QATs
Quality Assessment Tasks

NAME: ___________________________

UNITS 3 & 4 Practice Examination

VCE® Mathematical Methods

Written examination 1

Reading time: 15 minutes **Writing time: 1 hour**

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- A question and answer booklet of 11 pages.
- A double-sided page of formulas.
- Working space is provided throughout the question answer booklet.

Instructions
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination paper are **not** drawn to scale

Question 1 (4 marks)

a. One bag of sweets contains 4 red, 4 white and 2 green sweets. A second bag contains 2 red, 5 white and 3 green sweets. One sweet is chosen from each bag. Calculate the probability **(as a percentage)** that both sweets are the same colour. 2 marks

b. All sweets are put back in the bags. Two sweets are chosen, without replacement, from the **second** bag. Calculate the probability that exactly one **green** sweet is chosen. Express your answer in the form of $\frac{a}{b}$ where a and b are positive integers. 2 marks

Question 2 (6 marks)

If it rains on Monday, the probability that it rains on Tuesday as well is $\frac{1}{3}$. If it does not rain on Monday, the probability that it does not rain on Tuesday also is $\frac{2}{3}$. The probability that it rains on both days is r.

a. In terms of r, find the probability that it rains on Monday. 2 marks

b. Find the probability that it rains on Monday and does not rain on Tuesday. 2 marks

c. Find the maximal domain of possible values of r. 2 marks

Question 3 (5 marks)

a. $f(x) = e^{ax}\cos 2x$. Find $f'(x)$. 2 marks

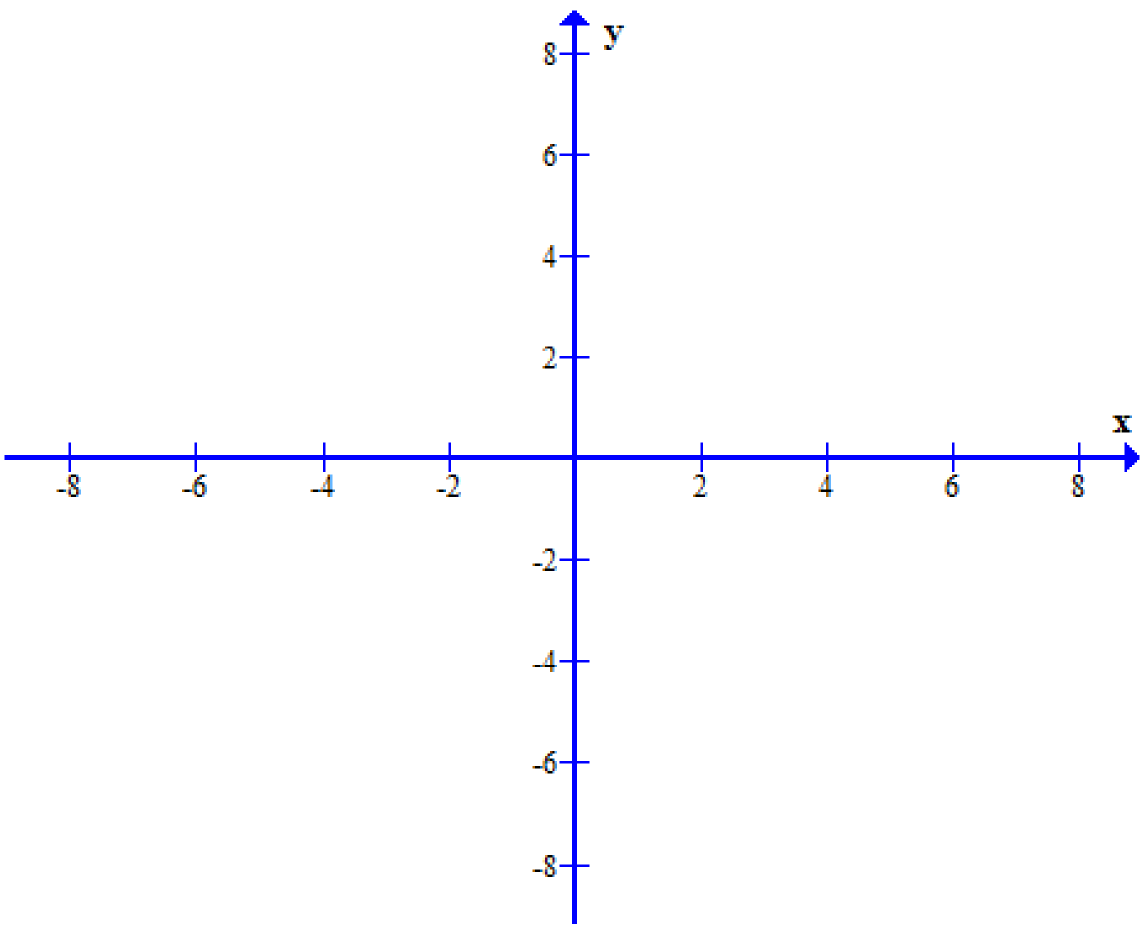
b. $f(x) = \dfrac{\log_e(2x+3)}{(\cos x - \sin x)}$. Find $f'(0)$. 3 marks

Question 4 (3 marks)

$f:[-\frac{1}{2},\frac{1}{4}]\rightarrow R;\ f(x)=4\sin 4\pi x-2\sqrt{3}$. Find all solutions for $f(x)=0$.

Question 5 (4 marks)

$g:R\rightarrow R; g(x)=\frac{2x-1}{3-x}$. Sketch the graph of *g(x)* and mark all axis intercepts and asymptotes.



Question 6 (5 marks)

Two functions *f* and *g* have the following rules:

$$f:(-\infty,\frac{2}{3})\to R: f(x)=\frac{1}{2-3x}$$

$$g:(-\infty,1]\to R: g(x)=\sqrt{1-x}$$

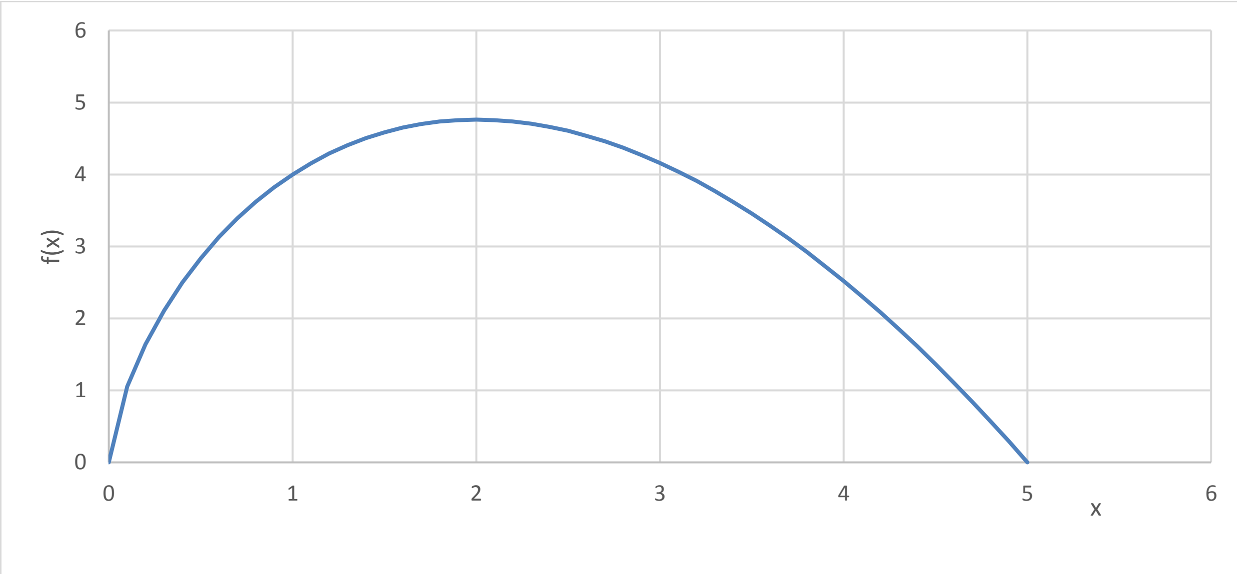
a. Find the average value of *f(x)* over the interval [-2,0]. Express your answer in the form $\frac{1}{a}\log_e b$ where *a* and *b* are positive integers. 2 marks

b. The function *h(x) = g{f(x)}.* Find the rule and maximal domain of *h.* 2 marks

c. Find the average rate of change of *h(x)* over the interval [-1, 0]. 1 mark

Question 7 (5 marks)

Let $f:[0,5]\to R: f(x)=x^{\frac{2}{3}}(5-x)$ The graph of *f(x)* is shown below:

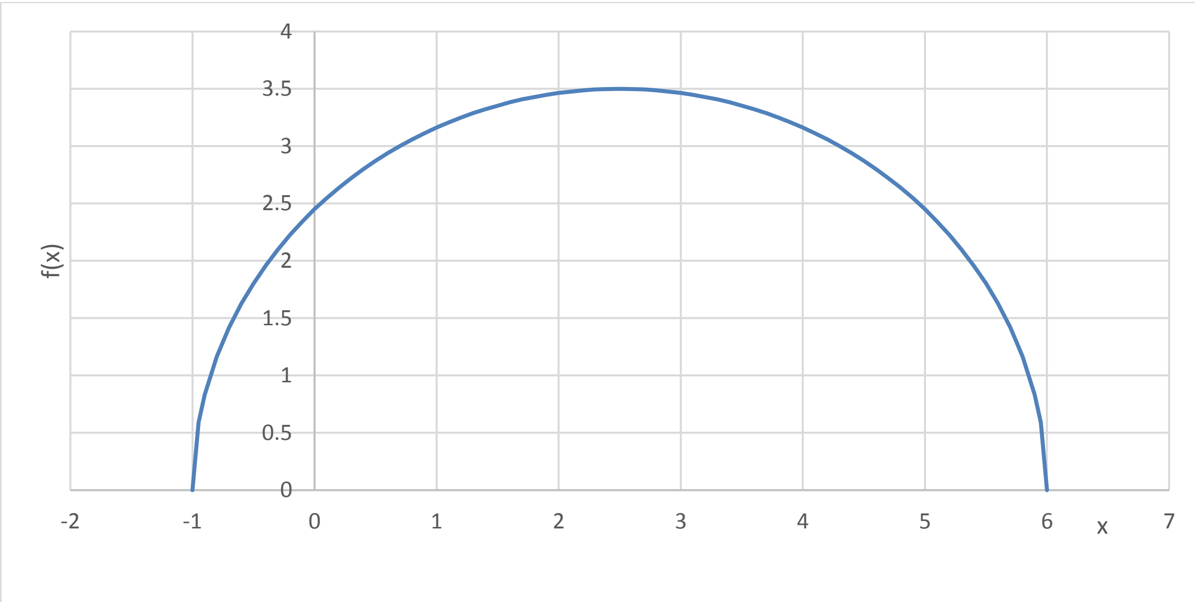


a. Find the area enclosed by *f(x)*, the *x* axis, and the line *x* = 1: Express your answer in the form $\frac{a}{b}$ where *a* and *b* are positive integers. 2 marks

b. Find the equation of the tangent to the curve of *f(x)* where *x* = 1. Express your answer in the form $y=\frac{1}{a}(bx+c)$ where *a, b* and *c* are positive integers. 3 marks

Question 8 (8 marks)

Let $f:[-1,6]\rightarrow R: f(x)=\sqrt{(6-x)(x+1)}$. The graph of *f(x)* is shown below:



At point A, the x coordinate is 2. Find the y coordinate. 1 mark

a. Find the area of the triangle OAB, where O is the origin and B is the point (6, 0). 1 mark

b. Find an expression for the distance of $f(x)$ from the origin and express it in its simplest form.

1 mark

c. Find the maximum value of $f(x)$ and give the coordinates of the point at which it occurs.

2 marks

d. Find the inverse function f^{-1} and state its domain and range. 3 marks

END OF QUESTION AND ANSWER BOOK

Mathematical Methods Formulas

Mensuration:

Area of a triangle: ½ *bc* sin A

Differential Calculus:

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$$

$$\frac{d}{dx}\log_e ax = \frac{1}{x}$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

$$\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integral Calculus:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C; n \neq -1$$

$$\int \frac{1}{x} dx = \log_e x + C$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + C$$

$$\int \sin ax dx = \frac{-1}{a}\cos ax + C$$

$$\int \cos ax dx = \frac{1}{a}\sin ax + C$$

$$\int \tan ax dx = \frac{1}{a}\log_e(\cos ax) + C$$

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Solution Pathway

NOTE: This task is sold on condition that it is NOT placed on any school network or social media site (such as Facebook, Wikispaces, etc.) at any time.

NOT FOR PRIVATE TUTOR USE.

Below are sample answers. Please consider the merit of alternative responses.

Mathematical Methods Exam 1: SOLUTIONS

1(a)	34% *1 mark for method, 1 for correct answer*	$\Pr(R,R)=\frac{4}{10}\times\frac{2}{10}=8\%$ $P\text{r}(W,W)=\frac{4}{10}\times\frac{5}{10}=20\%$ $\Pr(G,G)=\frac{2}{10}\times\frac{3}{10}=6\%$
1(b)	$\frac{7}{15}$ *1 mark for method, 1 for correct answer*	$\Pr(G,G')=\frac{3}{10}\times\frac{7}{9}=\frac{7}{30}$ $\Pr(G',G)=\frac{7}{10}\times\frac{6}{9}=\frac{7}{30}$
2(a)	$3r$ *1 mark for use of correct formula* *1 mark for correct answer*	$Pr\left(B\mid A\right)=\frac{1}{3};\ Pr\left(B'\mid A'\right)=\frac{2}{3}$ $\Pr(B\mid A)=\frac{\Pr(A\cap B)}{\Pr(A)}=\frac{r}{\Pr(A)}=\frac{1}{3}$ $\therefore \Pr(A)=3r$
2(b)	$2r$ *1 mark for correct deduction*	$\Pr(B'\mid A')=\frac{\Pr(A'\cap B')}{\Pr(A')}=\frac{2}{3}$ $\Pr(A')=1-3r$ $\therefore \Pr(A'\cap B')=\frac{2}{3}(1-3r)$

<table>
<tr>
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1 mark for correct answer

(2 marks if they did it this way)</td>
<td>
<table>
<tr><th>Pr</th><th>A</th><th>A'</th><th>Total</th></tr>
<tr><td>B</td><td>r</td><td></td><td></td></tr>
<tr><td>B'</td><td></td><td>⅔(1-3r)</td><td></td></tr>
<tr><td>Total</td><td>3r</td><td>1-3r</td><td>1</td></tr>
</table>
From the table it is clear that:
$\Pr(A \cap B') = 3r - r = 2r$

However, it is not necessary to have the chart. Another solution is:

$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B')$
</td>
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<tr>
<td>2(c)</td>
<td>(0, ⅓)

1 mark for completed chart

1 mark for correct domain</td>
<td>
Fill in the remainder of the chart:
<table>
<tr><th>Pr</th><th>A</th><th>A'</th><th>Total</th></tr>
<tr><td>B</td><td>r</td><td>⅓-r</td><td>⅓</td></tr>
<tr><td>B'</td><td>2r</td><td>⅔(1-3r)</td><td>⅔</td></tr>
<tr><td>Total</td><td>3r</td><td>1-3r</td><td>1</td></tr>
</table>
Since the probability of rain is never entirely zero, $r > 0$. Similarly, $1 - 3r > 0$.

Maximal domain is (0, ⅓)
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<tr>
<td>3(a)</td>
<td>$e^{ax}(a\cos 2x - 2\sin 2x)$

1 mark for use of correct formula
1 mark for correct answer</td>
<td>$u(x) = e^{ax}; v(x) = \cos 2x$
$f'(x) = uv' + vu' = e^{ax}(-2\sin 2x) + ae^{ax}\cos 2x$
$= e^{ax}(a\cos 2x - 2\sin 2x)$</td>
</tr>
<tr>
<td>3(b)</td>
<td>$\frac{2}{3} + \log_e 3$
1 mark for use of correct formula

1 mark

1 mark</td>
<td>$u(x) = \log_e(2x+3); v(x) = (\cos x - \sin x)$
$f'(x) = \frac{vu' - uv'}{v^2}$
$= \frac{2(\cos x - \sin x)/(2x+3) - (-\sin x - \cos x)\log_e(2x+3)}{(\cos x - \sin x)^2}$
$= \frac{2(\cos x - \sin x)/(2x+3) + (\sin x + \cos x)\log_e(2x+3)}{(\cos x - \sin x)^2}$
$\therefore f'(0) = \frac{\frac{2}{3} + \log_e 3}{1^2} = \frac{2}{3} + \log_e 3$</td>
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<tr><td>4</td><td>$\frac{-5}{12};\frac{-1}{3};\frac{1}{12};\frac{1}{6}$

1 mark

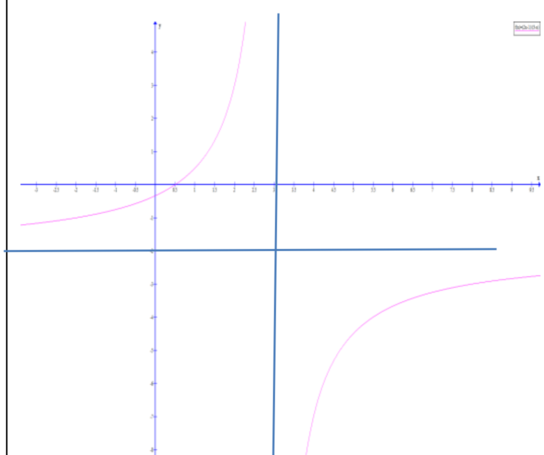
1 mark for correct domain

1 mark</td><td>$4\sin 4\pi x-2\sqrt{3}=0$
$\therefore \sin 4\pi x=\frac{\sqrt{3}}{2}$
$\therefore 4\pi x=\frac{-5\pi}{3};\frac{-4\pi}{3};\frac{\pi}{3};\frac{2\pi}{3}$
$\therefore x=\frac{-5}{12};\frac{-1}{3};\frac{1}{12};\frac{1}{6}$</td></tr>
<tr><td>5</td><td>Asymptotes at x = 3, y = -2

1 mark each

Intercepts at (0, -⅓) and (½, 0)

1 mark each

</td><td>Note that the graph appears in the 2nd and 4th quadrants rather than 1st and 3rd, because of the negative value of x in the denominator.</td></tr>
<tr><td>6(a)</td><td>$\frac{1}{3}\log_e 2$
1 mark for use of correct formula

1 mark for correct answer

($\frac{1}{6}\log_e 4$ would also be acceptable, although not as good)</td><td>$\frac{1}{0-(-2)}\int_{-2}^{0}\frac{dx}{2-3x}=\frac{1}{2}\times\frac{-1}{3}[\log_e(2-3x)]_{-2}^{0}$
$=\frac{-1}{6}[\log_e 2-\log_e 8]$
$=\frac{1}{6}[\log_e \frac{8}{2}]$
$=\frac{1}{3}\log_e 2$</td></tr>
<tr><td>6(b)</td><td>$(-\infty,\frac{1}{3}]$

1 mark for correct answer
1 mark for correct domain</td><td>$h(x)=\sqrt{1-\frac{1}{2-3x}}=\sqrt{\frac{2-3x-1}{2-3x}}=\sqrt{\frac{1-3x}{2-3x}}$
$1-3x\ge 0; 2-3x>0$</td></tr>
</table>

6(c)	$\frac{\sqrt{2}}{2}-\frac{2\sqrt{5}}{5}$ *1 mark*	
7(a)	$\frac{21}{8}$ *1 mark for correct expression* *1 mark for correct answer*	$A=\int_0^1 5x^{\frac{2}{3}}-x^{\frac{5}{3}}dx$ $=[3x^{\frac{5}{3}}-\frac{3}{8}x^{\frac{8}{3}}]_0^1=\frac{21}{8}$
7(b)	$y=\frac{1}{3}(5x+7)$ *1 mark* *1 mark* *1 mark*	$f(1)=(1)^{\frac{2}{3}}(5-1)=4$ $f'(x)=\frac{2}{3}\times 5x^{\frac{-1}{3}}-\frac{5}{3}x^{\frac{2}{3}}=\frac{5}{3}(2x^{\frac{-1}{3}}-x^{\frac{2}{3}})$ $f'(1)=\frac{5}{3}$ $y-4=\frac{5}{3}(x-1)$ $\therefore y=\frac{1}{3}(5x+7)$
8(a)	$(2, 2\sqrt{3})$ *1 mark*	A is at $x=2$, hence $f(2)=\sqrt{(6-2)(2+1)}=2\sqrt{3}$
8(b)	$6\sqrt{3}$ *1 mark*	$A=\frac{1}{2}\times 6\times 2\sqrt{3}=6\sqrt{3}$
8(c)	$\sqrt{5x+6}$ *1 mark*	$d(OP)=\sqrt{x^2+y^2}$ $=\sqrt{x^2+(6-x)(x+1)}$ $=\sqrt{x^2-x^2+5x+6}$ $=\sqrt{5x+6}$
8(d)	(2.5, 3.5) or $(\frac{5}{2},\frac{7}{2})$ *1 mark for correct derivative* *1 mark*	$f'(x)=\frac{1}{2}((6-x)(x+1))^{-0.5}\times(-2x+5)$ $f'(x)=0$ when $5-2x=0$, hence $x=2.5$. $f(\frac{5}{2})=\sqrt{\frac{49}{4}}=\frac{7}{2}$

8(e)	Domain: [0, 3.5] Range: [-1, 2.5] *1 mark* *1 mark* *1 mark for negative root*	$x=\sqrt{(6-y)(y+1)}$ $\therefore x^2=-y^2+5y+6$ $\therefore y^2-5y-6=-x^2$ $\therefore (y-\frac{5}{2})^2-(\frac{7}{2})^2=-x^2$ $\therefore (y-\frac{5}{2})^2=(\frac{7}{2})^2-x^2$ $\therefore y-\frac{5}{2}=\pm\sqrt{\frac{49}{4}-x^2}$ $\therefore y=\frac{5}{2}\pm\sqrt{\frac{49}{4}-x^2}$ When x = 0, y = -1, so we need the **<u>negative</u>** root. $\therefore y=\frac{5}{2}-\sqrt{\frac{49}{4}-x^2}$

