 **NAME:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 **UNITS 3 & 4 Practice Examination**

**VCE**®**Mathematical Methods**

**Written examination 1**

**Reading time: 15 minutes Writing time: 1 hour**

**QUESTION AND ANSWER BOOK**

**Structure of book**

|  |  |  |
| --- | --- | --- |
| *Number of**questions* | *Number of questions**to be answered* | *Number of**marks* |
| $$8$$ | $$8$$ | $$40$$ |

* Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
* Students are **not** permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

* A question and answer booklet of 11 pages.
* A double-sided page of formulas.
* Working space is provided throughout the question answer booklet.

**Instructions**

* All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination paper are **not** drawn to scale

**Question 1** (4 marks)

1. One bag of sweets contains 4 red, 4 white and 2 green sweets. A second bag contains 2 red, 5 white and 3 green sweets. One sweet is chosen from each bag. Calculate the probability **(as a percentage)** that both sweets are the same colour. 2 marks

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1. All sweets are put back in the bags. Two sweets are chosen, without replacement, from the **second** bag. Calculate the probability that exactly one **green** sweet is chosen. Express your answer in the form of  where *a* and *b* are positive integers. 2 marks

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**Question 2** (6 marks)

If it rains on Monday, the probability that it rains on Tuesday as well is ⅓. If it does not rain on Monday, the probability that it does not rain on Tuesday also is ⅔. The probability that it rains on both days is *r*.

1. In terms of *r*, find the probability that it rains on Monday. 2 marks

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1. Find the probability that it rains on Monday and does not rain on Tuesday. 2 marks

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1. Find the maximal domain of possible values of *r*. 2 marks

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**Question 3** (5 marks)

1. . Find . 2 marks

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1. . Find . 3 marks

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**Question 4** (3 marks)

. Find all solutions for.

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**Question 5 (**4 marks)

. Sketch the graph of *g(x)* and mark all axis intercepts and asymptotes.



**Question 6** (5 marks)

Two functions *f* and *g* have the following rules:





1. Find the average value of *f(x)* over the interval [-2,0]. Express your answer in the form  where *a* and *b* are positive integers. 2 marks

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1. The function *h(x) = g{f(x)}.* Find the rule and maximal domain of *h.* 2 marks

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1. Find the average rate of change of *h(x)* over the interval [-1, 0]. 1 mark

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**Question 7** (5 marks)

Let  The graph of *f(x)* is shown below:

1. Find the area enclosed by *f(x)*, the *x* axis, and the line *x* = 1: Express your answer in the form  where *a* and *b* are positive integers. 2 marks

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1. Find the equation of the tangent to the curve of *f(x)* where *x =* 1. Express your answer in the form  where *a, b* and *c* are positive integers. 3 marks

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**Question 8** (8 marks)

Let . The graph of *f(x)* is shown below:

At point A, the *x* coordinate is 2. Find the *y* coordinate. 1 mark

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1. Find the area of the triangle OAB, where O is the origin and B is the point (6, 0). 1 mark

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1. Find an expression for the distance of *f(x)* from the origin and express it in its simplest form. 1 mark

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1. Find the maximum value of  and give the coordinates of the point at which it occurs. 2 marks

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1. Find the inverse function  and state its domain and range. 3 marks

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**END OF QUESTION AND ANSWER BOOK**

**Mathematical Methods Formulas**

Mensuration:

Area of a triangle: ½ *bc* sin A

Differential Calculus: Integral Calculus:

 

 

 

 

 

 

 





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**Solution Pathway**

**NOTE: This task is sold on condition that it is NOT placed on any school network or social media site (such as Facebook, Wikispaces, etc.) at any time.**

**NOT FOR PRIVATE TUTOR USE.**

Below are sample answers. Please consider the merit of alternative responses.

**Mathematical Methods Exam 1: SOLUTIONS**

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| **1(a)** | 34%*1 mark for method, 1 for correct answer* |  |
| **1(b)** | *1 mark for method, 1 for correct answer* |   |
| **2(a)** | 3*r**1 mark for use of correct formula**1 mark for correct answer* |   |
| **2(b)** | 2*r**1 mark for correct deduction**1 mark for correct answer**(2 marks if they did it this way)* |

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| ***Pr*** | ***A*** | ***A'*** | ***Total*** |
| ***B*** | ***r*** |  |  |
| ***B'*** |  | **⅔(1-3*r*)** |  |
| ***Total*** | **3*r*** | **1-3*r*** | **1** |

From the table it is clear that:However, it is not necessary to have the chart. Another solution is:  |
| **2(c)** | (0, **⅓**)*1 mark for completed chart**1 mark for correct domain* | Fill in the remainder of the chart:

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| ***Pr*** | ***A*** | ***A'*** | ***Total*** |
| ***B*** | ***r*** | **⅓-*r*** | **⅓** |
| ***B'*** | **2*r*** | **⅔(1-3*r*)** | **⅔** |
| ***Total*** | **3*r*** | **1-3*r*** | **1** |

Since the probability of rain is never entirely zero, *r* > 0. Similarly, 1 – 3*r* > 0.Maximal domain is (0, **⅓**) |
| **3(a)** | *1 mark for use of correct formula**1 mark for correct answer* |  |
| **3(b)** | *1 mark for use of correct formula**1 mark**1 mark* |  |
| **4** | *1 mark**1 mark for correct domain**1 mark* |  |
| **5** | Asymptotes at *x* = 3, *y* = -2 *1 mark each*Intercepts at (0, -⅓) and (½, 0)*1 mark each* | Note that the graph appears in the 2nd and 4th quadrants rather than 1st and 3rd, because of the negative value of *x* in the denominator. |
| **6(a)** | *1 mark for use of correct formula**1 mark for correct answer*( would also be acceptable, although not as good) |  |
| **6(b)** |  *1 mark for correct answer**1 mark for correct domain* |   |
| **6(c)** | *1 mark* |  |
| **7(a)** | *1 mark for correct expression**1 mark for correct answer* |  |
| **7(b)** |  *1 mark**1 mark**1 mark* |     |
| **8(a)** | (2, 2√3) *1 mark* | A is at *x* = 2, hence   |
| **8(b)** | 6√3 *1 mark* |  |
| **8(c)** |  *1 mark* |  |
| **8(d)** | (2.5, 3.5) or ( ,*1 mark for correct derivative**1 mark* |  when 5 – 2*x* = 0, hence *x* = 2.5. |
| **8(e)** | Domain: [0, 3.5]Range: [-1, 2.5]*1 mark**1 mark**1 mark for negative root* |  When *x* = 0, *y* = -1, so we need the **negative** root. |