VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education 2019**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

**STUDENT NUMBER**

# MATHEMATICAL METHODS

## Written examination 1

**Wednesday 6 November 2019**

**Reading time: 9.00 am to 9.15 am (15 minutes)**
**Writing time: 9.15 am to 10.15 am (1 hour)**

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 9 | 9 | 40 |

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 13 pages
- Formula sheet
- Working space is provided throughout the book.

**Instructions**
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**At the end of the examination**
- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (4 marks)

Let $f:\left(\frac{1}{3},\infty\right)\to R,\ f(x)=\frac{1}{3x-1}$.

**a.** **i.** Find $f'(x)$. 1 mark

**ii.** Find an antiderivative of $f(x)$. 1 mark

**b.** Let $g: R\setminus\{-1\}\to R,\ g(x)=\frac{\sin(\pi x)}{x+1}$.

Evaluate $g'(1)$. 2 marks

**Question 2** (4 marks)

**a.** Let $f: R \setminus \left\{\frac{1}{3}\right\} \to R,\ f(x) = \frac{1}{3x-1}$.

Find the rule of $f^{-1}$. 2 marks

**b.** State the domain of $f^{-1}$. 1 mark

**c.** Let $g$ be the function obtained by applying the transformation $T$ to the function $f$, where

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

and $c, d \in R$.

Find the values of $c$ and $d$ given that $g = f^{-1}$. 1 mark

**Question 3** (3 marks)

The only possible outcomes when a coin is tossed are a head or a tail. When an unbiased coin is tossed, the probability of tossing a head is the same as the probability of tossing a tail.

Jo has three coins in her pocket; two are unbiased and one is biased. When the biased coin is tossed, the probability of tossing a head is $\frac{1}{3}$.

Jo randomly selects a coin from her pocket and tosses it.

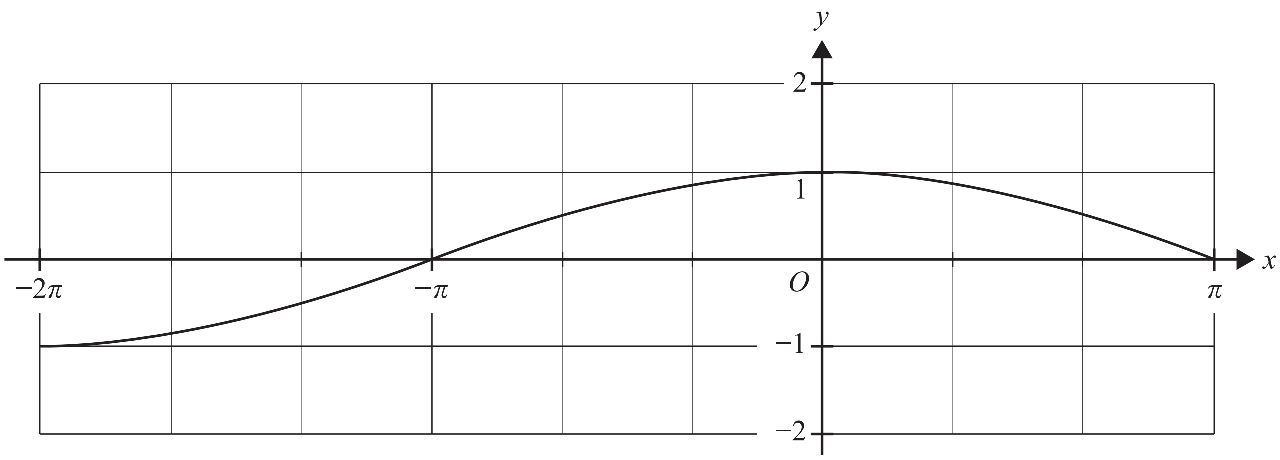
**a.** Find the probability that she tosses a head. 2 marks

**b.** Find the probability that she selected an unbiased coin, given that she tossed a head. 1 mark

**Question 4** (4 marks)

**a.** Solve $1-\cos\left(\frac{x}{2}\right)=\cos\left(\frac{x}{2}\right)$ for $x \in [-2\pi, \pi]$. 2 marks

**b.** The function $f: [-2\pi, \pi] \to R,\ f(x)=\cos\left(\frac{x}{2}\right)$ is shown on the axes below.




Let $g: [-2\pi, \pi] \to R,\ g(x) = 1 - f(x)$.

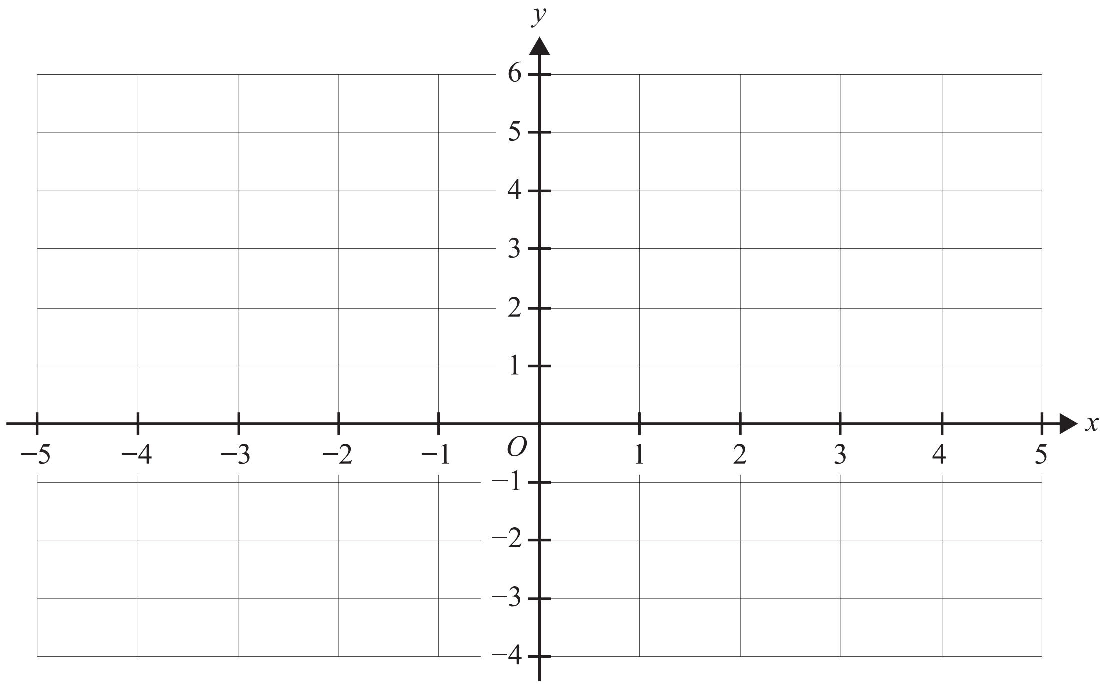
Sketch the graph of $g$ on the axes above. Label all points of intersection of the graphs of $f$ and $g$, and the endpoints of $g$, with their coordinates. 2 marks

**Question 5** (5 marks)

Let $f: R \setminus \{1\} \rightarrow R,\ f(x) = \dfrac{2}{(x-1)^2} + 1$.

**a.** **i.** Evaluate $f(-1)$. 1 mark

**ii.** Sketch the graph of $f$ on the axes below, labelling all asymptotes with their equations. 2 marks




**b.** Find the area bounded by the graph of $f$, the $x$-axis, the line $x = -1$ and the line $x = 0$. 2 marks

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**Question 6** (3 marks)

Fred owns a company that produces thousands of pegs each day. He randomly selects 41 pegs that are produced on one day and finds eight faulty pegs.

**a.** What is the proportion of faulty pegs in this sample? 1 mark

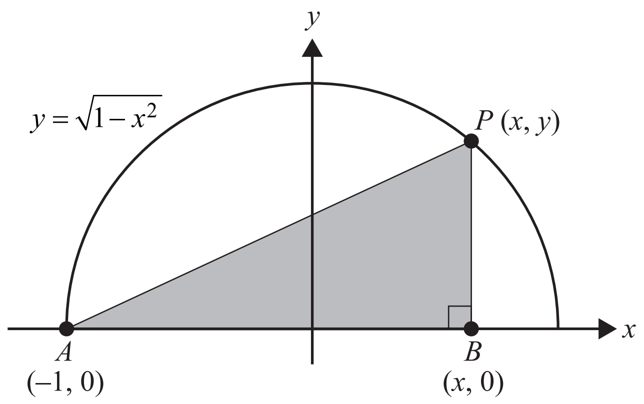
**b.** Pegs are packed each day in boxes. Each box holds 12 pegs. Let $\hat{P}$ be the random variable that represents the proportion of faulty pegs in a box.

The actual proportion of faulty pegs produced by the company each day is $\frac{1}{6}$.

Find $\Pr\left(\hat{P} < \frac{1}{6}\right)$. Express your answer in the form $a(b)^n$, where $a$ and $b$ are positive rational numbers and $n$ is a positive integer. 2 marks

**Question 7** (4 marks)

The graph of the relation $y = \sqrt{1 - x^2}$ is shown on the axes below. $P$ is a point on the graph of this relation, $A$ is the point $(-1, 0)$ and $B$ is the point $(x, 0)$.

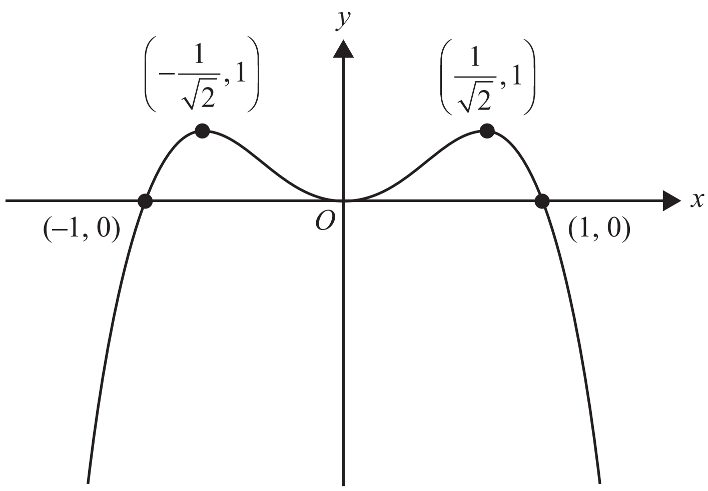



**a.** Find an expression for the length $PB$ in terms of $x$ only. 1 mark

**b.** Find the maximum area of the triangle $ABP$. 3 marks

**Question 8** (4 marks)

The function $f: R \to R$, $f(x)$ is a polynomial function of degree 4. Part of the graph of $f$ is shown below. The graph of $f$ touches the $x$-axis at the origin.




**a.** Find the rule of $f$. 1 mark

Let $g$ be a function with the same rule as $f$.

Let $h: D \to R$, $h(x) = \log_e(g(x)) - \log_e(x^3 + x^2)$, where $D$ is the maximal domain of $h$.

**b.** State $D$. 1 mark

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**Question 8** – continued

**c.** State the range of $h$. 2 marks

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**Question 9** (9 marks)

Consider the functions $f: R \to R,\ f(x) = 3 + 2x - x^2$ and $g: R \to R,\ g(x) = e^x$.

**a.** State the rule of $g\big(f(x)\big)$. 1 mark

**b.** Find the values of $x$ for which the derivative of $g\big(f(x)\big)$ is negative. 2 marks

**c.** State the rule of $f\big(g(x)\big)$. 1 mark

**d.** Solve $f\big(g(x)\big) = 0$. 2 marks

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**Question 9** – continued

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**e.** Find the coordinates of the stationary point of the graph of $f\left(g(x)\right)$. 2 marks

**f.** State the number of solutions to $g\left(f(x)\right)+f\left(g(x)\right)=0$. 1 mark

**END OF QUESTION AND ANSWER BOOK**

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AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2019**

# MATHEMATICAL METHODS

## Written examination 1

### FORMULA SHEET

**Instructions**

| |
|---|
| This formula sheet is provided for your reference.<br>A question and answer book is provided with this formula sheet. |

| |
|---|
| **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.** |

# Mathematical Methods formulas

## Mensuration

| | | | |
|---|---|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ | volume of a pyramid | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder | $2\pi rh$ | volume of a sphere | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder | $\pi r^2 h$ | area of a triangle | $\frac{1}{2}bc\sin(A)$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ | | |

## Calculus

| | |
|---|---|
| $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ | $\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$ |
| $\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$ | $\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$ |
| $\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$ | $\int e^{ax}dx=\frac{1}{a}e^{ax}+c$ |
| $\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$ | $\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$ |
| $\frac{d}{dx}\left(\sin(ax)\right)=a\ \cos(ax)$ | $\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$ |
| $\frac{d}{dx}\left(\cos(ax)\right)=-a\ \sin(ax)$ | $\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$ |
| $\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$ | |

| | | | |
|---|---|---|---|
| product rule | $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ | quotient rule | $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ |
| chain rule | $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ | | |

## Probability

| | | | |
|---|---|---|---|
| $\Pr(A) = 1 - \Pr(A')$ | | $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ | |
| $\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$ | | | |
| mean | $\mu = \mathrm{E}(X)$ | variance | $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$ |

| Probability distribution | | Mean | Variance |
|---|---|---|---|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \sum x\, p(x)$ | $\sigma^2 = \sum (x - \mu)^2\, p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x)dx$ | $\mu = \int_{-\infty}^{\infty} x\, f(x)dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$ |

## Sample proportions

| | | | |
|---|---|---|---|
| $\hat{P} = \dfrac{X}{n}$ | | mean | $\mathrm{E}(\hat{P}) = p$ |
| standard deviation | $\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$ | approximate confidence interval | $\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$ |

**END OF FORMULA SHEET**