MM1/2 Rates and Differential Calculus Mini Test 2019

Name: ANSWERS **Total Marks:**__________ **/ 26**

Notes or calculator NOT allowed **Time allowed: 40 minutes**

Multiple Choice – Circle the correct response

Question 1

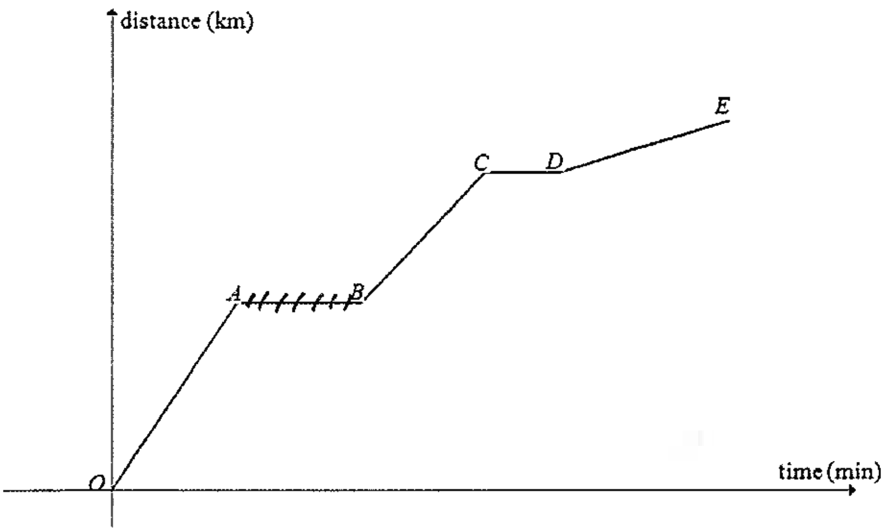
If $g(x) = 1 - 3x + x^2$, then $g(x+h) - g(x)$ is equal to:

A. $h(h+1)$
B. $h(h+2x-3)$
C. $1-3h+h^2$
D. $-3+2h$
E. $h(2x-3)$

$g(x+h) - g(x)$
$= 1 - 3(x+h) + (x+h)^2 - 1 + 3x - x^2$
$= \cancel{1} - \cancel{3x} - 3h + \cancel{x^2} + 2xh + h^2 - \cancel{1} + \cancel{3x} - \cancel{x^2}$
$= -3h + 2xh + h^2$
$= h(h+2x-3)$

Question 2

The graph below shows the movement of a train over a period of time.



For the section of the graph between A and B, the train is:

A. speeding up
B. slowing down
C. travelling east
D. travelling at a constant speed greater than zero
E. stationary

Question 3

If $y = x - 4(x+4)(x+1)$ then $\frac{dy}{dx}$ equals:

A. $-19 - 8x$
B. $3x^2 + 2x - 16$
C. $4x - 32$
D. $-2x - 4$
E. $-8x - 35$

$y = x - 4(x^2 + 5x + 4)$

$y = -4x^2 - 19x - 20$

$\frac{dy}{dx} = -8x - 19$

Question 4

For the function with the rule $f(x) = x^3 - 3x + 3$, the gradient of the chord that connects the points $x = 2$ and $x = 3$ is:

A. 21
B. 16
C. 5
D. 8
E. 4

$x = 2 \quad y = 5$

$x = 3 \quad y = 21$

gradient $= \frac{21-5}{3-2}$

$= 16$

Question 5

Given that $f(x) = 2x^3 + 3x^2 - 12x$, the value(s) of x for which $f'(x) = 0$ are:

A. 0 and 3
B. 2
C. 2 and -1
D. -2 and 1
E. 0, 1 and 2

$f'(x) = 6x^2 + 6x - 12$

$f'(x) = 0$

$6x^2 + 6x - 12 = 0$

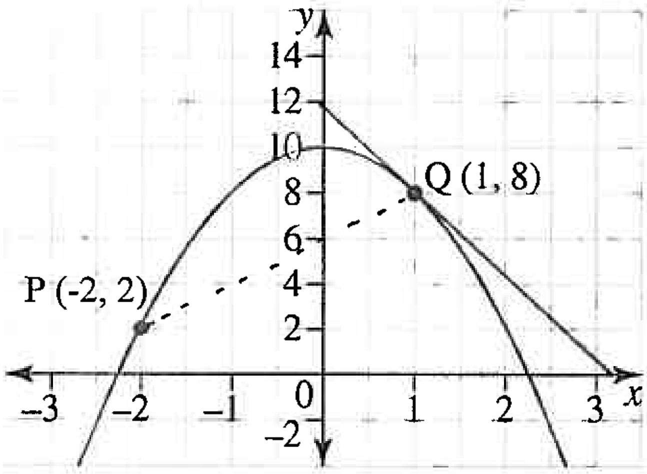
$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

Short Answer Questions
Write your answers in the spaces provided.

Question 6
The graph shows a function $y = f(x)$ on which the point $Q(1, 8)$ lies.



(a) Find the gradient of the secant connecting the points P(-2, 2) and Q(1, 8)

$M = \dfrac{8-2}{1--2}$ 1 attempt $\dfrac{y_2-y_1}{x_2-x_1}$ or $\dfrac{\text{rise}}{\text{run}}$

$= 2$ 1 A

(b) Draw the tangent to the curve at the point Q(1, 8) and find an estimate of the gradient of this function at the point Q.

$m = -\dfrac{12}{3}$ 1 tangent drawn at Q

$M = -4$ 1 'reasonable' answer

(2 + 2 = 4 marks)

Question 7

Evaluate:

$$\lim_{h\to 0}\frac{(2+h)^2-4}{h}$$

$= \lim_{h\to 0} \frac{\cancel{4}+4h+h^2-\cancel{4}}{h}$ ① A expand

$= \lim_{h\to 0} 4+h$

$= 4$ ① A

(2 marks)

Question 8

Given $f(x) = 2x^2 - 3x$, use **first principles** to show that $f'(x) = 4x - 3$

$f(x+h) = 2(x+h)^2 - 3(x+h)$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h$ ① A correct $f(x+h)$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2-\cancel{3x}-3h-\cancel{2x^2}+\cancel{3x}}{h}$ ① M using $f(x+h)-f(x)$

$= \lim_{h\to 0} \frac{4xh+2h^2-3h}{h}$ ① M simplify their

$= \lim_{h\to 0} \frac{\cancel{h}(4x+2h-3)}{\cancel{h}}$ ① A

$= \lim_{h\to 0} 4x+2h-3$

(4 marks)

$= 4x-3$

Question 9

Find the derivative of the following, giving positive powers only.

(a) $y = 5x^3 - x^2 + 2x - 4$

$$\frac{dy}{dx} = 15x^2 - 2x + 2$$ ① A

(b) $f(x) = x^4 + 6\sqrt{x} + \frac{3}{x^2}$

$$f(x) = x^4 + 6x^{1/2} + 3x^{-2}$$ ① A

$$f'(x) = 4x^3 + 3x^{-1/2} - 6x^{-3}$$

$$= 4x^3 + \frac{3}{x^{1/2}} - \frac{6}{x^3}$$ ① A

(1 + 2 = 3 marks)

Question 10

(a) Use the **product rule** to differentiate the function $f(x) = (x^2 - 4x)(2x^2 - 3)$. Expand and simplify your answer.

$u = x^2 - 4x \qquad v = 2x^2 - 3$

$u' = 2x - 4 \qquad v' = 4x$

$$f'(x) = (x^2 - 4x) \times 4x + (2x^2 - 3)(2x - 4)$$ ① M attempt product rule

$$= 4x^3 - 16x^2 + 4x^3 - 8x^2 - 6x + 12$$ ① M simplify

$$= 8x^3 - 24x^2 - 6x + 12$$ ① A

(b) Use the **chain rule** to differentiate $y = (4x^3 - 5x)^5$. **Do not** expand your answer.

$$\frac{dy}{dx} = 5(12x^2 - 5)(4x^3 - 5x)^4$$

1

1 answer

(3 + 2 = 5 marks)

Question 11

Given the function $f:(-\infty, \frac{1}{2}) \to R, \quad f(x) = \frac{2x-3}{3-6x}$.

Show that $f'(x) = \frac{-12}{(3-6x)^2}$

$u = 2x - 3 \qquad v = 3 - 6x$ 1A

$u' = 2 \qquad v' = -6$

$$f'(x) = \frac{(3-6x)\times 2 - (2x-3)\times -6}{(3-6x)^2}$$

1M attempt quotient rule (or product for (2x-3)(3-6x)

$$= \frac{6 - 12x + 12x - 18}{(3-6x)^2}$$

1M their expansion done correctly.

$$= \frac{-12}{(3-6x)^2}$$

(3 marks)