SOLVING TRIGONOMETRIC EQUATIONS

Step 1: Write all expressions in terms of one trigonometric function.

Step 2: Transpose the given equation so that the trigonometric expression (and the angle) is on one side of the equation, and the constants are located on the other side of the equation.

Step 3: Use the sign in front of the constant on the right hand side to determine the quadrants in which the solutions are to lie (use **CAST**).

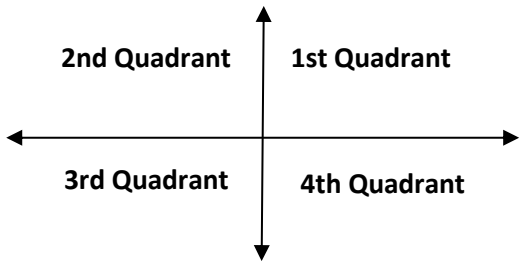
Step 4: Calculate the reference angle i.e. the first quadrant solution. If the exact value cannot be determined:

Press Inverse Sin, Cos or Tan of the number on the right hand side of the equation (but ignore the sign).

For example: $Sin^{-1}(number\ on\ RHS\ of\ equation\ but\ ignore\ the\ sign)$

(Ensure that the calculator is in Radian Mode).

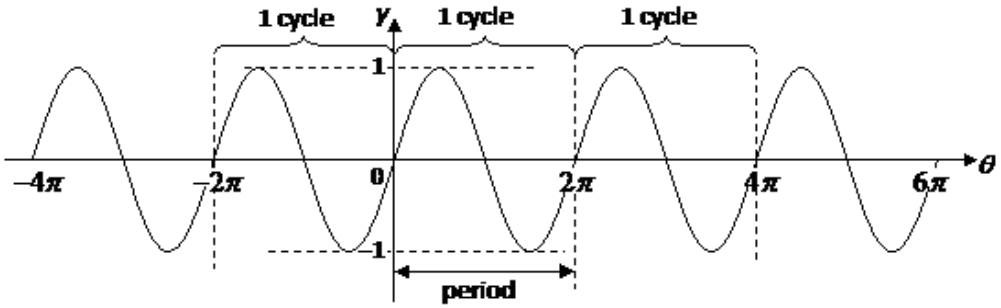
Step 5: Solve for the variable (usually x or θ). Let the angle equal the rule describing angles in the quadrants in which the solutions are to lie.



Note: First Quadrant Angle = FQA

Let angle = FQA if solution lies in 1st Quadrant.
Let angle = $\pi - FQA$ if solution lies in 2nd Quadrant.
Let angle = $\pi + FQA$ if solution lies in 3rd Quadrant.
Let angle = $2\pi - FQA$ if solution lies in 4th Quadrant.

Step 6: Evaluate all possible solutions by observing the given domain. This is done by adding or subtracting the **PERIOD** to each of the solutions, until the angles fall outside the given domain.



For sine and cosine functions: $Period = \dfrac{2\pi}{|The\ number\ in\ front\ of\ the\ variable|}$

For tangent functions: $Period = \dfrac{\pi}{|The\ number\ in\ front\ of\ the\ variable|}$

Always look closely at the brackets in the given domain and consider whether the upper and lower limits can be included in your solutions.

DO NOT discard any solution until the final step.

Step 7: Eliminate solutions that do not lie within the specified domain.

Note: Students may solve trigonometric equations by rearranging the domain.

QUESTION 1

Solve the following equation for x: $2\sin x = \sqrt{3}$, $x \in [0, 4\pi]$.

Solution

Transpose the given equation so that the trigonometric expression (and the angle) is on one side of the equation, and the constants are located on the other side of the equation:

$$\sin x = \frac{\sqrt{3}}{2}$$

Calculate the reference angle (the first quadrant solution):

$$x = Sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

Use the sign in front of the constant on the right hand side to determine the quadrants in which the solutions are to lie:

Solutions are to lie in the quadrants where sine is positive i.e. the 1st and 2nd quadrants:

S✓	A✓
T	C

Solve for the variable (usually x). Let the angle equal the rule describing angles in the quadrants in which the solutions are to lie:

$$x = \frac{\pi}{3} \quad \text{and} \quad x = \pi - 1st\ Quadrant\ Angle$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore \left\{x : x = \frac{\pi}{3}, \frac{2\pi}{3}\right\}$$

Evaluate all possible solutions by adding and/or subtracting the PERIOD to each of the calculated answers observing the given domain:

$$T = \frac{2\pi}{1} = 2\pi = \frac{6\pi}{3}$$

$$\left\{x : x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\right\}$$

SOLVING SIMPLE TRIGONOMETRIC EQUATIONS
WORKSHEET 1

QUESTION 1

Solve each equation over the interval $(-8, 8)$. State your answers correct to 4 decimal places.

(a) $\tan\theta = 4$

(b) $\cos\theta = 0.84$

(c) $\sin\theta = 0.63$

SOLUTIONS

QUESTION 1

(a)

$\tan\theta = 4$

$\theta = \tan^{-1}(4) = 1.32582$

S	A
T	C

Solutions to lie in Quadrants 1 + 3

$\theta = 1.32582, \pi + 1.32582$

$\theta = 1.32582, 4.46741$

Domain: $(-8, 8)$ $\therefore$ Add / subtract period $= \frac{\pi}{1} = \pi$

$\theta = -4.95737, -1.81577, 1.32582, 4.46741, 7.609$

$\therefore \theta = -4.9574, -1.8158, 1.3258, 4.4674, 7.6090$

(b)

$\cos\theta = 0.84$

$\cos^{-1}(0.84) = 0.573513$

S	A
T	C

Solutions to lie in Quadrants 1 + 4

$\theta = 0.573513, 2\pi - 0.573513$

$= 0.573513, 5.70967$

Domain: $(-8, 8)$ $\therefore$ Add / subtract period $= \frac{2\pi}{1} = 2\pi$

$\theta = 6.8567, 5.70967$
$0.573513, -0.573513$
$-5.70967, -6.8567$

$\therefore \theta = -6.8567, -5.7097, -0.5735, 0.5735, 5.7097, 6.8567$

(c)

$\sin\theta = 0.63$

$\sin^{-1}(0.63) = 0.681553$

S	A
T	C

Solutions to lie in Quadrants 1+2

$\theta = 0.681553, \pi - 0.681553$

$= 0.681553, 2.46004$

Domain: $(-8, 8)$ $\therefore$ Add / subtract period $= \frac{2\pi}{1} = 2\pi$

6.96474	~~8.74323~~
0.681553	2.46004
−5.6016 3	−4.12315

$\therefore \theta = -5.6016, -4.1232, 0.6816, 2.4600, 6.9647$