Victorian Certificate of Education – Free Trial Examinations

STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Free Trial Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{\sin(x)}{3x^2}$.

Find $\frac{dy}{dx}$. 2 marks

b. Let $f(x) = 4e^{\sqrt{x}}$.

Evaluate $f'(4)$. 2 marks

Question 2 (4 marks)

a. Let $g:(2,\infty)\rightarrow\mathbb{R},\ g(x)=\dfrac{x}{x-2}$.

i. Show that $\dfrac{x}{x-2}=1+\dfrac{2}{x-2}$. 1 mark

ii. Hence, find an antiderivative of $g(x)$ with respect to x. 1 mark

b. The derivative of $f:(0,\infty)\rightarrow\mathbb{R}$ with respect to x is given by the rule $f\,'(x)=\pi\cos(\pi x)+\dfrac{2}{\sqrt{x}}$.
Given that $f\left(\dfrac{1}{4}\right)=\dfrac{1}{\sqrt{2}}$, find $f(x)$ in terms of x. 2 marks

Question 3 (4 marks)

a. Find all solutions to the equation $2\sin(\pi x)-\sqrt{3}=0$ for $x\in(-2,1)$. 2 marks

b. Given that $2\log_e(a+3)-\log_e\left(b^2\right)=2$, where $a>-3$ and $b<0$, find a in terms of b. 2 marks

Question 4 (5 marks)

Lemons on a particular farm have masses which vary normally with a mean of 100 g and a standard deviation of 8 g. In this question, use the fact that $\Pr(Z > 1) = 0.16$, correct to two decimal places, where Z denotes the standard normal variable.

a. Find the probability that a randomly selected lemon has a mass of more than 108 g, correct to two decimal places. 1 mark

b. A randomly selected lemon is known to have a mass of less than 100 g.

Find the probability that this lemon has a mass of more than 92 g, correct to two decimal places. 2 marks

c. For samples of size n drawn from a large population, let $\hat{P}$ denote the random variable that represents the sample proportion of lemons from the farm that are of less than average mass.

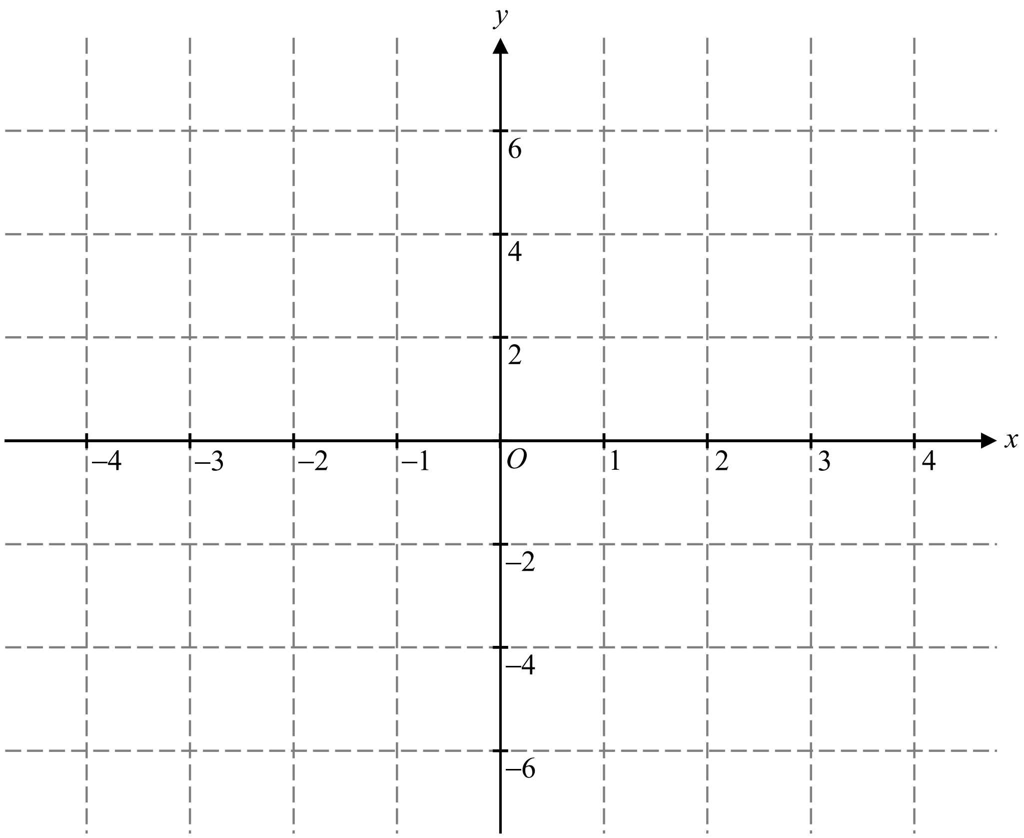
Find the smallest integer value of n such that $\text{sd}\left(\hat{P}\right) \le \frac{1}{48}$. 2 marks

Question 5 (5 marks)

Let $f:[-3,3]\to\mathbb{R},\ f(x)=\frac{1}{4}(x+2)(x-1)^2$. The rule of f can also be written as $f(x)=\frac{1}{4}\left(x^3-3x+2\right)$.

a. Find the coordinates of the stationary point(s) of f. 2 marks

b. Sketch the graph of f on the axes provided below. Label the endpoints, any stationary points and any intercepts with the coordinate axes with their respective coordinates. 2 marks



c. Using the features of the graph, state the average value of f over its domain. 1 mark

Question 6 (6 marks)

Sally has kept records of when she walks her dog for an extended period of time. Let the random variable X represent the number of times Sally walks her dog on a given day. Assume that the number of times Sally walks her dog on a given day is independent of the number of times she walks her dog on any other day.

The distribution of X is given in the table below.

x	0	1	2
$\Pr(X = x)$	0.1	m	n

a. Show that the variance of X, expressed in terms of m, is given by a quadratic expression of the form $am^2 + bm + c$, where $a, b, c \in \mathbb{R}$. Find the values of a, b and c. 3 marks

b. Hence, show that the maximum value of the variance of X occurs when $m = 0.3$. 1 mark

c. For $m = 0.3$, find the probability that Sally will walk her dog at least three times across a duration of two days. 2 marks

Question 7 (5 marks)

Let $f:[-5,\infty)\to R,\ f(x)=\sqrt{x+5}$, let $g:(-\infty,1]\to R,\ g(x)=x^2-2x$ and define $h(x)=f(g(x))$.

a. **i.** Show that the rule of h is given by $h(x)=\sqrt{(x-1)^2+4}$. 1 mark

ii. State the domain and range of h. 2 marks

b. Find the rule, domain and range of h^{-1}, the inverse function of h. 2 marks

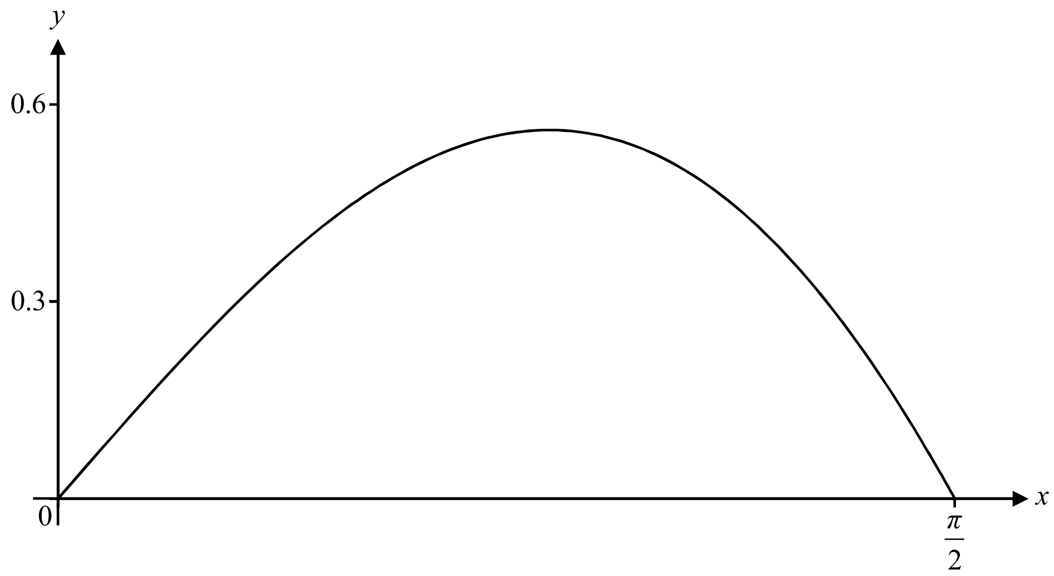
Question 8 (7 marks)

Let X be a continuous random variable with the probability density function

$$f(x)=\begin{cases} x\cos(ax) & 0 \le x \le b \\ 0 & \text{elsewhere} \end{cases},$$

where $a > 0$ and the value of b is the x-coordinate of the first positive x-axis intercept of $y = x\cos(ax)$.

The graph of $y = x\cos(x)$ for $x \in \left[0, \frac{\pi}{2}\right]$ is shown below.



a. **i.** State a sequence of simple transformations that maps the graph of $y = x\cos(x)$ onto the graph of $y = x\cos(ax)$. 2 marks

ii. Hence, or otherwise, show that $b = \frac{\pi}{2a}$. 1 mark

b. Find $\frac{d}{dx}[x\sin(ax)]$, and hence, find the value of b. Express your answer in the form $\frac{\pi}{\sqrt{m\pi - n}}$, where $m, n \in \mathbb{N}$. 4 marks

END OF QUESTION AND ANSWER BOOK