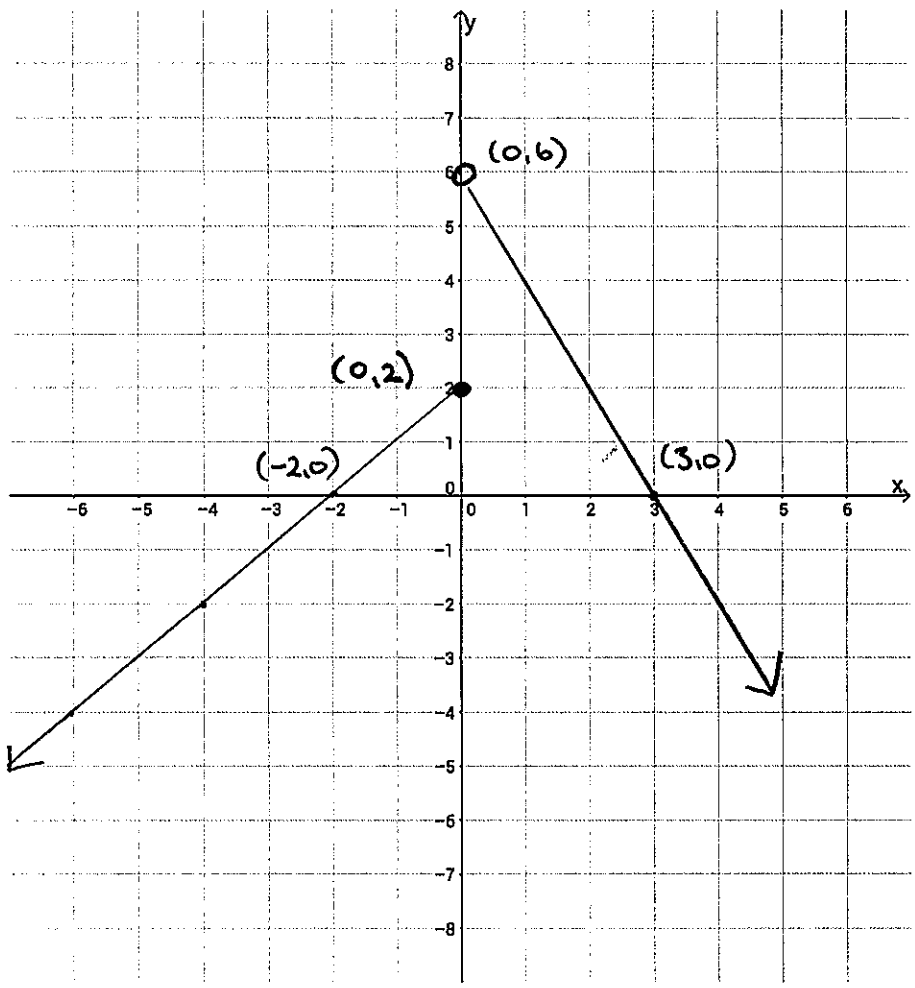
MM12 Cubics and Functions Test 2019

Skills - Part A	**/22**	**Name:**	ANSWERS
- Part B	**/3**		
Total Skills:	**/25**	**Analysis:**	**/23**
Part A: Short Answer		**No Calculator**	

Time Allowed: 20 minutes

1. (a) Sketch the graph of the following function. Clearly label any axial intercepts and end points with their coordinates.

$$f(x) = \begin{cases} x+2 & x \le 0 \\ -2x+6 & x > 0 \end{cases}$$



(b) State the range of the function.

range $y \in (-\infty, 6)$

(4+1 = 5 Marks)

2. For the function with the rule $f(x) = x^2 - 4x + 3$, find:

(a) $f(2)$ $= 4 - 8 + 3 = -1$

(b) $f(a) + f(-2a)$ $= a^2 - 4a + 3 + (-2a)^2 - 4(-2a) + 3$

$= 5a^2 + 4a + 6$

(c) $f(a+2) + f(-a-2)$ $= (a+2)^2 - 4(a+2) + 3 + (-a-2)^2 - 4(-a-2) + 3$

$= a^2 + 4a + 4 - 4a - 8 + 3 + a^2 + 4a + 4 + 4a + 8 + 3$

$= 2a^2 + 8a + 14$

(1+2+3 = 6 marks)

3. (a) **Show that** the circle with the equation $x^2 + y^2 - 2x - 4y - 20 = 0$ can be written in the form $(x-1)^2 + (y-2)^2 = 25$

$(x^2 - 2x + 1) + (y^2 - 4y + 4) - 20 - 1 - 4 = 0$

$(x-1)^2 + (y-2)^2 = 25$

(b) State the radius of the circle and the coordinates of its centre.

radius $= 5$ centre $(1,2)$

(c) Find the exact horizontal distance between the x-intercepts of the circle

x intercepts, $y = 0$

$(x-1)^2 + (-2)^2 = 25$

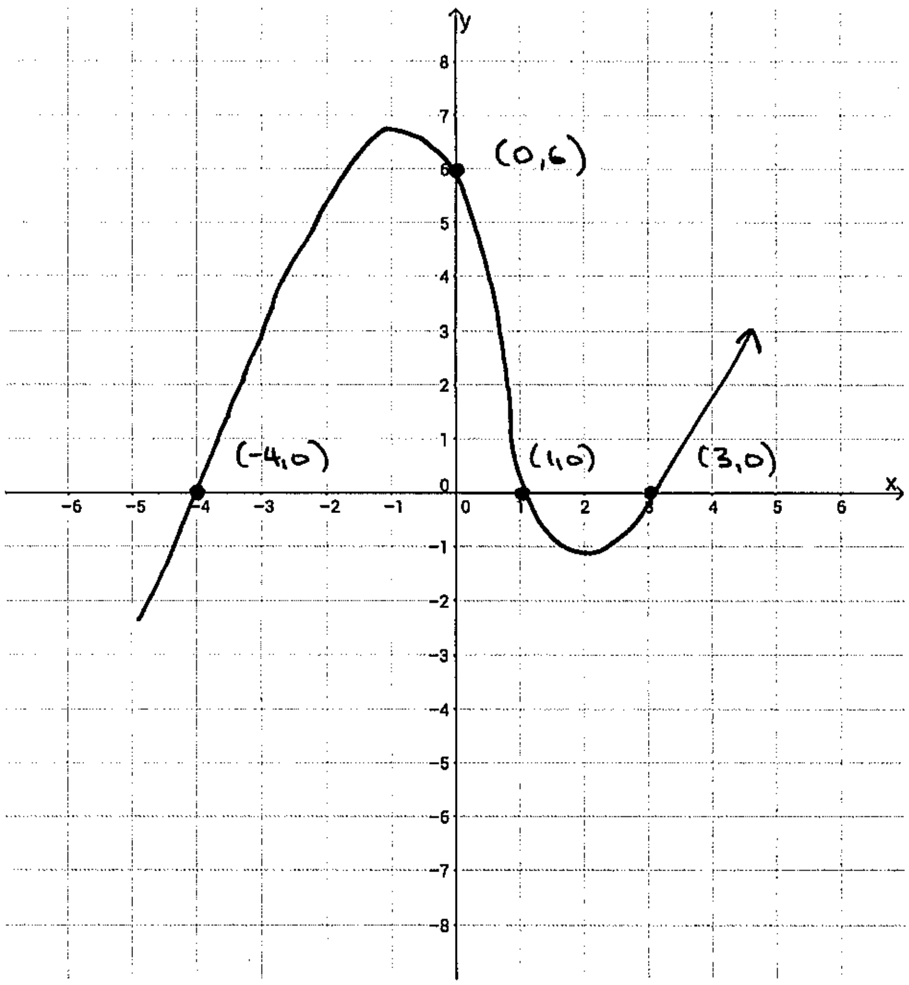
$(x-1)^2 = 21$

$x - 1 = \pm\sqrt{21}$

(2+2+2 = 6 marks)

$x = 1 \pm \sqrt{21}$ distance $= 2\sqrt{21}$

4. (a) Sketch the graph of $y = \frac{1}{2}(x+4)(x-1)(x-3)$. Label the coordinates of all axial intercepts. Coordinates of turning points do not need to be shown.



(b) Find $\left\{x: \frac{1}{2}(x+4)(x-1)(x-3) \geq 0\right\}$.

$x \in [-4, 1] \cup [3, \infty)$

1

(c) How many solutions will there be to the equation $\frac{1}{2}(x+4)(x-1)(x-3) = 1$

3

(2+2+1 = 5 marks)

End of Section A

Name: ____________________________

Section B: Multiple Choice Questions **Calculators and notes are allowed**

Circle the correct response **Time allowed: 25 minutes**

Question 1

A possible equation for the curve shown is

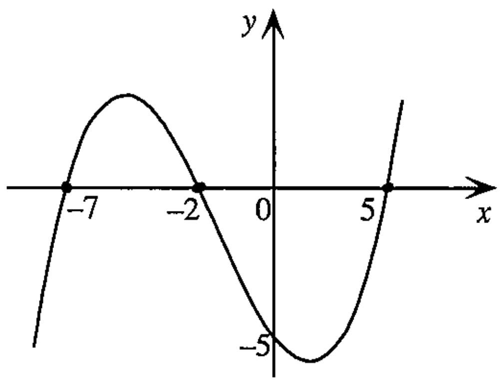
A. $y = (x + 7)(x + 2)(x - 5)$

B. $y = (x - 7)(x - 2)(x + 5)$

C. $y = \frac{1}{14}(x + 7)(x + 2)(x - 5)$

D. $y = \frac{1}{14}(x - 7)(x - 2)(x + 5)$

E. $y = -\frac{1}{14}(x + 7)(x + 2)(x - 5)$



Question 2

The graph of the equation $y = x^3$ is reflected in the x-axis and then translated 2 units to the right and 1 unit down. The equation of the new graph is

A. $y = -(x - 2)^3 - 1$

B. $y = -(x - 2)^3 + 1$

C. $y = (-x)^3 - 3$

D. $y = -(x)^3 - 1$

E. $y = -(x)^3 - 2 - 1$

Question 3

The range of the function $f: [-3, 5) \to \mathbb{R}, f(x) = 7 - x$ is

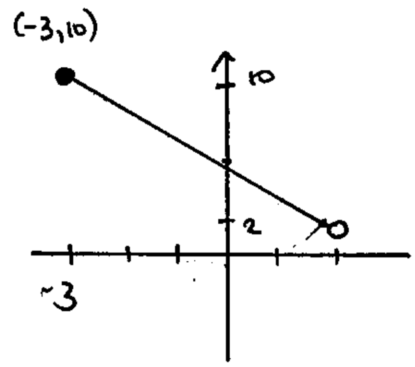
A. (2, 10]

B. [2, 10)

C. $(2, \infty)$

D. (2, 10)

E. $\mathbb{R}$

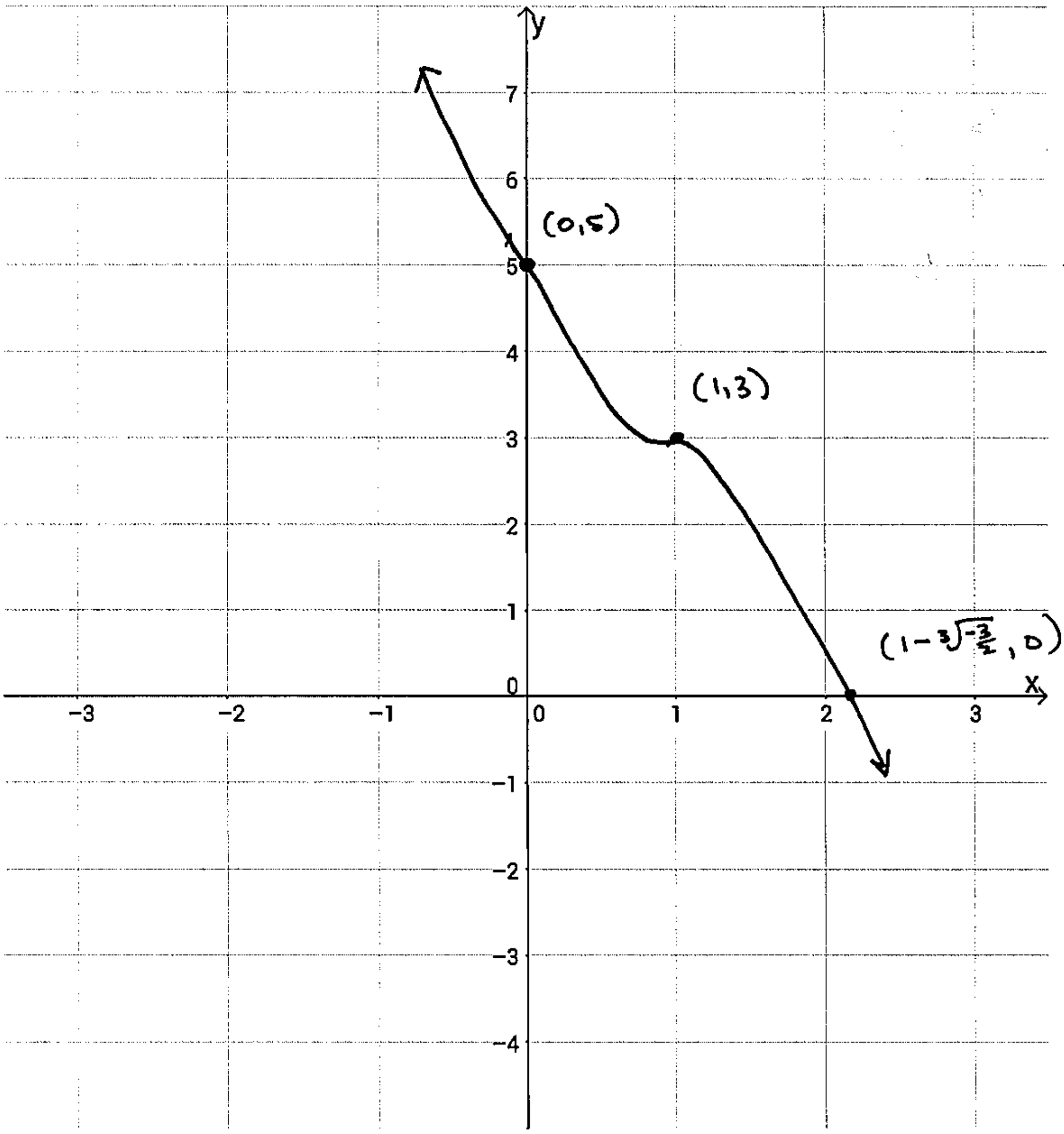


Section C: Extended Response

Exact answers should be given unless otherwise specified

Question 1

a. Sketch the graph of $y = 2(1-x)^3 + 3$ labelling any intercepts with the coordinate axes and the stationary point of inflexion with their **exact coordinates.**



b. If the domain was restricted to $x \in [0,3)$ what would be the range?

$(-13, 5]$

(3 + 1 = 4 marks)

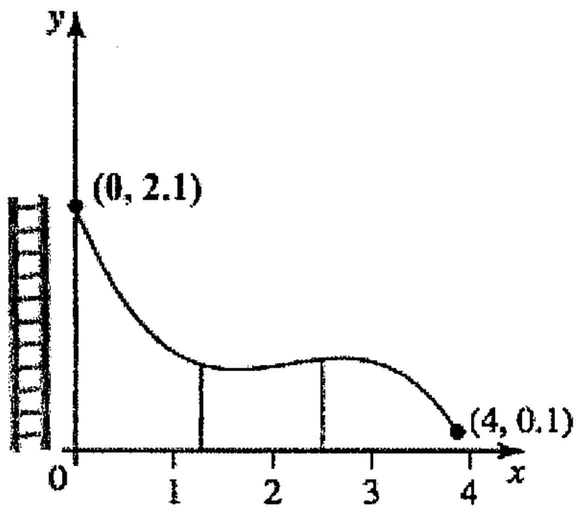
Question 2

A new playground slide is to be constructed at a local park. At the foot of the slide is a vertical ladder. The slide starts at a vertical height of 2.1 m above the ground.

The end point is 0.1 m above the ground and is 4 m horizontally from the foot of the slide.

A model for the slide is $h(x) = ax^3 + bx^2 + cx + d$ where h metres is the height of the slide above ground level at a horizontal distance, x m from the foot. The foot is at the origin.

The slide has two vertical supports, one is 1 m long and is 1.25 m horizontally from the foot and the other is 1.1 m long and 2.5 m horizontally from the foot.



(a) Give the coordinates of 4 points that lie on the cubic graph of the slide

(0, 2.1) (4, 0.1) (1.25, 1) (2.5, 1.1)

1 1

(b) State the value of d in the equation of the slide

$d = 2.1$ 1

Alternative

(c) Write three simultaneous equations that could be used to find the coefficients a, b, and c.

$h(4) = 0.1$
$h(1.25) = 1$
$h(2.5) = 1.1$

$0.1 = 64a + 16b + 4c + 2.1$ (1)

$1 = (1.25)^3a + (1.25)^2b + 1.25c + 2.1$

$1.1 = (2.5)^3a + (2.5)^2b + 2.5c + 2.1$

(d) The equation of the slide can be shown to be
$h(x) = -0.164x^3 + x^2 - 1.872x + 2.1$
Use this equation to find the length of a third vertical support at $x = 3.5\ m$. Give your answer correct to 2 decimal places.

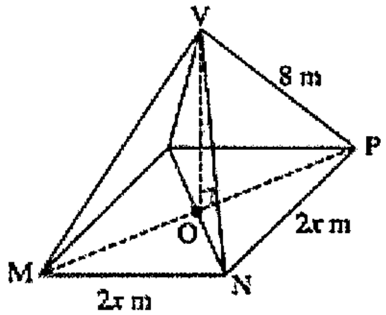
$h = 0.77$ m

(2+1+2+1 = 6 marks)

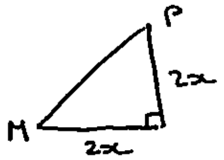
Question 3

A tent used by a group of campers is in the shape of a square-based right pyramid with a slant edge, VP, of 8 metres.

For the figure shown, let OV, the height of the tent be h metres and the edge of the square base be $2x$ metres.

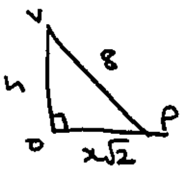


(a) Use Pythagoras' theorem and triangle MPN to show that the length of the diagonal (MP) of the square base is $2x\sqrt{2}$ metres



$$d(MP) = \sqrt{(2x)^2 + (2x)^2}$$
$$= \sqrt{8x^2}$$
$$= 2x\sqrt{2}$$

(b) Use Pythagoras' theorem and triangle VOP to show that $2x^2 = 64 - h^2$



$$8^2 = h^2 + (x\sqrt{2})^2$$
$$64 = h^2 + 2x^2$$
$$2x^2 = 64 - h^2$$

(c) The volume, V, of a pyramid is found using the formula $V = \frac{1}{3}Ah$, where A is the area of the base of the pyramid. Use this formula to show that the volume of the tent is given by $V = \frac{1}{3}(128h - 2h^3)$

$$V = \frac{1}{3} \times 4x^2 \times h$$
$$= \frac{1}{3} \times 2(64 - h^2) \times h$$
$$= \frac{1}{3}(128h - 2h^3)$$

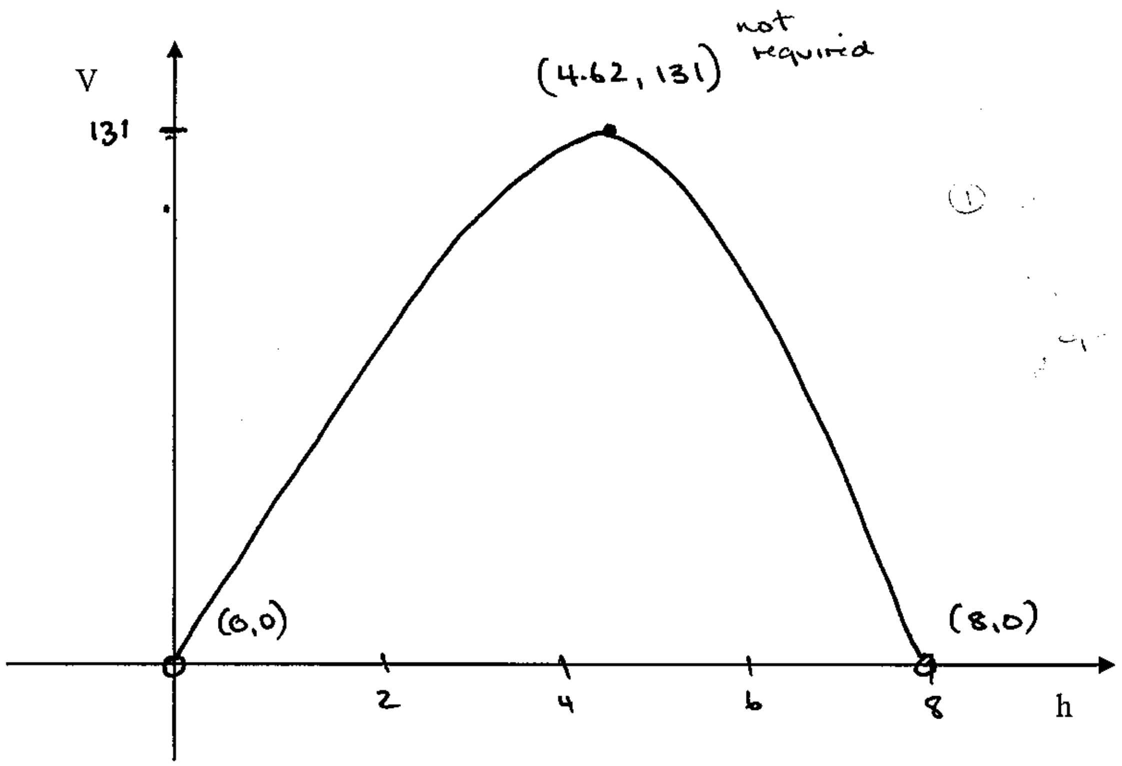
(d) What is the domain of the volume function

$$0 < h < 8$$

(e) If the height of the tent was 3 metres, what is the volume?

$$V = 110 \text{ m}^3$$

(f) Sketch a graph of $V = \frac{1}{3}(128h - 2h^3)$, showing the coordinates of any end points.



(g) What is the **maximum volume** of the tent (to the nearest whole number) and what **height** (correct to 1 d.p.) gives the maximum volume?

$V = 131 \text{ m}^3$

$h = 4.6 \text{ m}$

(2+2+2+1+1+3+2 = 13 marks)