2019 VCE Mathematical Methods Trial Examination 2

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Victorian Certificate of Education
2019

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 31 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple -choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Mark will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Which of the following functions has a period of b and an amplitude of b, where $b \in R^+$

A. $y = b\tan(bx)$

B. $y = b - b\tan\left(\frac{2\pi x}{b}\right)$

C. $y = b - b\sin\left(\frac{x}{b}\right)$

D. $y = b + b\sin\left(\frac{\pi}{b}\left(x - \frac{b}{2}\right)\right)$

E. $y = b - b\cos\left(\frac{2\pi x}{b}\right)$

Question 2

Let $p(x) = x^3 + bx^2 + cx + 5$, when $p(x)$ is divided by $x - 1$ the remainder is 6, and $p(x)$ has $x + 2$ as a factor, then

A. $b = c$

B. $b = \frac{1}{2}$ and $c = -\frac{1}{2}$

C. $b = -\frac{3}{2}$ and $c = -\frac{7}{2}$

D. $b = -\frac{1}{2}$ and $c = -\frac{11}{2}$

E. $b = -\frac{13}{2}$ and $c = \frac{13}{2}$

Question 3

Which of the following functions has a maximal domain of (a,∞) where $a \in R^+$

A. $f(x)=\sqrt{x-a}$

B. $f(x)=\dfrac{1}{x-a}$

C. $f(x)=\dfrac{1}{\sqrt{x^2-ax}}$

D. $f(x)=\log_e(x-a)$

E. $f(x)=\dfrac{1}{x^2-a^2}$

Question 4

The table below shows selected values of a differentiable and decreasing function *f* and its derivative. If g is the inverse of f, then $g'(2)$ is equal to

x	0	1	2
$f(x)$	5	2	−7
$f'(x)$	−2	−5	−14

A. $-\dfrac{1}{5}$

B. $\dfrac{1}{5}$

C. $-\dfrac{1}{14}$

D. -5

E. 5

Question 5

Let $f(x)=e^x+e^{-x}$ and $g(x)=e^x-e^{-x}$ Which of the following statements is **not** true?

A. $f'(x)=g(x)$

B. $g'(x)=f(x)$

C. $[f(x)]^2=f(2x)+2$

D. $[g(x)]^2=g(2x)+2$

E. $f(x)g(x)=g(2x)$

Question 6

Consider the function $f(x) = x^3 - bx$ where $b \in R$. Then which of the following is **not** true?

A. If there is a stationary point at $x = \pm 2$ then $b = 12$.

B. If the graph crosses the x-axis at $x = \pm 2\sqrt{3}$ then $b = 12$.

C. For $x \in \left(-\infty, -\frac{\sqrt{3b}}{3}\right)$ the function is one-one.

D. When $x \in \left(-\frac{\sqrt{3b}}{3}, \frac{\sqrt{3b}}{3}\right)$ the function is increasing.

E. If $b = 0$, there is a stationary point of inflexion at $x = 0$.

The data in the following table is used for Questions 7 and 8.

t minutes	0	4	8	12	16
$r(t)$ rotations per minute	70	78	86	68	60

Ashley rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at a time t minutes during Ashley's ride is modelled by a differentiable function $r(t)$ for $0 \le t \le 16$ minutes. Values of $r(t)$ for selected values of t are shown in the table above.

Question 7

An estimate of $r'(6)$ is equal to

A. 82

B. 41

C. 20.5

D. 4

E. 2

Question 8

An estimate of $\int_0^{16} r(t)\,dt$ using right rectangles, is equal to

A. 1208

B. 1168

C. 604

D. 584

E. 364

Question 9

Jared also rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at a time *t* minutes during Jared's ride is modelled by the function $r(t) = 30 + 35\pi\cos\left(\frac{\pi t}{32}\right)$ for $0 \le t \le 16$ minutes. The average number of rotations per minute of the wheel of the stationary bike for $0 \le t \le 16$, is equal to

A. 100

B. 1600

C. $\frac{35\pi}{16}$

D. 140

E. $\frac{35}{4}$

Question 10

The tangent to the graph of $y = \log_e(\cos(2x))$ is parallel to the line with the equation $y = 2x + 3$, when *x* is equal to

A. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

B. $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

C. $\frac{\pi}{8}(4n-1)$ where $n \in Z$

D. $\frac{\pi}{8}(1-4n)$ where $n \in Z$

E. $\frac{\pi}{8}(8n+1)$ where $n \in Z$

Question 11

If $f(x) = 2 - 3g(x)$ and $\int_4^{-1} g(x)\,dx = 3$ then $\int_{-1}^{4} \left(f(x) + g(x)\right)dx$ is equal to

A. 0

B. 2

C. 4

D. 14

E. 16

Question 12

Consider the function $f(x)=\begin{cases} x^2 & 0 \le x \le 2 \\ 5-x & 2 < x \le 5 \end{cases}$

Then

A. The function is continuous, and the average value does not exist.

B. The function is continuous, and the average value is equal to $\frac{17}{6}$.

C. The function is not continuous, and the average value is equal to $\frac{43}{30}$.

D. The function is not continuous, and the average value is equal to $\frac{17}{6}$.

E. The function is not continuous, and the average value does not exist.

Question 13

If $f(x)=\log_e\left(\frac{1}{\sqrt{g(x)}}\right)$, $f(4)=2$ and $f'(4)=-3$ then $g'(4)$ is equal to

A. $\frac{6}{e^4}$

B. $-\frac{6}{e^4}$

C. $-3e^4$

D. $\frac{3}{e^2}$

E. $3e^2$

Question 14

Consider the functions $f(x)=\sqrt{x-a}$ and $g(x)=x^2+b$, where $a, b \in R$.

For the function $f(g(x))$ to exist

A. $b+a \ge 0$

B. $b+a \le 0$

C. $a-b \ge 0$

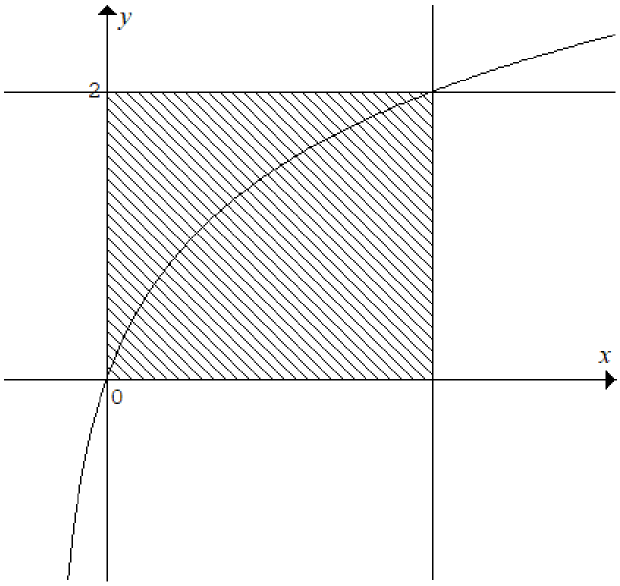
D. $b-a \ge 0$

E. $a+b=0$

Question 15

The diagram shows part of the graph of $y = \log_e(x+1)$ and the line $y = 2$.

Some students were considering the shaded areas.



Amanda stated that the area bounded by the graph of $y = \log_e(x+1)$ the *y*-axis and the line $y = 2$, is equal to $\int_0^2 (e^x - 1)\,dx$.

Breeanna stated that the area bounded by the graph of $y = \log_e(x+1)$ the *x*-axis and the vertical line, is equal to $\int_0^{e^2-1} \log_e(x+1)\,dx$.

Chloe stated that the total shaded area is equal to $2(e^2 - 1)$. Then

A. Only Amanda is correct.

B. Only Breeanna is correct.

C. Only Chloe is correct.

D. Only Amanda and Chloe are correct.

E. Amanda, Breeanna and Chloe are all correct.

Question 16

A random selection of 81 people were surveyed about increasing the minimum drinking age to 21. Ten per cent of the sample agreed with the proposal. A 95 percent confidence interval for the proportion of people who agreed with the proposal is closest to

A. $0.033, 0.167$

B. $0.035, 0.165$

C. $0.05, 0.15$

D. $0.067, 0.133$

E. $0.08, 0.12$

Question 17

The graph of $y=\sqrt{x}$ is transformed into the graph of $y=-2\sqrt{2-x}$ by

A. A dilation by a scale factor of 2 parallel to the *y*-axis, a reflection in the *x*-axis, and a translation of 2 units to the right parallel to the *x*-axis.

B. A dilation by a scale factor of 2 from the *y*-axis, a reflection in the *x*-axis, and a translation of 2 units to the left parallel to the *x*-axis.

C. A dilation by a scale factor of 2 parallel to the *y*-axis, a reflection in the *y*-axis, a reflection in the *x*-axis and a translation of 2 units to the right parallel to the *x*-axis.

D. A dilation by a scale factor of 2 parallel to the *y*-axis, a reflection in the *y*-axis, a reflection in the *x*-axis and a translation of 2 units to the left parallel to the *x*-axis.

E. A dilation by a scale factor of -2 parallel to the *y*-axis, a reflection in the *x*-axis, and a translation of 2 units from the *y*-axis.

Question 18

Given that $\tan\left(A-\frac{\pi}{2}\right)=\frac{-1}{\tan(A)}$, then transformation $T:R^2\to R^2$, which maps the curve with equation $y=\tan(x)$ to the curve with equation $y=4-\dfrac{3}{\tan\left(\frac{x}{2}\right)}$, has the rule

A. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}2&0\\0&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\pi\\4\end{bmatrix}$

B. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}2&0\\0&-3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\pi\\-4\end{bmatrix}$

C. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}\frac{1}{2}&0\\0&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}-\frac{\pi}{2}\\4\end{bmatrix}$

D. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}\frac{1}{2}&0\\0&-3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}-\frac{\pi}{2}\\-4\end{bmatrix}$

E. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}\frac{1}{2}&0\\0&-3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\frac{\pi}{2}\\4\end{bmatrix}$

Question 19

The random variable X has a binomial distribution with n independent trials and a probability of p of a success on any one trial, where $0 < p < 1$, then $\Pr(X = 1 | X > 0)$ is equal to

A. $(1-p)^{n-1}$

B. $\dfrac{np(1-p)^{n-1}}{1-(1-p)^n}$

C. $1-np(1-p)^{n-1}$

D. $\dfrac{np}{1-p}$

E. $np(1-p)^{n-1}$

Question 20

A discrete random variable X has the following probability distribution.

X	-1	0	1
$\Pr(X = x)$	a	b	c

Given that $E(X) = \frac{1}{4}$ and $\operatorname{var}(X) = \frac{3}{16}$ then

A. $a = \frac{1}{16}$, $b = \frac{5}{8}$, $c = \frac{5}{16}$

B. $a = \frac{1}{2}$, $b = 0$, $c = \frac{1}{2}$

C. $a = 0$, $b = \frac{3}{4}$, $c = \frac{1}{4}$

D. $a = \frac{1}{8}$, $b = \frac{1}{2}$, $c = \frac{3}{8}$

E. $a = \frac{3}{8}$, $b = 0$, $c = \frac{5}{8}$

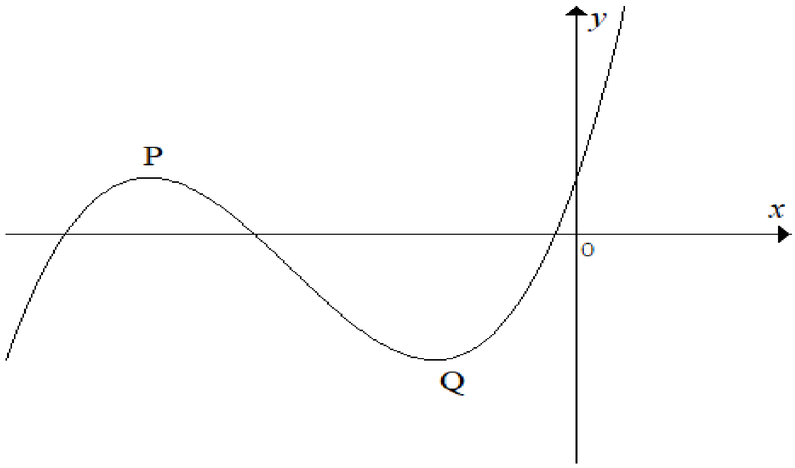
END OF SECTION A

SECTION B

Question 1 (12 marks)

Consider the function $f : R \to R$, $f(x) = x^3 + bx^2 + cx + d$, where $b, c, d \in R$.

The graph of the function f crosses the x-axis at three distinct points and has two stationary points as shown in the diagram below.



a. The function has a maximum turning point at $\mathrm{P}(p, f(p))$ and a minimum turning point at $\mathrm{Q}(q, f(q))$, where $q > p$ and $q < 0$. **Show** that $q = \dfrac{-b + \sqrt{b^2 - 3c}}{3}$ and find a similar expression for p in terms of b and c, stating restrictions between b and c.

2 marks

b. Given that $p=-6$ and $q=-2$, solve the equations to show that $b=12$ and $c=36$.

2 marks

c. Write down the coordinates of P and Q in terms of *d*, stating restrictions on *d*.

1 mark

d. Find in terms of *d*, the equation of the line *L* passing through the points P and Q.

1 mark

e. Let $\text{R}\left(r, f\left(r\right)\right)$ be the midpoint of the points P and Q.
Write down in terms of *d*, the coordinates of the point R.

1 mark

f. Show that the point R also lies on the graph of f.

1 mark

g. Determine the minimum value of the gradient of the curve $y = f\left(x\right)$.

h.i. The area A_1 between the graphs of the function *f* and the line *L*, and the points P and R is given by the definite integral $A_1 = \int_{x1}^{x2}\left(x^3 + 12x^2 + mx + n\right)dx$.
State the values of $x1, x2$, m and *n*.

1 mark

ii. Find the value of the area A_1.

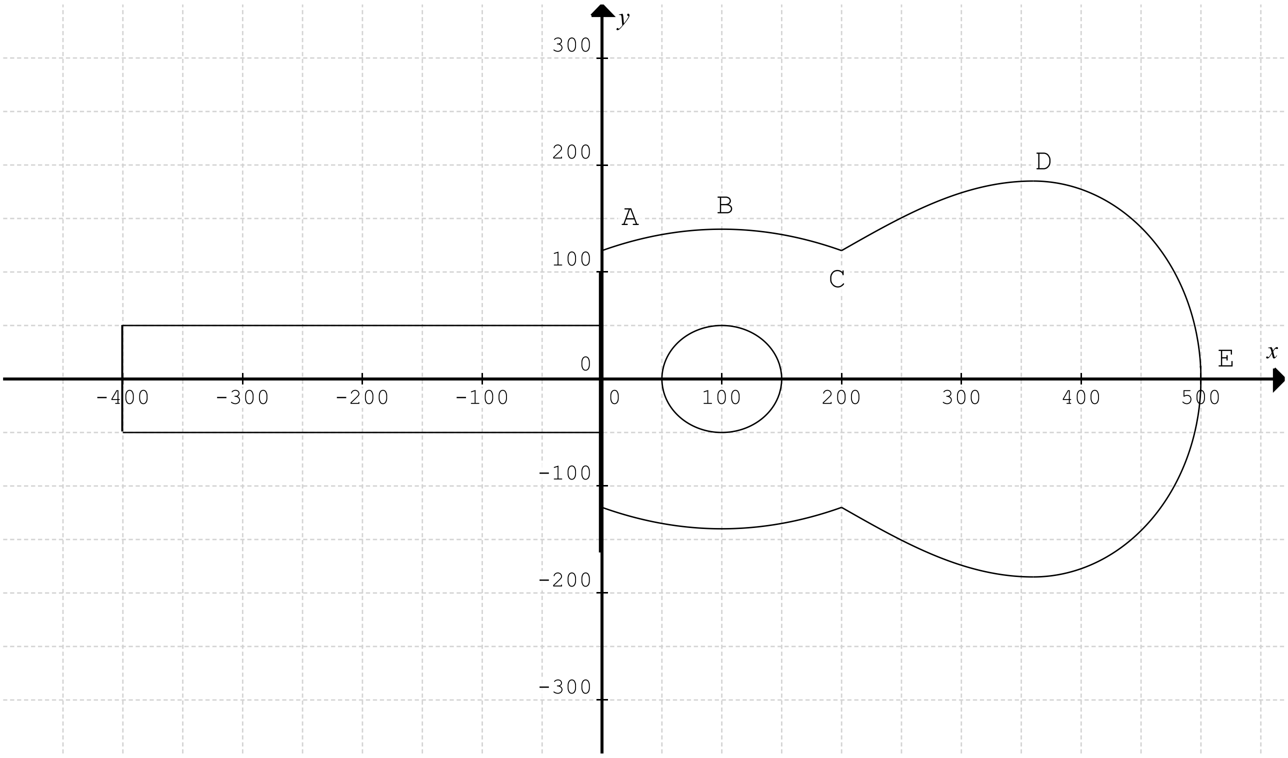
1 mark

j. Let A_2 be the area between the graph of the function *f* and the line *L*, and the points R and Q. Write down a definite integral for the area A_2, and find the value of the area A_2.

1 mark

Question 2 (13 marks)

The diagram below shows a cross-section of a guitar, placed on a Cartesian grid.
The top edge of the guitar is modelled by three curves, ABC, CD and DE which are symmetrical about the *x*-axis. The guitar's cross-section is to be constructed of wood, all dimensions shown are in centimetres.



The first section of the guitar ABC is modelled by the function $f(x)=a(x-h)^2+k$ for $0 \le x \le 200$. This curve passes through the points $A(0,120)$, $B(100,140)$ and $C(200,120)$.

a. Show that $a=-\frac{1}{500}$, $h=100$ and $k=140$. 2 marks

The second section of the guitar CD is modelled by the function $g(x) = p\sin\left(\frac{\pi}{n}(x-\alpha)\right)+r$ for $200 \le x \le 360$. This curve passes through the points $\text{C}(200,120)$ and $\text{D}(360,185)$ and is one-quarter of the sine wave, the point D is also the widest point on the guitar.

b. Show that $p = 65$, $n = 320$, $\alpha = 200$ and $r = 120$.

3 marks

The third section of the guitar DE is modelled by the function $h(x) = s\sqrt{c+dx-x^2}$ for $360 \le x \le 500$. This curve passes through the points $\text{D}(360,185)$ and $\text{E}(500,0)$ and is **smoothly** joined at D.

c. Write down simultaneous equations which can be solved for *s*, *c* and *d*, to show that $s = \frac{37}{28}$, $c = -110,000$ and $d = 720$.

3 marks

For acoustic purposes the guitar also has a circle cut out as shown, with its centre at $(100,0)$ and a radius of 50. The neck of the guitar is in the shape of a rectangle, which has a width of 100 cm and a length of 400 cm.

d. Write down the equation of the circle.

1 mark

e.i. Write down in terms of definite integrals the total cross-sectional area of the amount of wood required to construct the guitar in the first and fourth quadrants including the neck in the second and third quadrants.

1 mark

ii. Determine the total cross-sectional area of the wood required for the guitar in square centimetres. Give your answer correct to two decimal places.

1 mark

f. The strings on the guitar vibrate when plucked to play musical notes. The distance *x* metres at a time *t* seconds of a particular string is given by $x(t) = Re^{kt}\cos(m\pi t)$, where *t* is the time in seconds after being plucked and *x* the distance measured in metres from the equilibrium position. The string is plucked with an initial speed of 400 m/s and is initially one cm below the equilibrium position. The string vibrates with a frequency of 250 Hz, the frequency is the reciprocal of the period. Determine the values of *R*, *k* and *m*.

2 marks

Question 3 (20 marks)

a. The weights of packets of a certain brand of potato crisps are normally distributed with a mean of 106 grams with a standard deviation of 2.5 grams.

i. Find the probability correct to four decimal places, that a randomly selected packet of this brand of potato crisps, has a weight greater than 103 grams.

1 mark

ii. Find the probability correct to four decimal places that in a random sample of ten packets of this brand of potato crisps, at least eight of the packets, have weights greater than 103 grams.

1 mark

iii. Find the minimum number of packets, so that the probability of at least five packets, having weights greater than 103 grams, is at least 0.95.

1 mark

b.i. The weights of potatoes produced from a farm are classified in increasing weights as small, medium, large or extra large. It is also known that these weights are normally distributed. Of the potatoes produced from this farm, it is found that 21% are classified as large or extra large and have weights greater than 230 grams, 11% are small and have weights less than 138 grams. Find the mean and standard deviation weights of potatoes produced from this farm, giving your answers in grams correct to one decimal place.

3 marks

ii. 8% of the potatoes are classified as extra large, find the minimum weight of an extra large potato, giving your answer in grams correct to one decimal place.

1 mark

iii. The farm sells the potatoes. The revenue per potato for extra large potatoes is 15 cents, for large potatoes the revenue is 10 cents, for medium potatoes the revenue is 5 cents and for small potatoes the farm results in a loss of 3 cents. Complete the following table and find the expected revenue on selling the potatoes, correct to the nearest cent.

	small	medium	large	extra large
Probability				
potato weight, grams				
revenue cents				

1 mark

iv. A potato that is known not to be small, is selected at random.
What is the probability correct to three decimal places that it is a medium potato.

1 mark

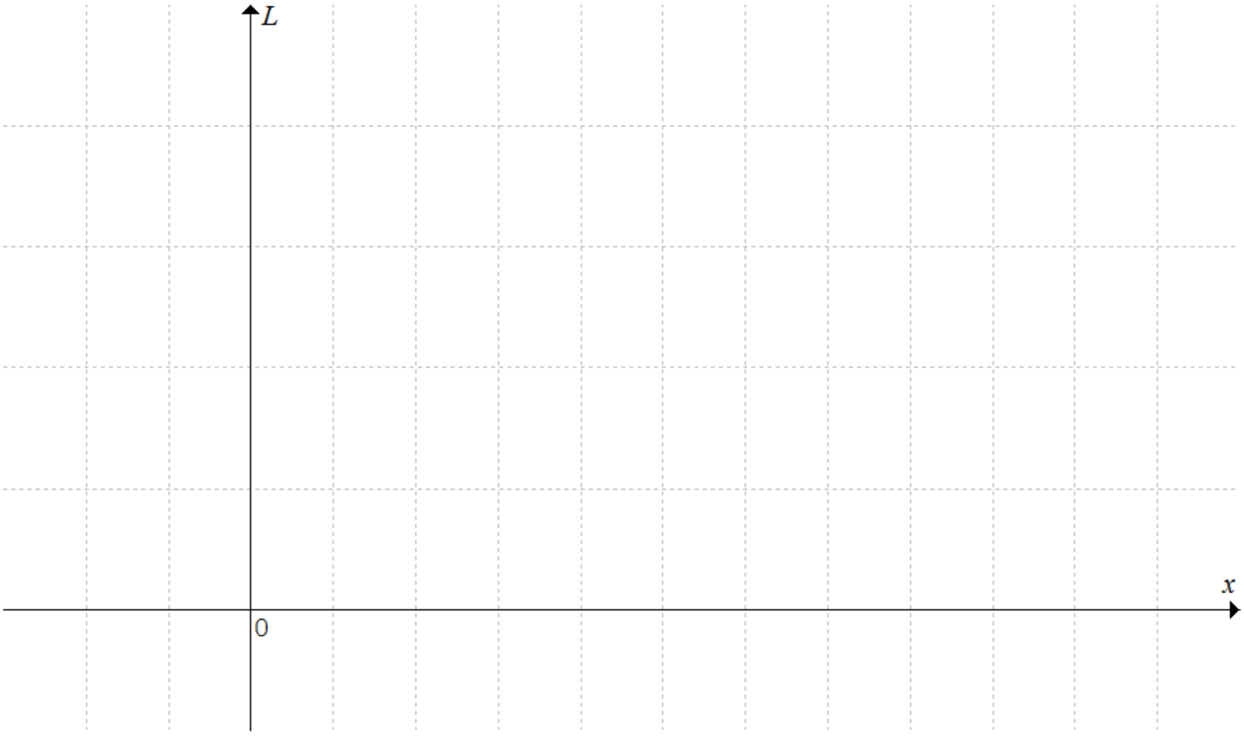
c. The potatoes are also made into French fries or chips. The lengths x in centimetres of the chips is found to satisfy a probability distribution $L(x)$ defined by

$$L(x)=\begin{cases} \dfrac{bx}{5} & 0 \le x \le 5 \\ b\cos\left(\dfrac{\pi(x-5)}{10}\right) & 5 < x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

i. **Show** that $b=\dfrac{2\pi}{5(\pi+4)}$. 2 marks

ii. Sketch the graph of $L(x)$ on the axes below, clearly labelling the scales and significant points.

1 mark



iii. Find $\mathrm{E}(L)$ and $\mathrm{Var}(L)$ giving your answers correct to three decimal places.

2 marks

iv. Find the median length in cm of the chips. Give your answer correct to three decimal places.

1 mark

d. A random sample of 100 potatoes produced from another farm B, has 64 potatoes that are classified as medium. Let $\hat{P}$ be the random variable of the sample proportion of medium potatoes produced from farm B.

i. Find the standard deviation of $\hat{P}$.
Give your answer correct to two significant figures.

1 mark

ii. Find the probability that the sample proportion of medium potatoes grown on this farm lies within one standard deviation of the mean. Give your answer correct to three decimal places. Do not use a normal approximation.

2 marks

e. A retailer buys potatoes from either farm A or farm B. 45% of the potatoes are purchased from farm A. It is known that 68% of the potatoes from farm A are medium, while 70% of the potatoes from farm B are medium. A particular potato is found not to be medium, find the probability correct to three decimal places that it came from farm B?

2 marks

Question 4 (15 marks)

Consider the function $f : B \to R$, $f(x) = \frac{1}{x+2} + 3$

a. State the coordinates where the graph of f crosses the x and y axis.

1 mark

b. State the maximal domain B **and** range of the function f.

1 mark

c. Let $\mathrm{P}(p, f(p))$ be a point on the graph of the function f. Find the value of p for which the distance from the origin to the point P is a minimum and state this minimum distance. Give both answers correct to three decimal places.

2 marks

d. Find f^{-1}, the inverse **function** *f*, giving your answer in the form $a+\dfrac{b}{x+c}$, where *a*, *b* and *c* are non-zero integers.

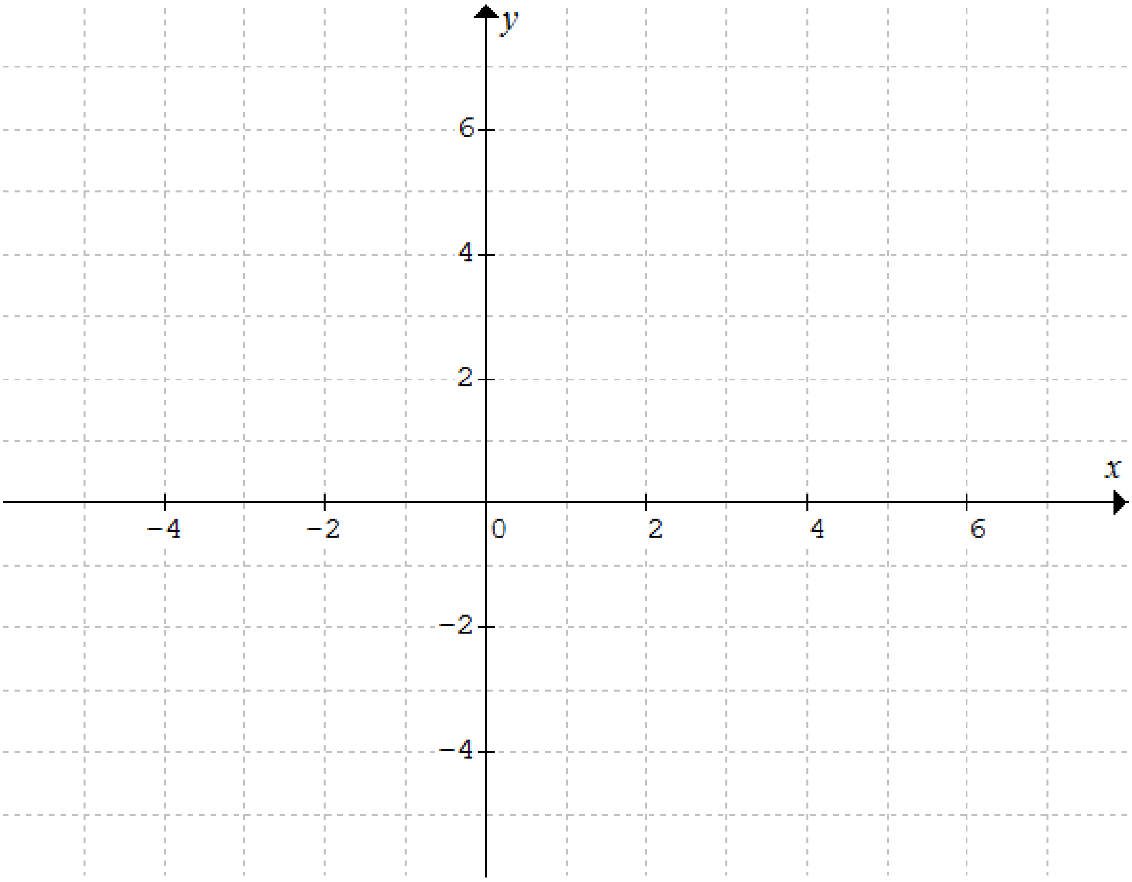
1 mark

e. Find all values of *u*, such that $f(u)=f^{-1}(u)$.

1 mark

f. Sketch the graphs of $y = f(x)$, $y = f^{-1}(x)$ and the graph of $y = x$ on the diagram below. Label all x and y intercepts and the intersection points.

2 marks



g. Write down two different equivalent statements involving definite integral which gives the area A bounded by the graphs of $y = f(x)$ and $y = f^{-1}(x)$ in the first quadrant and the coordinate axes. Determine the value of the area A giving your answer correct to three decimal places.

2 marks

Consider now the function with the rule $g(x)=\frac{1}{x+k}+3$, where $k>0$.

h. Find the coordinates where the graph of g crosses the x and y axis.

1 mark

i. Find the rule for g^{-1}, the inverse function g.

1 mark

j. Explain why the graphs of $y=g(x)$ and $y=g^{-1}(x)$ always intersect at two distinct points for any value of k.

2 marks

k. As the value of k gets larger, express the area $A(k)$ bounded by the graphs of $y=g(x)$ and $y=g^{-1}(x)$ in the first quadrant and the coordinate axes in terms of k, (**do not** write a definite integral or an equation involving logs) and hence determine $\lim_{k\to\infty} A(k)$.

1 mark

END OF SECTION B

EXTRA WORKING SPACE

End of question and answer book for the
2019 Kilbaha VCE Mathematical Methods Trial Examination 2

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MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\ ,\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

SIGNATURE ____________________________________

SECTION A

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
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5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
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8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
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