TRIUMPH PUBLICATIONS

Fortify Sample Exam 1B

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer booklet of 11 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \dfrac{\sin(2x)}{2 - x^2}$.

Find $\dfrac{dy}{dx}$.

2 marks

b. Differentiate $2x^3\mathrm{e}^{\frac{x}{3}}$ with respect to x.

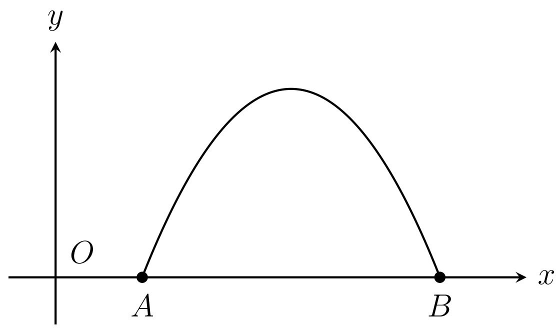
2 marks

Question 2 (3 marks)

Find m, given that $\displaystyle\int_{1}^{3} \frac{2}{x+3}\,dx = \log_{\mathrm{e}}(m)$.

Question 3 (4 marks)

A man standing at point A fires a model rocket into the air, which follows a trajectory modelled by the equation $h(x) = -2x^2 + 40x - 120$ where h is the height of the rocket above the ground and x is the horizontal distance from the man's house at O.



a. What is the maximum height that the rocket reaches?

1 mark

b. How far away from the man does the rocket land?

3 marks

Question 4 (3 marks)

Let X be a normally distributed random variable with mean 8 and variance 16. Let Z be the random variable which follows the standard normal distribution.

a. Find $\Pr(X < 8)$.

1 mark

b. Find the value of a, where $\Pr(X < 5) = \Pr(Z > a)$.

2 marks

Question 5 (5 marks)

The graphs of $y = 2\cos(x)$ and $y = \dfrac{a}{3}\sin(x)$, where a is a real constant, have a point of intersection at $x = \dfrac{\pi}{6}$.

a. Find the value of a.

2 marks

b. If $x \in \left[0, \dfrac{3\pi}{2}\right]$, find the coordinates of the two points of intersection between these graphs.

3 marks

Question 6 (3 marks)

At a particular restaurant, it is known that 70% of customers will order a drink with their meal. If three customers are chosen at random, what is the probability that:

a. all of them ordered a drink with dinner?

1 mark

b. exactly one of them ordered a drink with dinner?

2 marks

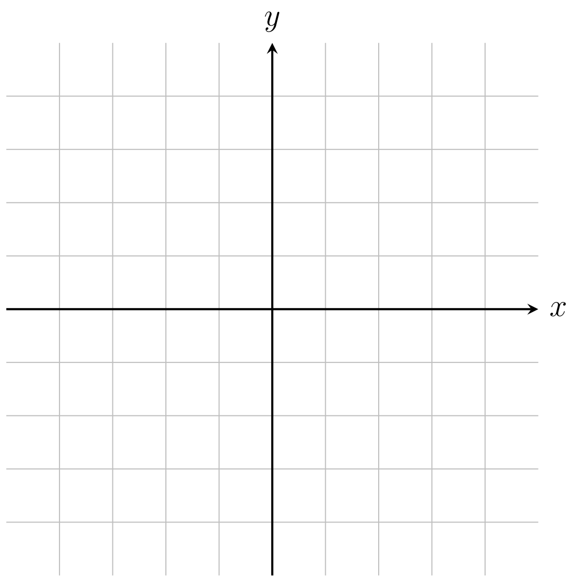
Question 7 (8 marks)

Let $g : R \to R,\ g(x) = \mathrm{e}^{3x} + 2$,

a. Find the rule and domain of the inverse function g^{-1}.

2 marks

b. On the axes below, sketch the graph of $y = g(g^{-1}(x))$. 3 marks



c. Find $g(-g^{-1}(3x))$ in the form $\dfrac{ax+b}{cx+d}$ where a, b, c and d are real constants.

3 marks

TURN OVER

Question 8 (7 marks)

Let $f : R \to R,\ f(x) = a\log_e(ax) - 2x$, where a is a positive real constant.

a. Find:

i. the x-coordinate of the stationary point of the graph of $y = f(x)$, in terms of a.

2 marks

ii. the range of values of a such that the stationary point of $y = f(x)$ lies above the x-axis.

2 marks

b. For a certain value of a, the tangent to the graph of $y = f(x)$ at $x = 2$ passes through the origin. Find this value of a.

3 marks

Question 9 (3 marks)

A continuous random variable X has a probability density function

$$f(x) = \begin{cases} e^x, & x \in [0,\ \log_e(2)] \\ 0, & \text{elsewhere} \end{cases}$$

Given that $\dfrac{d}{dx}(xe^x) = (x+1)e^x$, find $E(X)$.

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A questions and answer book is provided with this formula sheet.

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Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n \, dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n \, dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(\mathrm{e}^{ax}\right) = a\mathrm{e}^{ax}$	$\int \mathrm{e}^{ax} \, dx = \frac{1}{a}\mathrm{e}^{ax} + c$
$\frac{d}{dx}(\log_{\mathrm{e}}(x)) = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \log_{\mathrm{e}}(x) + c,\ x > 0$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) \, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) \, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$		

Formula Sheet

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\, (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\, dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{p(1-p)}{n}},\ \hat{p} + z\sqrt{\dfrac{p(1-p)}{n}}\right)$

END OF FORMULA SHEET