Neap:

Trial Examination 2019

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (4 marks)

a. $y = (3\log_e(x))^2$

Using the chain rule:

$\dfrac{dy}{dx} = 2(3\log_e(x)) \times 3 \times \dfrac{1}{x}$

$= \dfrac{18\log_e(x)}{x}$ A1

b. $f(x) = \dfrac{\cos(3x)}{x^2}$

Using the quotient rule:

Let $u = \cos(3x) \Rightarrow u' = -3\sin(3x)$

Let $v = x^2 \Rightarrow v' = 2x$ M1

$f'(x) = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{-3x^2\sin(3x) - 2x\cos(3x)}{x^4}$ A1

$f'\left(\dfrac{2\pi}{3}\right) = \dfrac{-3\left(\dfrac{2\pi}{3}\right)^2\sin(2\pi) - 2\left(\dfrac{2\pi}{3}\right)\cos(2\pi)}{\left(\dfrac{2\pi}{3}\right)^4}$

$= -\dfrac{27}{4\pi^3}$ A1

Question 2 (4 marks)

a. $f'(x) = 1 + \dfrac{2}{e^x}$

$f(x) = \int (1 + 2e^{-x})dx$

$= x - 2e^{-x} + c$ A1

$f(0) = 4 \Rightarrow 0 - 2 + c = 4$

$\therefore c = 6$

$f(x) = x - 2e^{-x} + 6$ A1

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b. $m_T = f'(0) = 1 + 2e^{-0}$

$= 3$ M1

$y - y_1 = m_T(x - x_1)$

$y - 4 = 3(x - 0)$

$y = 3x + 4$ A1

Question 3 (2 marks)

$\tan^2(2x) = 1$ for x, where $x \in [0, \pi]$.

$\tan(2x) = \pm 1$ for x, where $2x \in [0, 2\pi]$. M1

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$ A1

Question 4 (5 marks)

a. $5 - 2\log_e(x) = \log_e\left(x^{\frac{1}{2}}\right)$

$5 = \frac{1}{2}\log_e(x) + 2\log_e(x)$ M1

$\frac{5}{2}\log_e(x) = 5$

$\log_e(x) = 2$

$x = e^2$ A1

b. $5^x = 125^{k - x^2}$

$= 5^{3(k - x^2)}$

$x = 3(k - x^2)$ A1

$3x^2 + x - 3k = 0$

$\Delta = b^2 - 4ac$

$= 1 - 4(3)(-3k)$

$= 1 + 36k$ M1

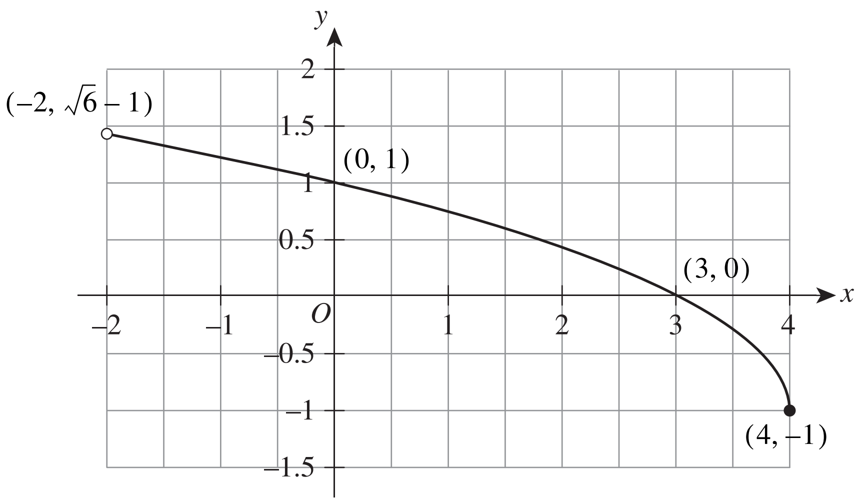
No solutions: $\Delta < 0$

$1 + 36k < 0$

$k < -\frac{1}{36}$ A1

Question 5 (6 marks)

a.



correct intercepts A1
correct endpoints A1
correct shape A1

b. $\text{area} = \int_0^3 (\sqrt{4-x} - 1)\,dx$ M1

$= \int_0^3 \left((4-x)^{\frac{1}{2}} - 1\right)dx$

$= \left[-\frac{2}{3}(4-x)^{\frac{3}{2}} - x\right]_0^3$ M1

$= \left(-\frac{2}{3} - 3\right) - \left(-\frac{2}{3} \times 8\right)$

$= -\frac{11}{3} + \frac{16}{3}$

$= \frac{5}{3}$ A1

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Question 6 (7 marks)

a. probability density function → area = 1

$$\int_0^1 (kx^2 - kx^3)\,dx = 1$$ M1

$$k\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 1$$

$$k\left(\frac{1}{3} - \frac{1}{4}\right) = 1$$

$$k \times \frac{1}{12} = 1$$

$k = 12$ as required A1

b. $$\text{mean} = \int_0^1 x \times 12x^2(1-x)\,dx$$ M1

$$= 12\int_0^1 (x^3 - x^4)\,dx$$

$$= 12\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$$

$$= 12\left[\frac{1^4}{4} - \frac{1^5}{5}\right]$$

$$= 12 \times \frac{1}{20}$$

$$= \frac{3}{5}$$ A1

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c. $g(x)$ is a probability density function.

$$\therefore q\int_0^p x^2(p-x)dx = 1$$

$$\frac{qp^4}{12} = 1 \qquad \textbf{(eq. 1)}$$

$$\text{mean} = \int_0^p x \times qx^2(p-x)dx$$

$$\frac{3}{10} = q\int_0^p x^3(p-x)dx$$

$$\frac{3}{10} = q\int_0^p (px^3 - x^4)dx$$

$$\frac{3}{10} = q\left[\frac{px^4}{4} - \frac{x^5}{5}\right]_0^p$$

$$\frac{3}{10} = q\left[\frac{p^5}{4} - \frac{p^5}{5}\right]$$

$$\frac{3}{10} = q \times \frac{p^5}{20}$$

$$q = \frac{6}{p^5} \qquad \textbf{(eq. 2)}$$ M1

Substitute **eq. 1** into **eq. 2**.

$$\frac{6}{p^5} \times \frac{p^4}{12} = 1$$

$$\frac{1}{2p} = 1$$

$$p = \frac{1}{2}$$ A1

Substitute $p = \frac{1}{2}$ into **eq. 2**.

$$q = \frac{6}{\left(\frac{1}{2}\right)^5}$$

$$= 192$$ A1

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Question 7 (5 marks)

a. $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{1}{8}$

$\Pr(A \cap B) = \dfrac{1}{8}\Pr(B)$

$\Pr(B|A) = \dfrac{\Pr(A \cap B)}{\Pr(A)} = \dfrac{1}{3}$

$\Pr(A \cap B) = \dfrac{1}{3}\Pr(A)$

$\Rightarrow \dfrac{1}{8}\Pr(B) = \dfrac{1}{3}\Pr(A)$

$\therefore \Pr(A) = \dfrac{3}{8}\Pr(B)$ as required A1

b. $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$, as A and B are independent

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A) \times \Pr(B)$

$\Pr(A) = \dfrac{3}{8}\Pr(B)$ from **part a.**

$\therefore \Pr(A \cup B) = \dfrac{3}{8}\Pr(B) + \Pr(B) - \dfrac{3}{8}\Pr(B) \times \Pr(B) = \dfrac{3}{4}$ M1

Let $b = \Pr(B)$.

$\dfrac{3}{8}b + b - \dfrac{3}{8}b \times b = \dfrac{3}{4}$

$\dfrac{11}{8}b - \dfrac{3}{8}b^2 = \dfrac{3}{4}$ M1

$3b^2 - 11b + 6 = 0$

$(3b - 2)(b - 3) = 0$

$b = \dfrac{2}{3}$ or $b = 3$

$\therefore \Pr(B) = \dfrac{2}{3}$, as $\Pr(B) < 1$ A1

c. $\Pr(A' \cap B) = \Pr(B) - \Pr(A \cap B)$

$= \Pr(B) - \Pr(A) \times \Pr(B)$

$= \Pr(B) - \dfrac{3}{8}\Pr(B) \times \Pr(B)$

$= \dfrac{2}{3} - \dfrac{3}{8} \times \dfrac{2}{3} \times \dfrac{2}{3}$

$= \dfrac{1}{2}$ A1

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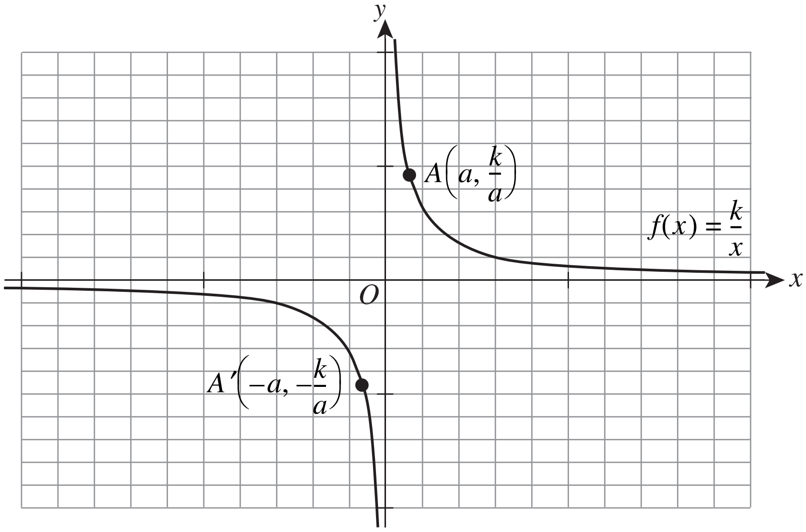
Question 8 (7 marks)

a.



A1

b. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a - (-a))^2 + \left(\frac{k}{a} - \left(-\frac{k}{a}\right)\right)^2}$ M1

$= \sqrt{(2a)^2 + \left(\frac{2k}{a}\right)^2}$

$= \sqrt{4a^2 + \frac{4k^2}{a^2}}$

$= \sqrt{\frac{4(a^4 + k^2)}{a^2}}$

$= \frac{2\sqrt{a^4 + k^2}}{a}$ A1

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c. **i.** Let distance = $d(a)$.

$$d(a) = \frac{2\sqrt{a^4 + k^2}}{a}$$

$$= \frac{\sqrt{4a^4 + 4k^2}}{\sqrt{a^2}}$$

$$= \sqrt{4a^2 + 4k^2a^{-2}}$$

Using the chain rule:

$$d'(a) = (8a - 8k^2a^{-3}) \times \frac{1}{2}(4a^2 + 4k^2a^{-2})^{-\frac{1}{2}}$$

$$= \frac{(8a - 8k^2a^{-3})}{2\sqrt{4a^2 + 4k^2a^{-2}}}$$

$$= \frac{(8a - 8k^2a^{-3})}{2\sqrt{4(a^2 + k^2a^{-2})}}$$

$$= \frac{8(a - k^2a^{-3})}{4\sqrt{a^2 + k^2a^{-2}}}$$

$$= \frac{2(a - k^2a^{-3})}{\sqrt{a^2 + k^2a^{-2}}}$$ M1

Let $d'(a) = 0$.

$a - k^2a^{-3} = 0$

$a^4 - k^2 = 0$ as $a > 0$

$a^4 = k^2$

$a = \sqrt{k}$ as $k > 0$ A1

$$d(\sqrt{k}) = \frac{2\sqrt{(\sqrt{k})^4 + k^2}}{\sqrt{k}}$$

$$= \frac{2\sqrt{2k^2}}{\sqrt{k}}$$

$$= 2\sqrt{2k}$$ A1

ii. $2\sqrt{2k} < 10$

$\sqrt{2k} < 5$

$k < \frac{25}{2}$

$\therefore 0 < k < \frac{25}{2}$ as $k > 0$ (stated in original question) A1