2019 VCE Mathematical Methods Trial Examination 2

Detailed Answers

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SECTION A

ANSWERS

1	A	B	C	D	**E**
2	A	**B**	C	D	E
3	A	B	C	**D**	E
4	**A**	B	C	D	E
5	A	B	C	**D**	E
6	A	B	C	**D**	E
7	A	B	C	D	**E**
8	A	**B**	C	D	E
9	**A**	B	C	D	E
10	A	B	**C**	D	E
11	A	B	C	D	**E**
12	A	B	**C**	D	E
13	**A**	B	C	D	E
14	A	B	C	**D**	E
15	A	B	C	D	**E**
16	A	**B**	C	D	E
17	A	B	**C**	D	E
18	**A**	B	C	D	E
19	A	**B**	C	D	E
20	A	B	**C**	D	E

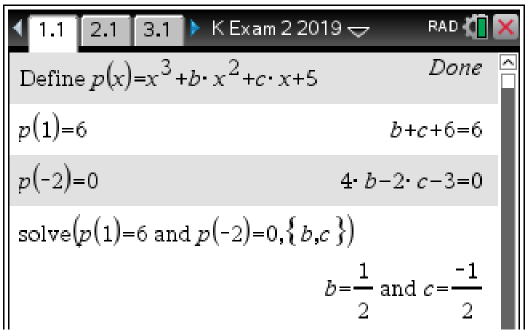
SECTION A

Question 1 **Answer E**

The graphs of tan do not have an amplitude. **A.** and **B.** are incorrect.
All of **C. D.** and **E.** have an amplitude of *b*.

Only **E.** $T = \frac{2\pi}{\frac{2\pi}{b}} = b$ has a period of *b*.

Question 2 **Answer B**



$p(x) = x^3 + bx^2 + cx + 5$

$p(1) = 6 \;\Rightarrow\; 6 = 1 + b + c + 5 = 6$

$(1)\; b + c = 0$

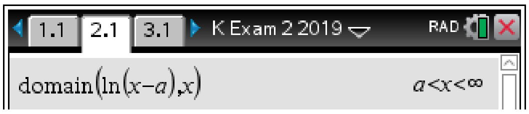
$p(-2) = 0 \Rightarrow -8 + 4b - 2c + 5 = 0$

$(2)\; 4b - 2c = 3$

$(2) + 2\times(1)\; 6b = 3$

$b = \frac{1}{2},\; c = -\frac{1}{2}$

Question 3 **Answer D**



$f(x) = \log_e(x-a)$

has a maximal domain $x > a = (a,\infty)$

Question 4 **Answer A**

$g(x) = f^{-1}(x)\;,\; f(1) = 2\;,\; g(2) = f^{-1}(2) = 1$

$f(g(x)) = f^{-1}(f(x)) = x$ differentiating using the chain rule

$g'(x)\, f'(g(x)) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{-1}{5}$

Question 5 **Answer D**

$f(x) = e^x + e^{-x}\;,\; f'(x) = e^x - e^{-x} = g(x)$ **A.** is true

$g(x) = e^x - e^{-x}\;,\; g'(x) = e^x + e^{-x} = f(x)$ **B.** is true

$\left[f(x)\right]^2 = \left(e^x + e^{-x}\right)^2 = e^{2x} + 2 + e^{-2x} = f(2x) + 2$ **C.** is true

$\left[g(x)\right]^2 = \left(e^x - e^{-x}\right)^2 = e^{2x} - 2 + e^{-2x} = f(2x) - 2$ **D.** is false

$f(x)g(x) = \left(e^x + e^{-x}\right)\left(e^x - e^{-x}\right) = e^{2x} - 2 + 2 + e^{-2x} = e^{2x} + e^{-2x} = g(2x)$ **E.** is true

Question 6 **Answer D**

$f(x) = x^3 - bx$

$f'(x) = 3x^2 - b = 0$ for stationary points $x = \pm\sqrt{\frac{b}{3}} = \pm\frac{\sqrt{3b}}{3}$

$f'(\pm 2) = 0 = 12 - b \Rightarrow b = 12$. **A.** is correct

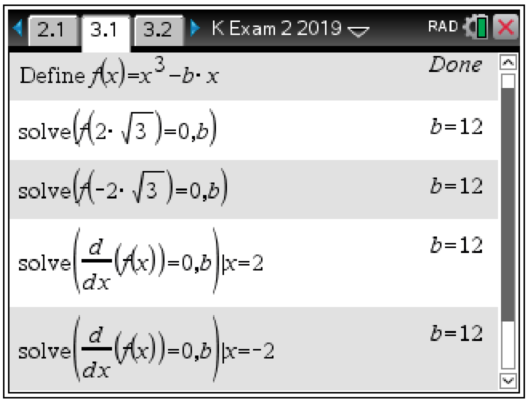
$f\left(2\sqrt{3}\right) = \left(2\sqrt{3}\right)^3 - 2\sqrt{3}b = 0 \Rightarrow b = 12$.

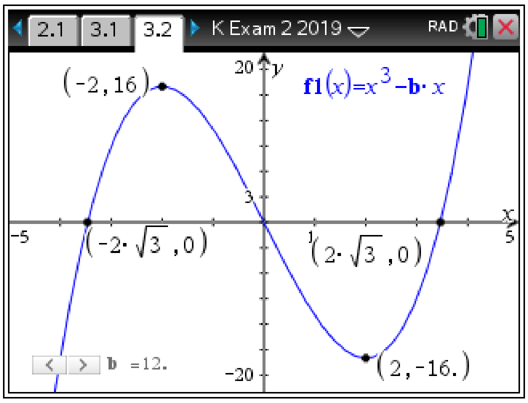
$f\left(-2\sqrt{3}\right) = \left(-2\sqrt{3}\right)^3 + 2\sqrt{3}b = 0 \Rightarrow b = 12$. **B.** is correct

There is a stationary point of inflexion at $x = 0$. **E.** is correct

For $x \in \left(-\infty, -\frac{\sqrt{3b}}{3}\right)$ the function is one-one. **C.** is correct

When $x \in \left(-\frac{\sqrt{3b}}{3}, \frac{\sqrt{3b}}{3}\right)$ the function is decreasing. **D.** is false





Question 7 **Answer E**

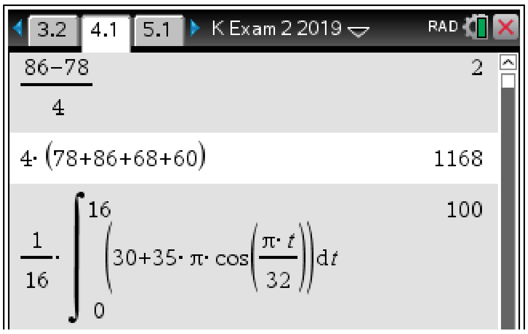
$$r'(6) = \frac{r(8) - r(4)}{8 - 4} = \frac{86 - 78}{4} = 2$$

Question 8 **Answer B**

$$\int_0^{16} r(t)\,dt = 4[78 + 86 + 68 + 60] = 1168$$

Question 9 **Answer A**

$$\bar{r} = \frac{1}{16 - 0}\int_0^{16}\left(30 + 35\pi\cos\left(\frac{\pi t}{32}\right)\right)dt = 100$$



Question 10 **Answer C**

$$y = \log_e\left(\cos(2x)\right)$$

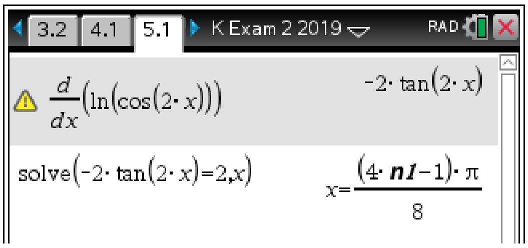
$$\frac{dy}{dx} = \frac{-2\sin(2x)}{\cos(2x)} = -2\tan(2x) = 2$$

$$\tan(2x) = -1$$

$$2x = n\pi - \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} - \frac{\pi}{8}$$

$$x = \frac{\pi}{8}(4n-1)$$



Question 11 **Answer E**

$f(x) = 2 - 3g(x)$ and $\int_4^{-1} g(x)\,dx = 3$ then

$$\int_{-1}^{4}\left(f(x)+g(x)\right)dx$$

$$= \int_{-1}^{4}\left(2-3g(x)+g(x)\right)dx = \int_{-1}^{4}\left(2-2g(x)\right)dx$$

$$= \left[2x\right]_{-1}^{4} - 2\int_{-1}^{4} g(x)\,dx = \left(8-(-2)\right) + 2\int_{4}^{-1} g(x)\,dx$$

$$= 10 + 2\times 3 = 16$$

Question 12 **Answer C**

$$f(x) = \begin{cases} x^2 & 0 \le x \le 2 \\ 5-x & 2 < x \le 5 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = 4 \quad , \quad \lim_{x\to 2^+} f(x) = 3$$

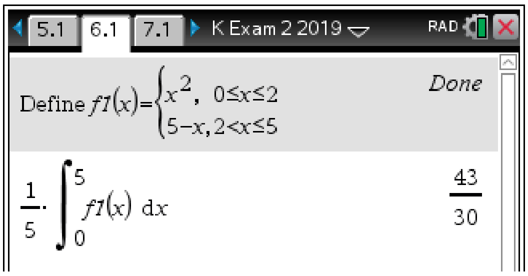
The function is not continuous at $x = 2$.

The average value does exist.

$$\bar{f} = \frac{1}{5-0}\left[\int_0^5 f(x)\,dx\right]$$

$$= \frac{1}{5}\left[\int_0^2 x^2\,dx + \int_2^5 (5-x)\,dx\right].$$

$$= \frac{1}{5}\left(\frac{8}{3}+\frac{9}{2}\right) = \frac{43}{30}$$



Question 13 **Answer A**

$$f(x)=\log_e\left(\frac{1}{\sqrt{g(x)}}\right)=-\frac{1}{2}\log_e\left(g(x)\right)$$

$$f'(x)=-\frac{g'(x)}{2g(x)}$$

$$g'(x)=-2g(x)\,f'(x)$$

$$g'(4)=-2g(4)\,f'(4)$$

Now $f(4)=2=-\frac{1}{2}\log_e\left(g(4)\right)$, $g(4)=e^{-4}$

$$g'(4)=-2e^{-4}\times{}^{-}3=6e^{-4}=\frac{6}{e^4}$$

Question 14 **Answer D**

$f(x)=\sqrt{x-a}$ and $g(x)=x^2+b$

	$f(x)$	$g(x)$
domain	$[a,\infty)$	R
range	$[0,\infty)$	$[b,\infty)$

For the function $f\left(g(x)\right)$ to exist range $g\subseteq$ domain f $\Rightarrow$ $[b,\infty)\subseteq[a,\infty)$

So that $b\geq a$ or $b-a\geq 0$

Question 15 **Answer E**

The function, $y=\log_e(x+1)$, when $y=2$, $2=\log_e(x+1)$ $\Rightarrow x=e^2-1$

The inverse function $f^{-1}:$ $x=\log_e(y+1)$ $\Rightarrow$ $y=f^{-1}(x)=e^x-1$

The area bounded by the graph of $y=\log_e(x+1)$

the y-axis and the line $y=2$, is equal to $\int_0^2\left(e^x-1\right)dx$.

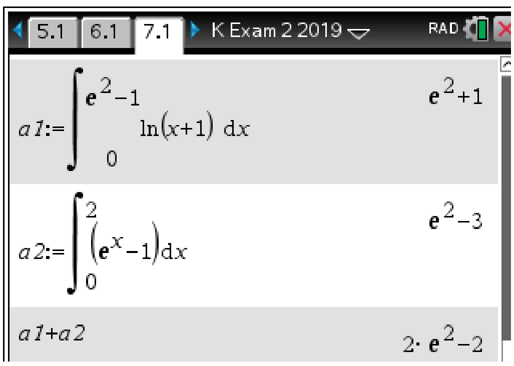
The area bounded by the graph of $y=\log_e(x+1)$

the x-axis and the vertical line, is equal to

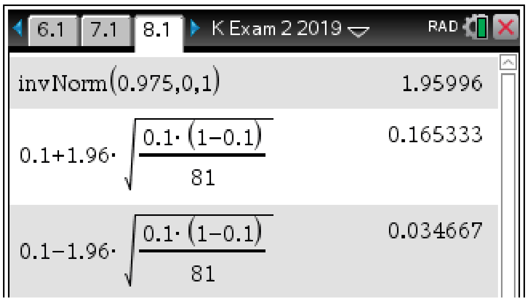
$\int_0^{e^2-1}\log_e(x+1)\,dx$.

The total shaded area is the area of the rectangle and is equal to $2\left(e^2-1\right)$.

Amanda, Breeanna and Chloe are all correct.



Question 16 **Answer B**



$n=81$, $p=0.1$, 95% $z=1.96$

$$p \pm z\sqrt{\frac{p(1-p)}{n}}$$

$$0.1 \pm 1.96\sqrt{\frac{0.1 \times 0.9}{81}}$$

$$(0.035, 0.165)$$

Question 17 **Answer C**

$y=\sqrt{x}$ is transformed into

$y=2\sqrt{x}$ by a dilation by a scale factor of 2 parallel to the y-axis,

$y=-2\sqrt{x}$ by a reflection in the x-axis,

$y=-2\sqrt{-x}$ by a reflection in the y-axis,

$y=-2\sqrt{-(x-2)}=-2\sqrt{2-x}$ by translation of 2 units to the right parallel to the x-axis.

Question 18 **Answer A**

The original curve $y=\tan(x)$ is transformed into the image curve with equation

$$y'=4-\frac{3}{\tan\left(\frac{x'}{2}\right)}=4+3\tan\left(\frac{x'}{2}-\frac{\pi}{2}\right)$$

$$\frac{y'-4}{3}=\tan\left(\frac{x'}{2}-\frac{\pi}{2}\right)$$

$$y=\frac{y'-4}{3} \quad , \quad x=\frac{x'}{2}-\frac{\pi}{2}$$

$$y'=3y+4 \quad ., \quad x'=2x+\pi$$

in matrix form this becomes $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} \pi \\ 4 \end{bmatrix}$

Question 19 **Answer B**

$X \stackrel{d}{=} \text{Bi}(n, p)$, $\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

and $\Pr(X = 0) = (1-p)^n$, $\Pr(X = 1) = np(1-p)^{n-1}$

$$\Pr(X = 1 | X > 0) = \frac{\Pr(X = 1 \cap X > 0)}{\Pr(X > 0)} = \frac{\Pr(X = 1)}{\Pr(X \geq 1)}$$

$$= \frac{\Pr(X = 1)}{1 - \Pr(X = 0)} = \frac{np(1-p)^{n-1}}{1-(1-p)^n}$$

Question 20 **Answer C**

$\sum \Pr(X = x) = 1$

$(1)\quad a + b + c = 1$

$E(X) = \sum x \Pr(X = x)$

$(2)\quad -a + c = \frac{1}{4}$

$E(X^2) = \sum x^2 \Pr(X = x)$

$E(X^2) = a + c$

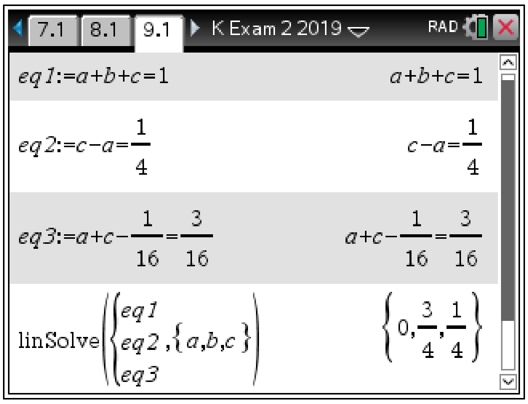
$\text{var}(X) = E(X^2) - (E(X))^2$

$a + c - \left(\frac{1}{4}\right)^2 = \frac{3}{16}$

$(3)\quad a + c = \frac{1}{4}$

$(2) + (3) \Rightarrow 2c = \frac{1}{2}$

$c = \frac{1}{4}$, $a = 0$, $b = \frac{3}{4}$



END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a. $f(x) = x^3 + bx^2 + cx + d$

$f'(x) = 3x^2 + 2bx + c = 0$ for stationary points

$\Delta = (2b)^2 - 4 \times 3 \times c = 4b^2 - 12c = 4\left(b^2 - 3c\right)$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{-2b \pm 2\sqrt{b^2 - 3c}}{6}$ M1

$x = \dfrac{-b \pm \sqrt{b^2 - 3c}}{3}$ since $q > p$, $q < 0$ so

$q = \dfrac{-b + \sqrt{b^2 - 3c}}{3}$, $p = \dfrac{-b - \sqrt{b^2 - 3c}}{3}$ and $b^2 > 3c$ A1

b. $q = -2 = \dfrac{-b + \sqrt{b^2 - 3c}}{3}$, $p = -6 = \dfrac{-b - \sqrt{b^2 - 3c}}{3}$

$(1) \quad -6 = -b + \sqrt{b^2 - 3c}$ A1

$(2) \quad -18 = -b - \sqrt{b^2 - 3c}$

$(1) + (2) \quad -24 = -2b$, $b = 12$

$\sqrt{b^2 - 3c} = -6 + b = 6$

$144 - 3c = 36$

$3c = 144 - 36 = 108$

$c = \dfrac{108}{3} = 36$ A1

c. $f(x) = x^3 + 12x^2 + 36x + d$

$f(-2) = d - 32$, $f(-6) = d$

$\text{P}(-6, d)$, $\text{Q}(-2, d - 32)$

Since the graph crosses the *x*-axis three times, $d > 0$ and $d - 32 < 0$ so that
$0 < d < 32$ A1

Define $f1(x) = x^3 + 12 \cdot x^2 + 36 \cdot x + d$	*Done*
$\dfrac{d}{dx}(f1(x))$	$3 \cdot x^2 + 24 \cdot x + 36$
$\text{solve}\left(\dfrac{d}{dx}(f1(x)) = 0, x\right)$	$x = ^{-}6$ or $x = ^{-}2$
$f1(^{-}6)$	d
$f1(^{-}2)$	$d - 32$

d. $m(\text{PQ}) = \dfrac{d-(d-32)}{-6-{}^{-}2} = \dfrac{32}{-4} = -8$

$L(x): \quad y - d = -8(x+6)$

$L(x): \quad y = -8x + d - 48$ A1

e. $R\left(-\dfrac{6+{}^{-}2}{2}, \dfrac{d+d-32}{2}\right)$, $R(-4, d-16)$ A1

f. $f(-4) = (-4)^3 + 12\times(-4)^2 + 36\times{}^{-}4 + d = -64 + 192 - 144 + d$

$f(-4) = d - 16$, so R lies on the graph of f. A1

$f1(-4)$	$d-16$
Define $f2(x)=^{-}8\cdot x+d-48$	*Done*
solve$(f1(x)=f2(x),x)$	$x=^{-}6$ or $x=^{-}4$ or $x=^{-}2$

g. The point R is the point where a tangent crosses the curve.
The line joining PQ is not the tangent to the curve at R, but the minimum gradient occurs at R.

Let $m = f'(x) = 3x^2 + 24x + 36$, $\dfrac{dm}{dx} = 6x + 24 = 0$ for maximum or minimum gradient

$\Rightarrow \quad x = -4$, $f'(-4) = 3\times(-4)^2 + 24\times{}^{-}4 + 36 = 48 - 96 + 36 = -12$

The minimum gradient is -12 A1

h.i. $A_1 = \int_{xp}^{xr} \left(f(x) - L(x)\right) dx$

$A_1 = \int_{-6}^{-4} \left(x^3 + 12x^2 + 44x + 48\right) dx$

$x1 = -6$, $x2 = -4$, $m = 44$, $n = 48$ A1

h.ii. $A_1 = 4$ A1

j. $A_2 = \int_{xr}^{xp} \left(L(x) - f(x)\right) dx$

$A_2 = -\int_{-4}^{-2} \left(x^3 + 12x^2 + 44x + 48\right) dx = 4$ or by symmetry A1

$f1(x)-f2(x)$	$x^3+12\cdot x^2+44\cdot x+48$
$\int_{-6}^{-4} \left(x^3+12\cdot x^2+44\cdot x+48\right) dx$	4
$-\int_{-4}^{-2} \left(x^3+12\cdot x^2+44\cdot x+48\right) dx$	4

Question 2

a. $f(x)=a(x-h)^2+k$ for $0\le x\le 200$,

$A(0,120)$, $f(0)=120 \Rightarrow 120=ah^2+k$

$B(100,140)$ since it is the maximum turning point,

$f(100)=140 \Rightarrow h=100$, $k=140$ A1

$C(200,120)$, $f(200)=120 \Rightarrow 120=a(100)^2+140$

$a(100)^2=-20$

$a=\dfrac{-20}{10,000}=-\dfrac{1}{500}$ A1

b. $\dfrac{1}{4}$ of a period $\dfrac{1}{4}\times\dfrac{2\pi}{\frac{\pi}{n}}=\dfrac{n}{2}=160 \Rightarrow n=320$ A1

the gradient function $g'(x)=\dfrac{p\pi}{n}\cos\left(\dfrac{\pi}{n}(x-\alpha)\right)$ has a maximum at $D(360,185)$

$g'(360)=0 \Rightarrow \cos\left(\dfrac{\pi}{320}(360-\alpha)\right)=0=\cos\left(\dfrac{\pi}{2}\right)$

$360-\alpha=160$

$\alpha=200$ A1

$g(x)=p\sin\left(\dfrac{\pi}{320}(x-200)\right)+r$

$C(200,120)$, $g(200)=120 \Rightarrow 120=p\sin(0)+r=r$ so $r=120$

$D(360,185)$, $g(360)=185$, $185=p\sin\left(\dfrac{\pi}{2}\right)+120=p+120$

$p=185-120$

$p=65$ A1

c. $D(360,185)$, $h(360)=185 \Rightarrow 185=s\sqrt{c+360d-360^2}$

$E(500,0)$, $h(500)=0 \Rightarrow 0=s\sqrt{c+500d-500^2}$

since it is smoothly joined at D, $h'(360)=g'(360)=0$ M1

now $h'(x)=\dfrac{s(d-2x)}{2\sqrt{c+dx-x^2}}$, $h'(360)=0 \Rightarrow d-2\times 360=0$

$d=720$ A1

substitute $d=720$ into $s\sqrt{c+500d-500^2}=0 \Rightarrow c=500^2-500\times 720=-110,000$

substitute $d=720$ and $c=-110,000$ into $185=s\sqrt{c+360d-360^2}$ gives

$s=\dfrac{185}{\sqrt{-110,000+360\times 720-360^2}}=\dfrac{37}{28}$ A1

d. $\left(x-100\right)^2+y^2=50^2$ A1

e.i. $f\left(x\right)=-\frac{1}{500}\left(x-100\right)^2+140$, $g\left(x\right)=65\sin\left(\frac{\pi}{320}\left(x-200\right)\right)+120$

$h\left(x\right)=\frac{37}{28}\sqrt{720x-110,000-x^2}$

the neck is a rectangle length 400 by 100, the area of the circle is $\pi\times\left(50\right)^2$

the total amount of wood required is

$$A=400\times100+2\left[\int_0^{200}f\left(x\right)dx+\int_{200}^{360}g\left(x\right)dx+\int_{360}^{500}h\left(x\right)dx\right]-\pi\times50^2$$ A1

$$=2\left[\int_0^{200}\left(-\frac{1}{500}\left(x-100\right)^2+140\right)dx+\int_{200}^{360}\left(65\sin\left(\frac{\pi}{320}\left(x-200\right)\right)+120\right)dx+\int_{360}^{500}\left(\frac{37}{28}\sqrt{720x-110,000-x^2}\right)dx\right]$$

$$+\ 40,000-2,500\pi$$

ii. $177,804.67\text{ cm}^2$ A1

$$f2(x) \quad \left\{\frac{-x^2}{500}+\frac{2\cdot x}{5}+120, 0\le x\le 200\right.$$

$$f1(x) \quad \left\{120-65\cdot\sin\left(\frac{\pi\cdot x}{320}+\frac{3\cdot\pi}{8}\right), 200\le x\le 360\right.$$

$$f3(x) \quad \left\{\frac{37\cdot\sqrt{-x^2+720\cdot x-110000}}{28}, 360\le x\le 500\right.$$

$$2\cdot\left(\int_0^{200}f2(x)\,dx+\int_{200}^{360}f1(x)\,dx+\int_{360}^{500}f3(x)\,dx\right)-\pi\cdot50^2+400\cdot100 \quad 177804.67$$

f. $x\left(t\right)=Re^{kt}\cos\left(m\pi t\right)$

$x\left(0\right)=R=-\frac{1}{100}=-0.01$

$v\left(t\right)=\frac{dx}{dt}=Re^{kt}\left(k\cos\left(m\pi t\right)-m\pi\sin\left(m\pi t\right)\right)$

$v\left(0\right)=Rk=400 \Rightarrow k=\frac{400}{-0.01}=-40,000$ A1

$T=\frac{2\pi}{m\pi}=\frac{2}{m}=\frac{1}{f}=\frac{1}{250}$

$R=\frac{1}{100}=-0.01$, $m=500$ A1

Question 3

a.i. $P \overset{d}{=} N\left(\mu = 106, \sigma^2 = 2.5^2\right)$

$\Pr\left(P > 103\right) = 0.8849$ A1

ii. $P_{10} \overset{d}{=} \text{Bi}\left(n = 10, p = 0.8849\right)$

$\Pr\left(P_{10} \geq 8\right) = 0.9015$ A1

iii. $P_n \overset{d}{=} \text{Bi}\left(n = ?, p = 0.8849\right)$

$\Pr\left(P_n \geq 5\right) \geq 0.95$ to find the value of n, using trial and error

$n = 7$ A1

normCdf(103,∞,106,2.5)	0.8849
p:=0.88493	0.8849
binomCdf(10,0.88493,8,10)	0.9015
binomCdf(n,0.88493,5.,n)|n=6	0.8549
binomCdf(n,0.88493,5.,n)|n=7	0.9627

b.i. $P \overset{d}{=} N\left(\mu = ?, \sigma^2 = ?^2\right)$

large or extra large, $\Pr\left(P > 230\right) = 0.21 \quad \Rightarrow \quad \Pr\left(P \leq 230\right) = 0.79$ M1

$(1) \quad \dfrac{230 - \mu}{\sigma} = 0.8064$

small, $\Pr\left(P < 138\right) = 0.11$ M1

$(2) \quad \dfrac{138 - \mu}{\sigma} = -1.2265$

solving (1) and (2) $\mu = 193.5$, $\sigma = 45.3$ A1

invNorm(0.79,0,1)	0.8064
$\frac{230-m}{s}=0.8064$	$\frac{230-m}{s}=0.8064$
invNorm(0.11,0,1)	⁻1.2265
solve$\left(\frac{230-m}{s}=0.806421 \text{ and } \frac{138-m}{s}=^{-}1.226528,\{m,s\}\right)$	s=45.2545 and m=193.5059

b.ii. $\Pr(P \ge L) = 0.08$, $\Pr(P \le L) = 0.92$

$\frac{L-193.5}{45.25} = 1.4051$ solving $L = 257.1$

minimum weight for extra large potato $L = 257.1$ A1

$\text{invNorm}(0.92,0,1)$	1.4051
$\text{invNorm}(0.92,193.5059,45.25)$	257.0854
$\text{solve}\left(\frac{l-193.5}{45.25}=1.4051,l\right)$	$l=257.0808$

iii. completing the table

	small	medium	large	extra large
Probability	0.11	0.68	0.13	0.08
potato weight, w grams	$w<138$	$138 \le w \le 230$	$230 < w < 257.1$	$w \ge 257.1$
revenue cents	–0.03	0.05	0.10	0.15

expected revenue $E(R) = 0.08 \times 0.15 + 0.13 \times 0.10 + 0.68 \times 0.05 - 0.11 \times 0.03 = 0.06$

$= 6$ cents A1

iv. $\Pr(138 \le P \le 230 \mid P \ge 138) = \frac{\Pr(138 \le P \le 230)}{\Pr(P \ge 138)} = \frac{0.68}{1-0.11} = \frac{0.68}{0.89}$

$= 0.764$ A1

c.i. $b\left[\int_0^5 \frac{x}{5}\,dx + \int_5^{10} \cos\left(\frac{\pi(x-5)}{10}\right)dx\right] = 1$ total area under the curve is one.

$$b\left[\left[\frac{x^2}{10}\right]_0^5 + \left[\frac{10}{\pi}\sin\left(\frac{\pi(x-5)}{10}\right)\right]_5^{10}\right] = 1$$

M1

$$b\left[\left(\frac{25}{10}-0\right) + \frac{10}{\pi}\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right)\right] = 1$$

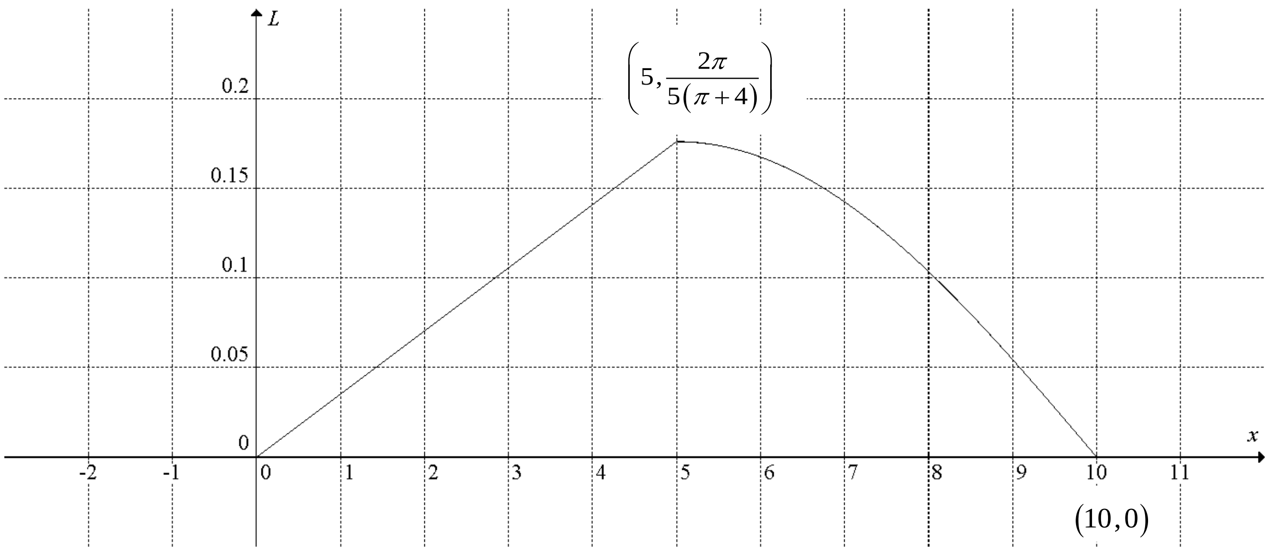
$$b\left(\frac{5}{2} + \frac{10}{\pi}\right) = b\left(\frac{5\pi+20}{2\pi}\right) = 1$$

A1

$$b = \frac{2\pi}{5(\pi+4)}$$

ii. The function is continuous at $x = 5$ and $\frac{2\pi}{5(\pi+4)} \approx 0.176$. correct scaling, shape,

zero elsewhere. G1



iii. $E(L) = \frac{2\pi}{5(\pi+4)}\left[\int_0^5 \frac{x^2}{5}\,dx + \int_5^{10} x\cos\left(\frac{\pi(x-5)}{10}\right)dx\right] = 5.284$ A1

$$E(L^2) = \frac{2\pi}{5(\pi+4)}\left[\int_0^5 \frac{x^3}{5}\,dx + \int_5^{10} x^2\cos\left(\frac{\pi(x-5)}{10}\right)dx\right] = 32.330$$

$\text{Var}(L) = E(L^2) - (E(L))^2 = 32.33018 - 5.28448^2 = 4.404$ A1

$b:=\frac{2\cdot\pi}{5\cdot(\pi+4)}$	$\frac{2\cdot\pi}{5\cdot(\pi+4)}$
Define $f1(x)=\frac{b\cdot x}{5}$	*Done*
Define $f2(x)=b\cdot\cos\left(\frac{\pi\cdot(x-5)}{10}\right)$	*Done*
Define $f3(x)=\begin{cases} f1(x), & 0\le x\le 5 \\ f2(x), & 5\le x\le 10\end{cases}$	*Done*
$\int_0^{10} f3(x)\,dx$	1
$\int_0^{10} (x\cdot f3(x))\,dx$	5.284
$\int_0^{10} (x^2\cdot f3(x))\,dx$	32.330
$32.330184313499-(5.2844767120181)^2$	4.404

iv. Since $\frac{2\pi}{5(\pi+4)}\left[\int_0^5 \frac{x}{5}\,dx\right]=0.44$ the median satisfies

$$\frac{2\pi}{5(\pi+4)}\left[\int_5^m \cos\left(\frac{\pi(x-5)}{10}\right)dx\right]=0.5-0.440=0.06$$

solving gives $m=5.342$ A1

$\int_0^5 f1(x)\,dx$	0.440
$\text{solve}\left(\int_5^m f2(x)\,dx=0.5-0.44,m\right)\|5<m<10$	$m=5.342$
$\text{solve}\left(\int_0^m f3(x)\,dx=0.5,m\right)$	$m=5.342$

d.i. $E(\hat{P})=p=0.64$

$$\text{sd}(\hat{P})=\sqrt{\frac{p(1-p)}{n}}$$

$$\text{sd}(\hat{P})=\sqrt{\frac{0.64\times(1-0.64)}{100}}$$

$\text{sd}(\hat{P})=0.048$ A1

ii. $\Pr(0.64-0.048\le\hat{P}\le 0.64+0.048)$

$=\Pr(0.592\le\hat{P}\le 0.688)$

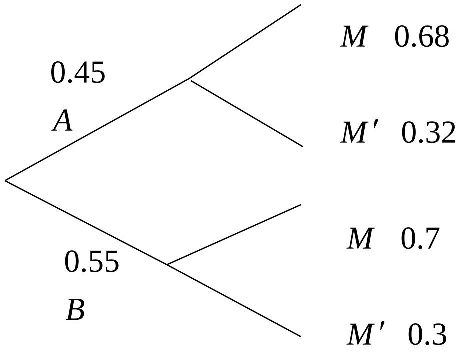
for 100 potatoes, we require $\Pr(59.2\le P_{100}\le 68.8)$ M1

Since it is discrete, $P_{100}\overset{d}{=}\text{Bi}(n=100,\ p=0.64)$

$\Pr(60\le P_{100}\le 68)=0.651$ A1

$\sqrt{\frac{0.64\cdot 0.36}{100}}$	0.048
0.64+0.048	0.688
0.64−0.048	0.592
binomCdf(100,0.64,59.2,68.8)	0.651
binomCdf(100,0.64,60,68)	0.651

e. tree diagram



$\Pr(M') = \Pr(M' \cap A) + \Pr(M' \cap B) = 0.45 \times 0.32 + 0.55 \times 0.3 = 0.309$ M1

$\Pr(B \mid M') = \dfrac{0.55 \times 0.3}{0.309} = 0.534$ A1

$\dfrac{0.55 \cdot 0.3}{0.55 \cdot 0.3 + 0.45 \cdot 0.32}$ 0.534

Question 4

a. $f(x) = \dfrac{1}{x+2} + 3$

crosses the x-axis $y = 0$, $0 = \dfrac{1}{x+2} + 3 \Rightarrow x = -\dfrac{7}{3}$

crosses the y-axis $x = 0$, $y = \dfrac{1}{2} + 3 \Rightarrow y = \dfrac{7}{2}$, the cordinates are required

$\left(-\dfrac{7}{3}, 0\right), \left(0, \dfrac{7}{2}\right)$ A1

b. domain $B = R \setminus \{-2\}$ range $R \setminus \{3\}$ A1

Define $f1(x) = \dfrac{1}{x+2} + 3$ Done

solve$(f1(x) = 0, x)$ $x = \dfrac{-7}{3}$

$f1(0)$ $\dfrac{7}{2}$

domain$(f1(x), x)$ $x \neq -2$

c. $s = d(OP) = \sqrt{p^2 + (f(p))^2} = \sqrt{p^2 + \left(\frac{1}{p+2} + 3\right)^2} = \frac{\sqrt{p^4 + 4p^3 + 13p^2 + 42p + 49}}{p+2}$

$\frac{ds}{dp} = 0 \;\Rightarrow\; p = -2.310 \text{ or } p = 0.530$ M1

$s(-2.310) = 2.321 \text{ and } s(0.530) = 3.436$

minimum value of p is $p = -2.310$ and the minimum distance is 2.321 A1

Define $s(p)=\sqrt{p^2+(f1(p))^2}$	*Done*
$s(p)$	$\frac{\sqrt{p^4+4\cdot p^3+13\cdot p^2+42\cdot p+49}}{\lvert p+2\rvert}$
$\frac{d}{dp}(s(p))$	$\frac{p^4+6\cdot p^3+12\cdot p^2+5\cdot p-7}{(p+2)\cdot\sqrt{p^4+4\cdot p^3+13\cdot p^2+42\cdot p+49}\cdot\lvert p+2\rvert}$
solve$\left(\frac{d}{dp}(s(p))=0,p\right)$	$p=^-2.31032$ or $p=0.530301$
$s(^-2.3103)$	2.32101
$s(0.53)$	3.43637

d. $f: \quad y = \frac{1}{x+2} + 3$

$f^{-1}: \quad x = \frac{1}{y+2} + 3 \;,\; x - 3 = \frac{1}{y+2} \;,\; y + 2 = \frac{1}{x-3}$

$f^{-1}: R \backslash \{3\} \to R \;,\; f^{-1}(x) = -2 + \frac{1}{x-3} \;,\; a = -2,\, b = 1,\, c = -3$ A1

solve$(f1(y)=x,y)$	$y=\frac{^-(2\cdot x-7)}{x-3}$
propFrac$\left(y=\frac{^-(2\cdot x-7)}{x-3}\right)$	$y=\frac{1}{x-3}-2$

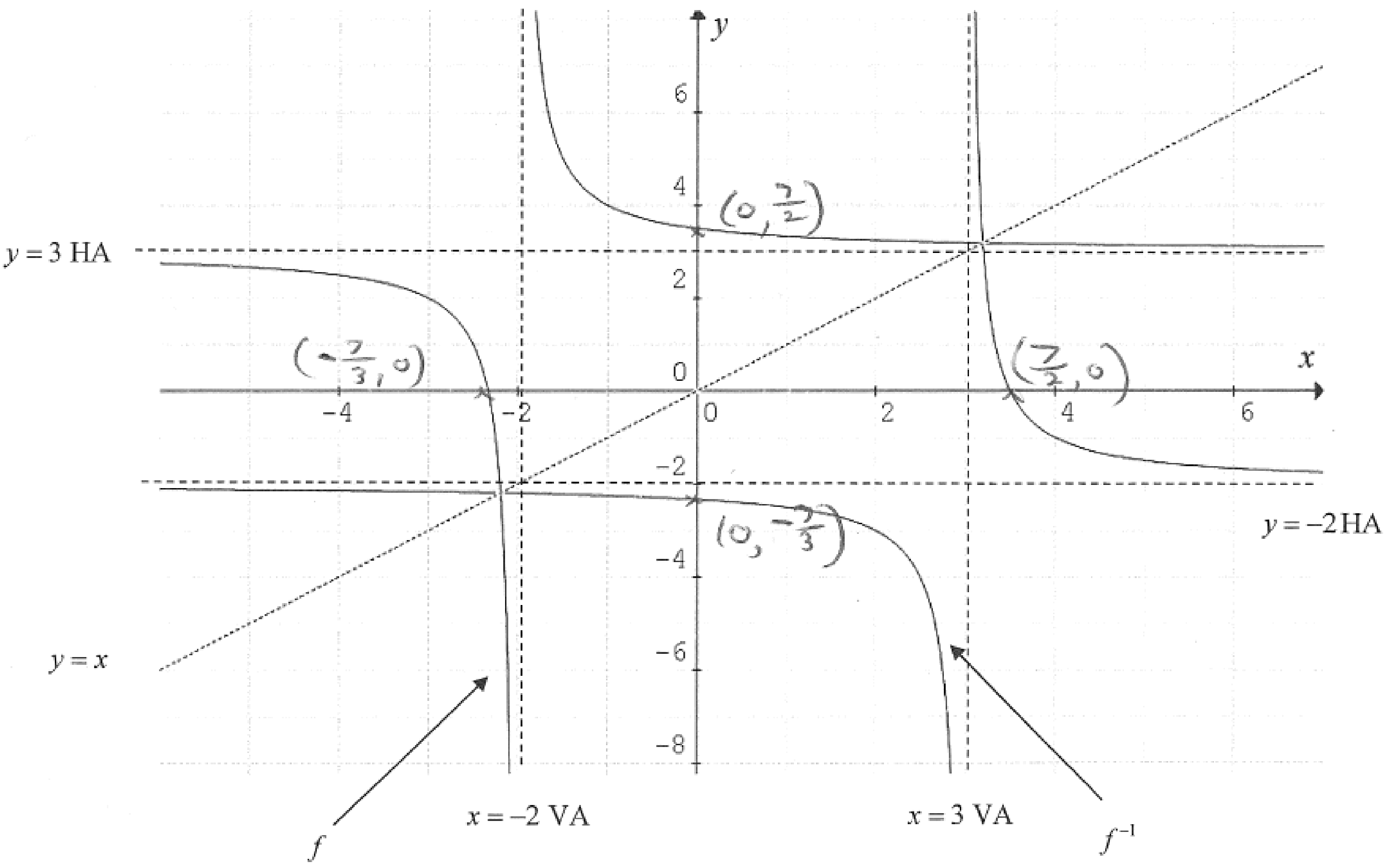
e. solving $f(x) = x$ or $f^{-1}(x) = x$ or $f(x) = f^{-1}(x)$

$\frac{1}{u+2} + 3 = u \text{ or } \frac{1}{u-3} - 2 = u \text{ or } \frac{1}{u+2} + 3 = \frac{1}{u-3} - 2$

all give $u = \frac{1 \pm \sqrt{29}}{2}$ A1

Define $f2(x)=\frac{1}{x-3}-2$	*Done*
solve$(f1(x)=f2(x),x)$	$x=\frac{-(\sqrt{29}-1)}{2}$ or $x=\frac{\sqrt{29}+1}{2}$
solve$(f1(x)=x,x)$	$x=\frac{-(\sqrt{29}-1)}{2}$ or $x=\frac{\sqrt{29}+1}{2}$
solve$(f2(x)=x,x)$	$x=\frac{-(\sqrt{29}-1)}{2}$ or $x=\frac{\sqrt{29}+1}{2}$

f. G2



g. Note that $u=\dfrac{1+\sqrt{29}}{2}\approx 3.19<3.5$, must give two of the following three possible areas,

$$A=2\left[\int_0^{\frac{1+\sqrt{29}}{2}}\left(f(x)-x\right)dx\right]=2\left[\int_0^{\frac{1+\sqrt{29}}{2}}\left(\frac{1}{x+2}+3-x\right)dx\right]$$

or $$A=\int_0^{\frac{1+\sqrt{29}}{2}} f(x)\,dx+\int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}} f^{-1}(x)\,dx=\int_0^{\frac{1+\sqrt{29}}{2}}\left(\frac{1}{x+2}+3\right)dx+\int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}}\left(\frac{1}{x-3}-2\right)dx$$

A1

or $$A==2\left[\int_0^{\frac{1+\sqrt{29}}{2}} x\,dx+\int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}} f^{-1}(x)\,dx\right]=2\left[\int_0^{\frac{1+\sqrt{29}}{2}} x\,dx+\int_{\frac{1+\sqrt{29}}{2}}^{\frac{7}{2}}\left(\frac{1}{x-3}-2\right)dx\right]$$

$A=10.871$ A1

$u:=\dfrac{\sqrt{29}+1}{2}$ $\qquad \dfrac{\sqrt{29}+1}{2}$

$2\cdot\displaystyle\int_0^u \left(f1(x)-x\right)dx$ $\qquad 10.8711$

$\displaystyle\int_0^u f1(x)\,dx+\int_u^{\frac{7}{2}} f2(x)\,dx$ $\qquad 10.8711$

$2\cdot\left(\displaystyle\int_0^u x\,dx+\int_u^{\frac{7}{2}} f2(x)\,dx\right)$ $\qquad 10.8711$

h. $g(x)=\dfrac{1}{x+k}+3$

crosses the x-axis $y=0$, $0=\dfrac{1}{x+k}+3 \Rightarrow x=-\dfrac{1}{3}-k$

crosses the y-axis $x=0$, $y=\dfrac{1}{k}+3$

$\left(-\left(k+\dfrac{1}{3}\right),0\right),\ \left(0\ ,\ \dfrac{1}{k}+3\right)$ A1

i. $g^{-1}(x)=\dfrac{1}{x-3}-k$ A1

j. solving $g(x)=x$ or $g^{-1}(x)=x$ or $g(x)=g^{-1}(x)$

$$\frac{1}{x+k}+3=x \qquad \frac{1}{x-3}-k=x$$

$$\frac{1}{x+k}=x-3 \qquad \frac{1}{x-3}=k+x$$

both give

$$1=(k+x)(x-3)$$

$$1=x^2+(k-3)x-3k$$

$$x^2+(k-3)x-(3k+1)=0$$

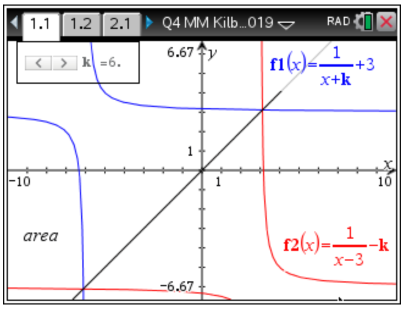
$$\Delta=(k-3)^2+4(3k+1)$$

$$\Delta=k^2-6k+9+12k+4=k^2+6k+13$$

$$x=\frac{-k\pm\sqrt{(k+3)^2+4}}{2}$$ M1

$\Delta=(k+3)^2+4$ since $\Delta>0$ for any value of k, A1

the graphs always intersect at two distinct points.



k. The area approaches the area of a rectangle

with side lengths $\frac{1}{k}+3$

$$A(k)=\left(\frac{1}{k}+3\right)^2=\frac{1}{k^2}+\frac{6}{k}+9$$

$$\lim_{k\to\infty} A(k)=9$$ A1

END OF SECTION B SUGGESTED ANSWERS

End of detailed answers for the
2019 Kilbaha VCE Mathematical Methods Trial Examination 2

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