MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

TSSM™
Creating VCE Success

2019 Trial Examination

SOLUTIONS

Question 1 (4 marks)

a. $u = 2x - 4 \qquad u' = 2$

$v = x^2 \qquad v' = 2x$

$\frac{dy}{dx} = \frac{2x^2 - 2x(2x-4)}{(x^2)^2}$ (1M)

$\frac{dy}{dx} = \frac{2x^2 - 4x^2 + 8x}{x^4}$

$\frac{dy}{dx} = \frac{8x - 2x^2}{x^4}$

$\frac{dy}{dx} = \frac{8 - 2x}{x^3}$ (1A)

2 marks

b. $u = 1 + x^2 \qquad u' = 2x$

$v = \cos x \qquad v' = -\sin x$

$f'(x) = -(1 + x^2)\sin x + 2x\cos x$ (1M)

$f'(-\pi) = -(1 + (-\pi)^2)\sin(-\pi) + 2(-\pi)\cos(-\pi)$

$f'(-\pi) = -(1 + (-\pi)^2) \times 0 + 2(-\pi) \times -1$

$f'(-\pi) = 2\pi$ (1A)

2 marks

Question 2 (2 marks)

$f(x) = \int \frac{4}{(2-x)^2}\,dx$

$f(x) = \frac{-4}{x-2} + c$ (1M)

$5 = \frac{-4}{-2-2} + c$

$c = 4$

$f(x) = \frac{-4}{x-2} + 4$ (1A)

2 marks

Question 3 (4 marks)

a. $u = x$ $u' = 1$

$v = \cos(3x)$ $v' = -3\sin(3x)$

$\frac{dy}{dx} = \cos(3x) - 3x\sin(3x)$

1 mark

b. $\int \cos(3x) - 3x\sin(3x)\,dx = x\cos(3x) + c$ (1M)

$\int \cos(3x)\,dx - \int 3x\sin(3x)\,dx = x\cos(3x) + c$

$-3\int \sin(3x)dx = x\cos(3x) - \int \cos(3x)\,dx + c$

$\int x\sin(3x)\,dx = -\frac{1}{3}x\cos(3x) + \frac{1}{3}\int \cos(3x)\,dx + c$ (1M)

$2\int x\sin(3x)\,dx = -\frac{2}{3}x\cos(3x) + \frac{2}{3}\int \cos(3x)\,dx + c$

$\int 2x\sin(3x)\,dx = -\frac{2}{3}x\cos(3x) + \frac{2}{9}\sin(3x) + c$ (1A)

3 marks

Question 4 (5 marks)

a. Let $y = \frac{-1}{x+2} + 3$

For inverse swap x and y

$x = \frac{-1}{y+2} + 3$ (1M)

$x - 3 = \frac{-1}{y+2}$

$y + 2 = \frac{-1}{x-3}$

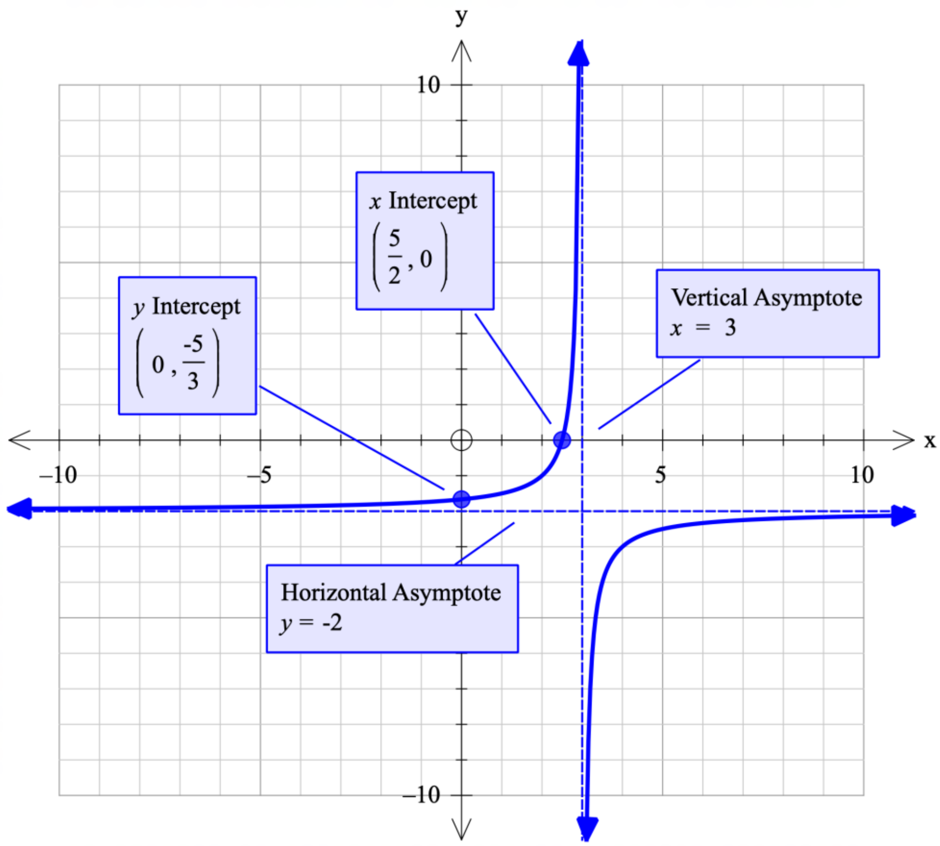
$y = \frac{-1}{x-3} - 2$

$f^{-1}(x) = \frac{-1}{x-3} - 2$

$f^{-1}: R\backslash\{3\} \to R, f^{-1}(x) = \frac{-1}{x-3} - 2$ (1A)

3 marks

b.



(½M each intercept, ½M each asymptote)
2 marks

Question 5 (4 marks)

a. $\cos(x)^2 + 2\cos(x)\tan(x) + \tan(x)^2 - (\cos(x)^2 - 2\cos(x)\tan(x) + \tan(x)^2)$ (1M)

$\cos(x)^2 - \cos(x)^2 + \tan(x)^2 - \tan(x)^2 + 2\cos(x)\tan(x) + 2\cos(x)\tan(x)$

$4\cos(x)\tan(x)$

$4\cos(x) \times \frac{\sin(x)}{\cos(x)}$ (1M)

$4\sin(x)$

2 marks

b. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (1M)

$\frac{x}{2} = \frac{\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{3}, \frac{11\pi}{3}$

For domain of $0 \le x \le 3\pi$

$x = \frac{\pi}{3}$ (1A)

2 marks

Question 6 (3 marks)

$g'(x) = 3x^2 + 8x - 3$
$0 = 3x^2 + 8x - 3$ (1M)
$0 = (x+3)(3x-1)$
$x = -3$ or $x = \frac{1}{3}$
$g(-3) = (-3)^3 + 4(-3)^2 - 3(-3) - 2 = 16, (-3, 16)$
$g\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) - 2 = -\frac{68}{27}, \left(\frac{1}{3}, -\frac{68}{27}\right)$

$g'(x)$	-4	-3	0	$\frac{1}{3}$	1
Sign	+	0	−	0	+
Slope	/	−	\	−	/

$(-3, 16)$ is a maximum (1A)
$\left(\frac{1}{3}, -\frac{68}{27}\right)$ is a minimum (1A)

3 marks

Question 7 (3 marks)
Probability Map to support question 7 (bold is given in question)

	$\boldsymbol{A}$	$\boldsymbol{A'}$	
$\boldsymbol{B}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$
$\boldsymbol{B'}$	$\boldsymbol{\frac{1}{5}}$	$\frac{2}{5}$	$\boldsymbol{\frac{3}{5}}$
	$\boldsymbol{\frac{3}{10}}$	$\frac{7}{10}$	1

a. $\Pr(A' \cap B') = \Pr(B') - \Pr(A \cap B')$
$\Pr(A' \cap B') = \frac{3}{5} - \frac{1}{5}$
$\Pr(A' \cap B') = \frac{2}{5}$

1 mark

b. $\Pr(B|A') = \frac{\Pr(B \cap A')}{\Pr(A')}$ (1M)
$\Pr(B|A') = \frac{\frac{3}{10}}{\frac{7}{10}}$
$\Pr(B|A') = \frac{3}{7}$ (1A)

2 marks

Question 8 (4 marks)

a. $\int_1^2 (ax^2 - ax)\, dx = 1$

$\left[\frac{ax^3}{3} - \frac{ax^2}{2}\right]_1^2 = 1$ (1M)

$\left[\frac{a\times 2^3}{3} - \frac{a\times 2^2}{2}\right] - \left[\frac{a\times 1^3}{3} - \frac{a\times 1^2}{2}\right] = 1$

$\frac{2a}{3} - \frac{-a}{6} = 1$

$\frac{5a}{6} = 1$ (1M)

$a = \frac{6}{5}$

2 marks

b. $\int_1^b \left(\frac{6}{5}x^2 - \frac{6}{5}x\right) dx = \frac{1}{5}$

$\left[\frac{2x^3}{5} - \frac{3x^2}{5}\right]_1^b = \frac{1}{5}$

$\left[\frac{2\times b^3}{5} - \frac{3\times b^2}{5}\right] - \left[\frac{2\times 1^3}{5} - \frac{2\times 1^2}{5}\right] = \frac{1}{5}$

$\frac{2b^3}{5} - \frac{3b^2}{5} + \frac{1}{5} = \frac{1}{5}$ (1M)

$\frac{2b^3}{5} - \frac{3b^2}{5} = 0$

$2b^3 - 3b^2 = 0$

$b^2(2b - 3) = 0$

$b = 0$ or $b = \frac{3}{2}$

As $b > 1$, then $b = \frac{3}{2}$ (1M)

2 marks

Question 9 (12 marks)

a. Gradient $= \tan\theta$, therefore $m = \tan\frac{\pi}{4} = 1$ (1M)

$\frac{f(a)-f(b)}{a-b} = 1$

To find b, let $f(x) = 0$

$0 = x^3 - 3x^2 + 4$

Using factor theorem $f(2) = 0$ and $f(-1) = 0$, therefore $b = -1$ (1A)

To find a, sub in $(-1, 0)$

$\frac{f(a)-0}{a--1} = 1$

$\frac{f(a)}{a+1} = 1$

$f(a) = a + 1$

$a^3 - 3a^2 + 4 = a + 1$

$a^3 - 3a^2 - a + 3 = 0$

Using factor theorem $f(1) = 0$, $f(-1) = 0$ and $f(3) = 0$, therefore $a = 1$ (1A)

3 marks

b. $f'(x) = 3x^2 - 6x$
$1 = 3x^2 - 6x$ (1M)
$0 = 3x^2 - 6x - 1$
$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times -1}}{2 \times 3}$
$x = \frac{6 \pm \sqrt{36+12}}{6}$
$x = \frac{6 \pm 4\sqrt{3}}{6}$
$x = \frac{3 \pm 2\sqrt{3}}{3}$ (1A)

2 marks

c. Equation of line segment $m = 1, f(1) = 2$
$y - 2 = 1(x - 1)$
$y = x + 1$ (1A)
Enclosed area
$\int_{-1}^{1}(f(x) - (x+1))dx + \int_{1}^{3}\big((x+1) - f(x)\big)dx$ (1M)
$\left[\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x\right]_{-1}^{1} + \left[-\frac{x^4}{4} + x^3 + \frac{x^2}{2} - 3x\right]_{1}^{3}$
$\left[\frac{7}{4} - \frac{-9}{4}\right] + \left[\frac{9}{4} - \frac{-7}{4}\right]$
8 square units (1A)

3 marks

d. $-2x^2 + 4 = -\frac{1}{2}$
$x^2 = \frac{9}{4}, x = \pm\frac{3}{2}$
As $x < 0, x = -\frac{3}{2}$

1 mark

e. $\int_{-\sqrt{\frac{3}{2}}}^{1}(g(x) - (x+1))dx$ (1M)
$= \frac{11}{6} - \left(-\sqrt{6} - \frac{3}{4}\right)$
$= \frac{31}{12} + \sqrt{6}$ (1M)
Difference
$= \frac{31}{12} + \sqrt{6} - 8$
$= \frac{31}{12} + \sqrt{6} - \frac{96}{12}$
$= -\frac{65}{12} + \sqrt{6}$ (1A)

marks
3 marks

