cos(\pi) + \sin(\pi)\right)\right) - \left(\cos(0) + \sin(0)\right)$

$A = \frac{1}{4}\left(e^{-\pi} + 1\right)$ A1

**End of detailed answers for the**
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