TSSM
Creating VCE Success

2019 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1 (4 marks)

a. If $y = \frac{2x-4}{x^2}$, find $\frac{dy}{dx}$. 2 marks

b. Let $f(x) = (1 + x^2)\cos x$. Evaluate $f'(-\pi)$. 2 marks

TURN OVER

Question 2 (2 marks)

The derivative with respect to x of the function $f: R\backslash\{2\} \rightarrow R$ has the rule $f'(x) = \frac{4}{(2-x)^2}$

Given that the function $f(x)$ passes through the point $(-2, 5)$, find $f(x)$ in terms of x.

Question 3 (4 marks)

Let $y = x\cos(3x)$

a. Find $\frac{dy}{dx}$. 1 mark

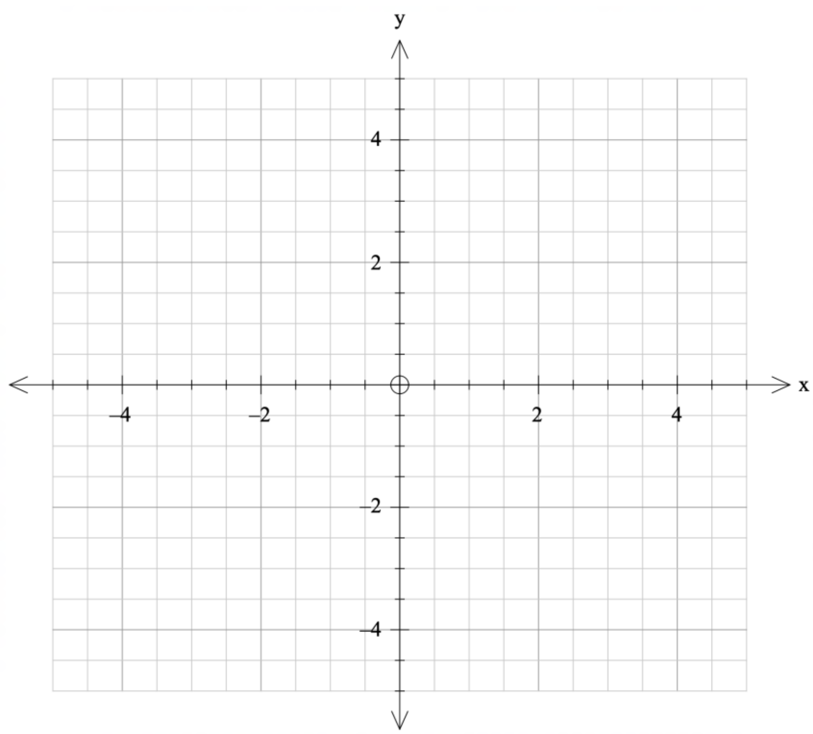
b. Hence, find $\int 2x\sin(3x)\,dx$. 3 marks

Question 4 (4 marks)

Let $f: R\backslash\{-2\} \to R, f(x) = \frac{-1}{(x+2)} + 3$

a. Define f^{-1}, the inverse function of $f(x)$ 2 marks

b. Sketch the graph of the function f^{-1} on the axes below. Label all axial intercepts and asymptotes.
2 marks



TURN OVER

Question 5 (4 marks)

a. Show that $(\cos(x) + \tan(x))^2 - (\cos(x) - \tan(x))^2 = 4\sin(x)$, 2 marks

b. Solve the equation $2\cos\left(\frac{x}{2}\right) - \sqrt{3} = 0$ for $0 \leq x \leq 3\pi$ 2 marks

Question 6 (3 marks)

Let $g(x) = x^3 + 4x^2 - 3x - 2$

Find the coordinates of the stationary points and their nature for the function $g(x)$

Question 7 (3 marks)

For the events A and B from a sample space, $\Pr(A \cap B') = \frac{1}{5}$ and $2\Pr(A) = \Pr(B') = \frac{3}{5}$, where B' denotes the complement of B. Calculate

a. $\Pr(A' \cap B')$ 1 mark

b. $\Pr(B|A')$ 2 marks

TURN OVER

Question 8 (4 marks)

The probability density function of a continuous random variable X is given by

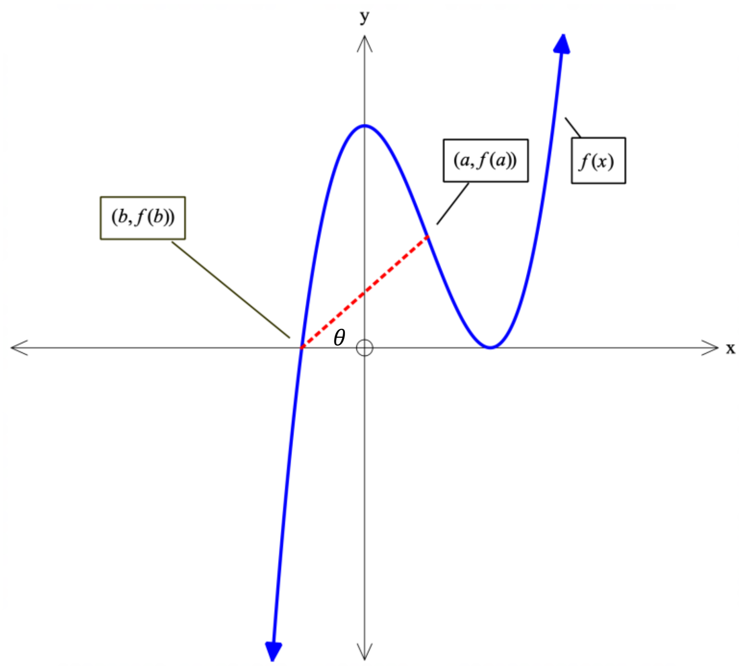
$$f(x) = \begin{cases} ax(x-1), & 1 \le x \le 2 \\ 0, & elsewhere \end{cases}$$

a. Show that $a = \frac{6}{5}$ 2 marks

b. Find the value of b such that $\Pr(X \le b) = \frac{1}{5}$ 2 marks

Question 9 (12 marks)

There is a line segment that joins the two points, a and b, on the graph of the function $f: R \to R, f(x) = x^3 - 3x^2 + 4$. This is shown in the diagram below.



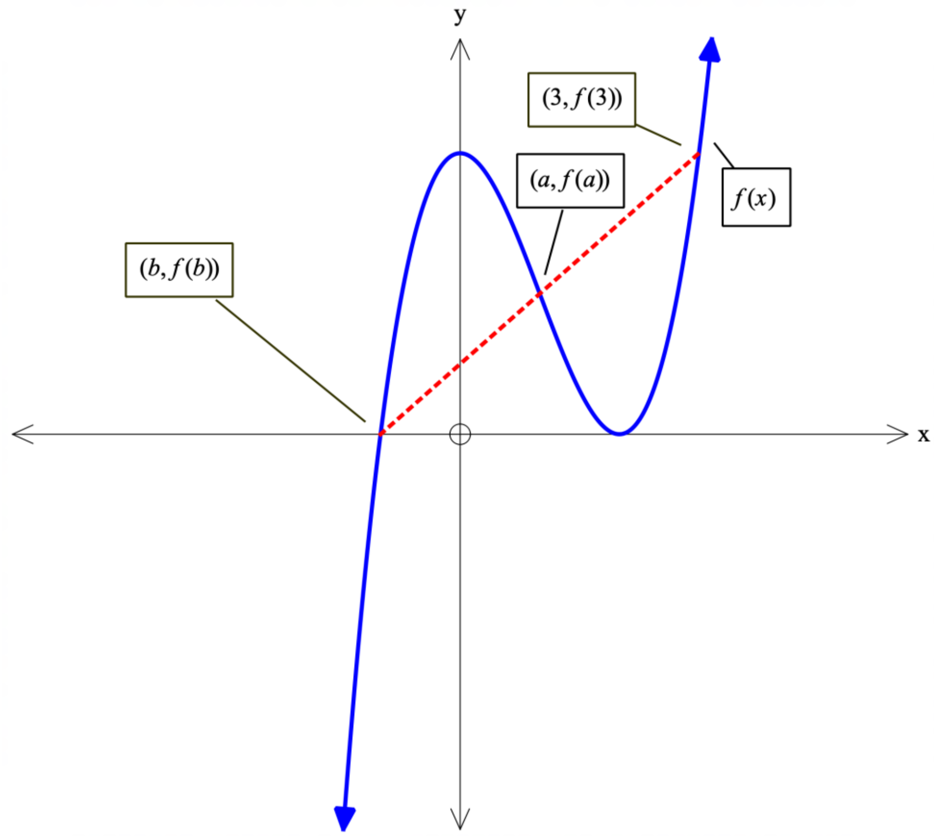
The angle θ, which is the angle that the line segment makes with the positive direction of the x-axis, is equal to $\frac{\pi}{4}$.

a. Determine the values of a and b, where $-2 < b < 0 < a < 2$ 3 marks

TURN OVER

b. Find the exact x-value of the point(s) where the gradient of the curve is equal to the gradient of the line segment 2 marks

The line segment is extended so that it touches $f(x)$ at $x = 3$, as shown in the diagram below.



c. Determine the area enclosed between the line segment and $f(x)$ 3 marks

The section of $f(x)$ between the points a and b, can be roughly modelled by the function

$$g: D \to R, g(x) = -2x^2 + 4, \text{ where } D \text{ is the domain of } g(x)$$

d. If the line segment touches $g(x)$ at $\left(x_g, -\frac{1}{2}\right)$, determine the value of x_g 1 mark

TURN OVER

e. The graph of $g(x)$ is to be used to calculate the area between the line segment and $f(x)$.
How much more or less area does this model produce? 3 marks

END OF QUESTION AND ANSWER BOOK