The Mathematical Association of Victoria

Trial Examination 2019

MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

a. $\frac{d}{dx}\left(\frac{\cos(x)}{x}\right) = \frac{-x\sin(x)-\cos(x)}{x^2}$ **1M, 1A**

b. $f(x) = 5x^2 \tan(3x)$

$f'(x) = 10x\tan(3x) + 15x^2\sec^2(3x)$ **1M**

$f'(\pi) = 10\pi\tan(3\pi) + 15\pi^2\sec^2(3\pi)$

$= 15\pi^2$ **1A**

Question 2

$2e^x + 5 = 3e^{-x}$

$2e^x + 5 - 3e^{-x} = 0$

$2e^{2x} + 5e^x - 3 = 0$ **1M**

Let $a = e^x$

$2a^2 + 5a - 3 = 0$

$(2a-1)(a+3) = 0$ **1M**

$a = \frac{1}{2}$, $a = e^x \neq -3$

$e^x = \frac{1}{2}$

$x = \log_e\left(\frac{1}{2}\right) = -\log_e(2)$ either form **1A**

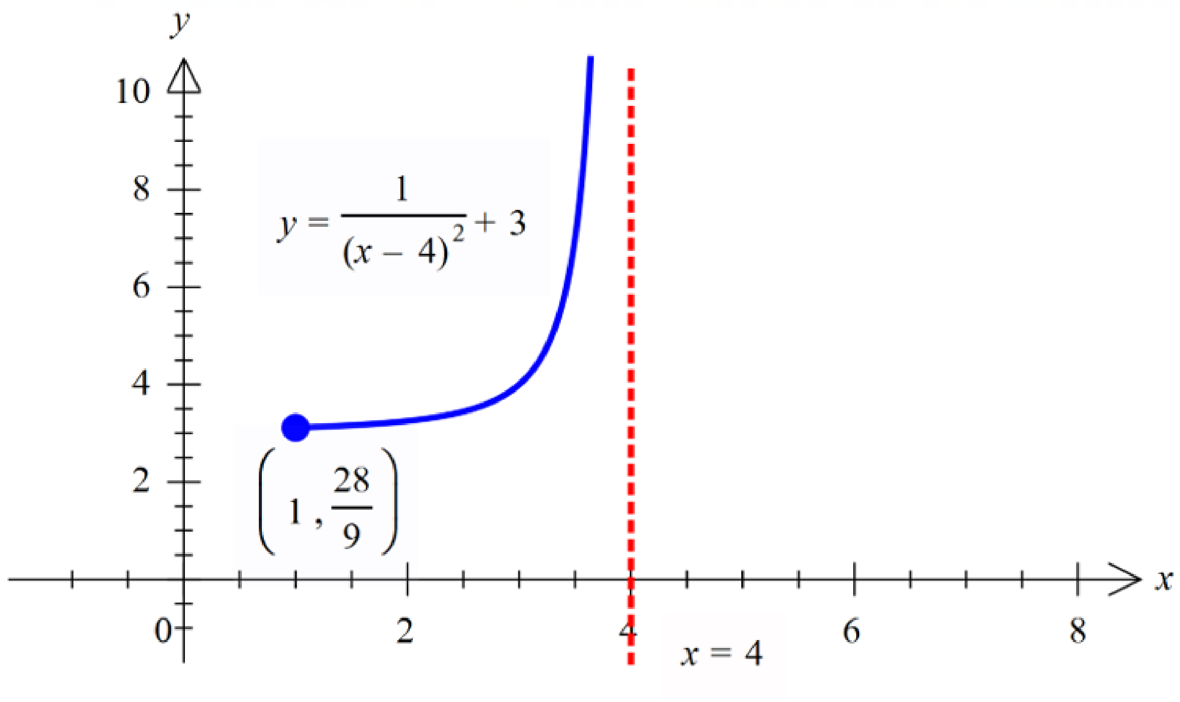
Question 3

a. The range of *f*, $(0,9]$, is a subset of the domain of *g*, $R\setminus\{0\}$. **1A**

b. $g(f(x)) = \frac{1}{(x-4)^2} + 3$ **1A**

Domain $[1, 4)$ **1A**

c. Shape, including correct domain **1A**
Asymptote **1A**



Question 4

a. $h:[-1,\infty) \to R, h(x) = -\sqrt{x+1}$

Let $y = -\sqrt{x+1}$.

Inverse swap x and y

$x = -\sqrt{y+1}$

$x^2 = y+1$

$h^{-1}(x) = x^2 - 1$ **1A**

Domain $(-\infty, 0]$ **1A**

OR

$h^{-1}:(-\infty,0] \to R, h^{-1}(x) = x^2 - 1$ **2A**

b. Solve $-\sqrt{x+1} = x$ for x.

$x + 1 = x^2$

$x^2 - x - 1 = 0$ **1M**

$x = \frac{1-\sqrt{5}}{2}, x \neq \frac{1+\sqrt{5}}{2}$

$\left(\frac{1-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}\right)$ **1A**

$(-1,0), (0,-1)$ **1A**

OR

Solve $-\sqrt{x+1} = x^2 - 1$ for x.

$x + 1 = (x^2 - 1)^2$

$0 = x^4 - 2x^2 - x$ **1M**

$0 = x(x^3 - 2x - 1)$

$0 = x(x+1)(x^2 - x - 1)$

$x = 0, x = -1, x = \frac{1-\sqrt{5}}{2}, x \neq \frac{1+\sqrt{5}}{2}$

$\left(\frac{1-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}\right)$ **1A**

$(-1, 0), (0, -1)$ **1A**

Question 5

a. $\frac{d}{dx}\left(x\log_e(x) - x\right)$

$= \log_e(x) + 1 - 1$

$= \log_e(x)$ **1A**

b.i. $g(x) = 2\log_e(x-1)$

A dilation by a factor of 2 from the x-axis. **1A**

A translation of 1 unit to the right. **1A**

b.ii. $\int_2^3 \left(2\log_e(x-1)\right) dx$ **1A**

$= 2\int_1^2 \left(\log_e(x)\right) dx$

$= 2\left[x\log_e(x) - x\right]_1^2$ **1M**

$= 2\left((2\log_e(2) - 2) - (-1)\right)$

$= 4\log_e(2) - 2$ **1A**

OR

$\int_2^3 \left(2\log_e(x-1)\right) dx$ **1A**

$= 2\left[(x-1)\log_e(x-1) - (x-1)\right]_2^3$ **1M**

$= 2\left((2\log_e(2) - 2) - (-1)\right)$

$= 4\log_e(2) - 2$ **1A**

Question 6

a. $\frac{1}{2} \times \frac{6}{16} \times \frac{5}{15} + \frac{1}{2} \times \frac{4}{9} \times \frac{3}{8}$ **1M**

$= \frac{1}{16} + \frac{1}{12} = \frac{7}{48}$ **1A**

b. $\Pr(B_A \mid 2W) = \frac{\Pr(B_A \cap 2W)}{\Pr(2W)} = \frac{\frac{1}{16}}{\frac{7}{48}} = \frac{3}{7}$ **1A**

Question 7

$$\int_{-1}^{0}\left((x+1)^{\frac{1}{2}}\right)dx$$

$$=\left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_{-1}^{0}$$ **1M**

$$=\frac{2}{3}$$ **1A**

Solve $\int_{0}^{a}\left(\frac{2}{x+2}\right)dx=\frac{1}{3}$ for a.

$$2\left[\log_e(x+2)\right]_0^a=\frac{1}{3}$$ **1M**

$$\log_e(a+2)-\log_e(2)=\frac{1}{6}$$

$$\log_e\left(\frac{a+2}{2}\right)=\frac{1}{6}$$

$$e^{\frac{1}{6}}=\frac{a+2}{2}$$

$$a=2e^{\frac{1}{6}}-2$$ **1A**

Question 8

a. $\sqrt{3}\tan\left(2x-\frac{\pi}{2}\right)+2=1$

$$\tan\left(2x-\frac{\pi}{2}\right)=-\frac{1}{\sqrt{3}}$$

$$2x-\frac{\pi}{2}=\frac{5\pi}{6}...$$ **1A**

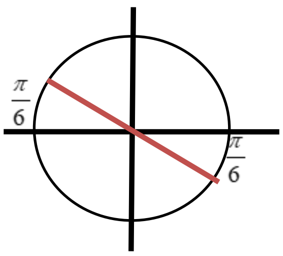
$$2x=\frac{8\pi}{6}...$$

$x=\frac{8\pi}{12}...$ (add and subtract the period, $\frac{\pi}{2}$) **1A**

$$x=-\frac{\pi}{3},\frac{\pi}{6}$$ **1A**

OR

Unit circle location 2nd/4th quadrant with basic angle

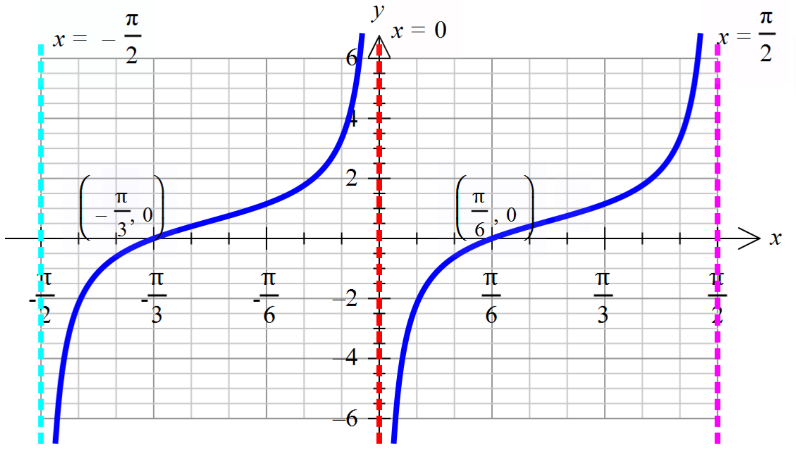


and/or adjustment of domain i.e. $2x-\frac{\pi}{2}\in\left(-\frac{3\pi}{2},\frac{\pi}{2}\right)$ **1A**

$2x-\frac{\pi}{2}=-\frac{7\pi}{6},-\frac{\pi}{6}$ **1A**

$x=-\frac{\pi}{3},\frac{\pi}{6}$ **1A**

b. Shape **1A**
Asymptotes **1A**
Intercepts **1A**



Question 9

a. $f(x)=x^3+2x$

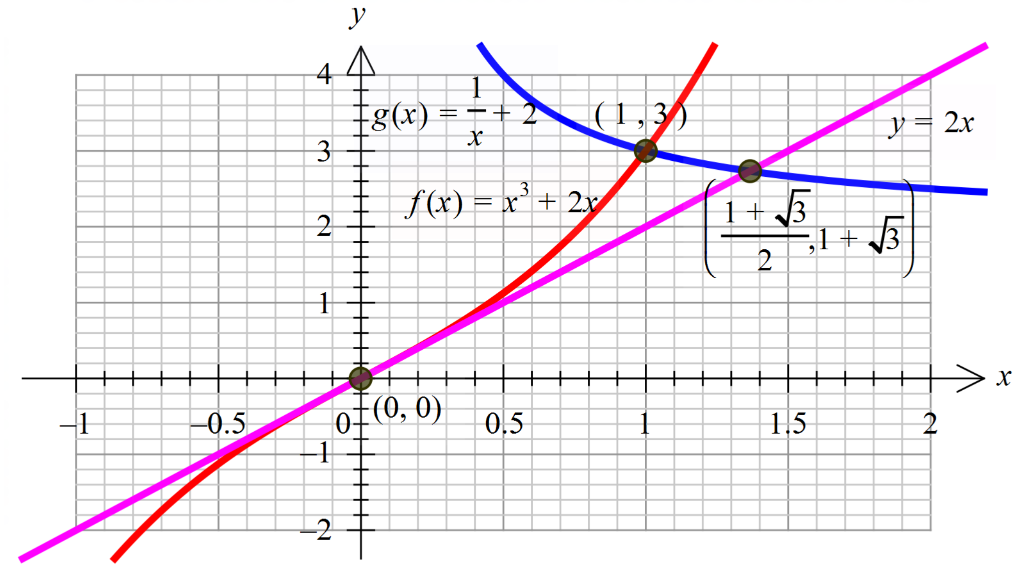
$f'(x)=3x^2+2$

$m=f'(0)=2$

$f(0)=0$

$y=2x$ **1M Show that**

b.



$x^3+2x=\frac{1}{x}+2$

$x^4+2x^2-2x-1=0$

$x=1$ is a solution **1A**

As there is only one positive solution (curvature of graphs) there is no need to investigate further solutions.

$2x = \frac{1}{x} + 2$

$2x^2 - 2x - 1 = 0$

$x = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}$

$x = \frac{1+\sqrt{3}}{2}, x > 0$ **1A**

$\int_0^1 \left(f(x) - 2x\right) dx + \int_1^{\frac{1+\sqrt{3}}{2}} \left(g(x) - 2x\right) dx$ **1A**

END OF SOLUTIONS