2019 VCE Mathematical Methods 2 examination report

General comments

The 2019 Mathematical Methods 2 examination was accessible to most students. There were some excellent responses to each question in Section B. Many students found Questions 2b., 3e. and 5f. challenging.

Advice to students

- Do not cross work out unless it is replaced with another solution.
- Read questions carefully to ensure all parts of the question are answered. Question 1bii. required the values of *x* and the maximum value. Question 1d. required the minimum distance and the *m* value. Question 2b. was asking for the set of values when the gradient of the hill was strictly decreasing, not for when the hill was strictly decreasing.
- Check that answers are reasonable. For example, in Question 2d. by looking at the graph, the average gradient of the hill had to be negative; in Question 4fiii., $E\left(\hat{P}\right)$ cannot be greater than one.
- Use technology effectively. Errors could have been avoided in Question 1 if a graph of the function had been sketched. In Question 5a., the equation of the tangent could have been readily found using technology. There was no need to do this manually, as errors were often made when re-arranging the equation.
- Be careful transcribing results obtained using technology. The main transcription errors occurred in Question 1a., Question 2d. and Question 5a.
- If the question asks for a rule or an equation, as was the case in Question 1ci., Question 2c., Question 5a. and Question 5d., make sure the answer is given in the required form.
- Be familiar with mathematical conventions and language. In Question 1bi. some students did not know how to 'state the nature of the stationary point'. In Question 4, some students were confused by the wording 'live longer than two weeks' and 'lives at least two weeks'.
- Be familiar with the different types of probability distributions and their properties. In Question 4, some students thought that the life span, *X*, was a discrete random variable.
- If a maximum or minimum value is asked for, make sure the value is explicitly identified in the response. Do not just write down the coordinates of the point. This was the case in Question 1bii. and Question 3c. Likewise, if the *x* or *t* values are asked for, as in Question 3b. and Question 5c., there is no need to write down the coordinates.
- Appropriate working must be shown for questions worth more than one mark. Some students did not show their method in Question 3d., Question 4a., Question 4fii. and Question 5c.
- Provide exact answers unless otherwise stated. Approximate answers were seen in Questions 1b., 1ci., 2d., 3d., 3f., 4a., 5e. and 5g.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding, resulting in a total more or less than 100 per cent.

Section A – Multiple-choice questions

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Qu	% A	% B	% C	% D	% E	% No answer	Comments
1	3	89	4	4	1	0	
2	10	59	15	6	9	0	
3	4	9	2	5	80	0	
4	6	8	75	7	3	0	
5	3	5	90	1	1	0	
6	63	9	7	12	8	1	
7	5	4	6	82	3	0	
8	9	11	71	5	4	0	
9	9	19	8	6	57	0	
10	2	6	33	55	4	0	
11	30	23	16	12	19	1	$\Pr(A)=p$ $\Pr(B\mid A)=\frac{\Pr(A\cap B)}{\Pr(A)}=\frac{\Pr(A)\times\Pr(B)}{\Pr(A)}=\Pr(B)=m$ $\Pr(B\mid A')=\frac{\Pr(A'\cap B)}{\Pr(A')}=\frac{\Pr(A')\times\Pr(B)}{\Pr(A')}=\Pr(B)=n$ Hence $m=n$.
12	10	26	13	13	38	0	$\int_1^4 f(x)dx=4,\ \int_2^4 f(x)dx=-2$ $\int_1^2 (f(x)+x)dx=\int_1^2 f(x)dx+\int_1^2 (x)dx$ $=\int_1^2 f(x)dx+\left[\frac{x^2}{2}\right]_1^2=\int_1^2 f(x)dx+\frac{3}{2}$ $\int_1^4 f(x)dx=\int_1^2 f(x)dx+\int_2^4 f(x)dx$ $=\int_1^2 f(x)dx-2=6-2=4$ $\int_1^2 f(x)dx+\frac{3}{2}=6+\frac{3}{2}=\frac{15}{2}$

Qu	% A	% B	% C	% D	% E	% No answer	Comments
13	12	11	65	8	3	0	
14	14	67	8	7	3	1	
15	55	26	8	5	4	0	
16	63	7	7	9	14	0	
17	8	14	21	43	14	0	Let *R* represent a red marble and *G* a green marble. $\Pr(RR)+\Pr(GG)$ $=\frac{k}{n}\times\frac{k-1}{n-1}+\frac{n-k}{n}\times\frac{n-k-1}{n-1}$ $=\frac{k(k-1)+(n-k)(n-k-1)}{n(n-1)}$
18	16	17	26	27	13	1	Average value $=\frac{1}{a+b}\int_{-a}^{b}p(x)dx=\frac{3}{4}$ The area under curve = area of the triangle + area of the trapezium $=a^2+\frac{b(2a+b)}{2}$ $\frac{3}{4}(a+b)=\int_{-a}^{b}p(x)dx=a^2+\frac{b(2a+b)}{2}=1$ Solve $\frac{3}{4}(a+b)=1$ and $a^2+\frac{b(2a+b)}{2}=1$ for *a*, $a=\frac{\sqrt{2}}{3}$, $\Pr(X>0)=1-a^2=1-\left(\frac{\sqrt{2}}{3}\right)^2=\frac{7}{9}$
19	4	12	27	30	25	1	$\tan(\alpha)=d,\ d>0,\ 0<\alpha<\frac{\pi}{2}$ $\tan(2x)=d,\ 0<x<\frac{5\pi}{4}$ Tan is positive in the first and third quadrants. $2x=\alpha,\ \pi+\alpha,\ 2\pi+\alpha...$ So the solutions to $\tan(2x)=d,\ 0<x<\frac{5\pi}{4}$ are $x=\frac{\alpha}{2},\ x=\frac{\pi+\alpha}{2},\ x=\frac{2\pi+\alpha}{2}$. The sum of the solutions is $\frac{\alpha}{2}+\frac{\pi+\alpha}{2}+\frac{2\pi+\alpha}{2}=\frac{3\alpha+3\pi}{2}$.

Qu	% A	% B	% C	% D	% E	% No answer	Comments
20	10	18	16	47	8	1	$\log_x(y)+\log_y(z)$ $=\dfrac{\log_y(y)}{\log_y(x)}+\dfrac{\log_z(z)}{\log_z(y)}$ $=\dfrac{1}{\log_y(x)}+\dfrac{1}{\log_z(y)}$

Section B

Question 1a.

Marks	0	1	Average
%	6	94	**1.0**

$f: R \to R, f(x)=x^2e^{-x^2}$, $f'(x)=2xe^{-x^2}-2x^3e^{-x^2}$

Other equivalent forms are acceptable.

This question was answered well. Some students appeared to transcribe the output from technology incorrectly: $f'(x)=2x^3e^{-x^2}-2xe^{-x^2}$ and $f'(x)=2e^{-x^2}-2x^3e^{-x^2}$ were occasionally seen. Others tried to find the derivative by hand or further engaged with the output from technology and made errors.

Question 1bi.

Marks	0	1	Average
%	28	72	**0.7**

Minimum

This question was answered well. Some students did not understand what the term 'nature of the stationary point' meant. Common incorrect answers were point of inflection, stationary points and turning points. Some gave the coordinates of the turning point, (0, 0).

Question 1bii.

Marks	0	1	2	Average
%	10	22	67	**1.6**

Solve $f'(x)=0$ for *x*, $x=-1$ or $x=1$, Maximum $f(1)=\dfrac{1}{e}$

Some students included $x=0$ or only gave one answer for *x*. Others did not find the maximum value. Some gave the approximate answer for the maximum value. An exact answer was required.

Question 1biii.

Marks	0	1	Average
%	65	35	**0.4**

$d < -\frac{1}{e}$

This question was not answered well. Common incorrect answers were $d = -\frac{1}{e}$, $d \le -\frac{1}{e}$, $d > \frac{1}{e}$, $d > -\frac{1}{e}$, $d < \frac{1}{e}$ or $d < -x^2 e^{-x^2}$. Some students wrote $\left(-\frac{1}{e}, -\infty\right)$. Others did not attempt the question.

Question 1ci.

Marks	0	1	Average
%	21	79	**0.8**

$y = \frac{1}{e}$

This question was answered well. An equation was required.

Question 1cii.

Marks	0	1	2	Average
%	32	12	56	**1.3**

$$\text{Area of rectangle} - \text{integral of function} = \frac{2}{e} - \int_{-1}^{1}\left(x^2 e^{-x^2}\right)dx \text{ OR } \int_{-1}^{1}\left(\frac{1}{e} - x^2 e^{-x^2}\right)dx$$

Area = 0.3568 correct to four decimal places

Most students were able to subtract *f* from their tangent. Common incorrect methods were

$\int_{0}^{1}\left(\frac{1}{e} - x^2 e^{-x^2}\right)dx$ and $\int_{-1}^{1}\left(x^2 e^{-x^2}\right)dx$ and $\int_{-1}^{1}\left(x^2 e^{-x^2} - \frac{1}{e}\right)dx$.

Question 1d.

Marks	0	1	2	3	Average
%	46	23	7	24	**1.1**

$d = \sqrt{(0-m)^2 + (e - f(m))^2}$, Solve $d'(m) = 0$ for *m*, or, $\text{m}_{\text{Tangent}} = -(m^3 - 2m)e^{-m^2}$,

$\text{m}_{\text{Perpendicular}} = \frac{-1}{-(m^3 - 2m)e^{-m^2}} = \frac{1}{(m^3 - 2m)e^{-m^2}}$, $\text{m}_{\text{Perpendicular}} = \frac{y_2 - y_1}{x_2 - x_1}$, $\frac{1}{(m^3 - 2m)e^{-m^2}} = \frac{m^2 e^{-m^2} - e}{m - 0}$,

$m = 0.783$ correct to three decimal places, $d(0.738...) = 2.511$ correct to three decimal places

Many students were able to use the distance formula. Others found *m* but not the distance. Some gave their answers correct to two decimal places.

Question 2a.

Marks	0	1	Average
%	7	93	1.0

Let $f(x) = \frac{3x(x-30)^2}{2000}$, $f'(x) = \frac{3(-30+x)^2}{2000} + \frac{3x(-30+x)}{1000} = \frac{9(x-30)(x-10)}{2000}$

Other equivalent forms were acceptable.

This question was answered well. Common incorrect answers were $\frac{9x(x-30)(x-15)}{500}$ and $\frac{9(x^2-40x+30)}{2000}$.

Question 2b.

Marks	0	1	Average
%	97	3	0.1

$x \in (0, 20]$

This question was not done well. Most students interpreted the question as asking where the function modelling the hill was strictly decreasing, rather than the gradient of the hill and so the most common incorrect response was $[10,\ 30]$ or a combination of round and square brackets with those two values.

Question 2c.

Marks	0	1	Average
%	38	62	0.6

$h(x) = \frac{3x(x-30)^2}{2000} + 3 = \frac{3x^3}{2000} - \frac{9x^2}{100} + \frac{27x}{20} + 3$

This question was generally well done. Some students did not give an equation. Others added 10, instead of 3, to *f*.

Question 2d.

Marks	0	1	2	3	Average
%	41	13	9	37	1.4

$$\text{Average Gradient} = \frac{f(30)-f(10)}{30-10} = \frac{1}{30-10}\int_{10}^{30} h'(x)\,dx = -\frac{3}{10},\ f'(x) = \frac{9(x-30)(x-10)}{2000} = -\frac{3}{10},$$

$$x = \frac{10}{3}\left(6 \pm \sqrt{3}\right) = 20 \pm \frac{10}{\sqrt{3}} = 20 \pm \frac{10\sqrt{3}}{3} = \frac{60 \pm 10\sqrt{3}}{3}$$

A common incorrect answer for the average gradient was $\frac{3}{10}$.

Some students used $\frac{1}{30-10}\int_{10}^{30} h(x)\,dx$ instead of $\frac{1}{30-10}\int_{10}^{30} h'(x)\,dx$.

Some students gave approximate answers for the *x* values, 14.23 and 25.77.

Other students did not use brackets correctly, giving $x = \frac{\pm 10\left(\sqrt{3}+6\right)}{3}$ as their answer. Another common incorrect answer was $\frac{6 \pm 10\sqrt{3}}{3}$.

Question 2ei.

Marks	0	1	Average
%	48	52	0.5

$$\frac{9a^2}{2000} - \frac{9a}{50} + \frac{27}{20} \text{ or } \frac{h(a)-10}{a} = \frac{\frac{3a(a-30)^2}{2000} + 3 - 10}{a} = \frac{3a^2}{2000} - \frac{9a}{100} - \frac{7}{a} + \frac{27}{20}$$

Common incorrect answers were $\frac{9a^2}{2000} - \frac{9a}{50} + \frac{27}{50}$, $\frac{3}{2000}a^2 - \frac{9}{100}a - \frac{10}{a} + \frac{27}{20}$ and $\frac{3a^3 - 180a^2 + 2700a - 20000}{2000a}$.

Some students used $\frac{f(a)-10}{a}$ instead of $\frac{h(a)-10}{a}$. Other students wrote $\frac{b-10}{a}$.

The answer had to be given in terms of *a*.

Question 2eii.

Marks	0	1	2	3	Average
%	69	9	6	15	0.7

$$\frac{3a^2}{2000}-\frac{9a}{100}-\frac{7}{a}+\frac{27}{20}=\frac{9(a-30)(a-10)}{2000} \text{ or } \frac{9(a-30)(a-10)}{2000}a+10=\frac{3a(a-30)^2}{2000}+3,$$

$(11.12,\ 8.95)$ correct to two decimal places

Many students did not equate the correct expressions. Some students found the value of *a* but not the value of *b*. Other students rounded their answers incorrectly, giving (11.11, 8.94).

Question 2eiii.

Marks	0	1	Average
%	80	20	**0.2**

$$h'(11.1...)=\frac{9(a-30)(a-10)}{2000}=-0.1 \text{ correct to one decimal place}$$

Students who obtained the correct value for *a* in Question 2eii. were generally successful with this question.

Question 3a.

Marks	0	1	Average
%	24	76	**0.8**

Period = 12

This question was answered well. Common incorrect answers were 6, 18 and 12*t*.

Question 3b.

Marks	0	1	Average
%	24	76	**0.8**

$t=0,\ 4,\ 6$

This question was answered well. Some students only gave two values, either 0, 4 or 4, 6.

Question 3c.

Marks	0	1	Average
%	27	73	**0.8**

Maximum strength is 1.76 correct to two decimal places

This question was answered well. Common incorrect answers were 1.73 and 1.79.

Question 3d.

Marks	0	1	2	Average
%	32	6	62	**1.3**

$\int_0^4 f(t)dt - \int_4^6 f(t)dt = \frac{15}{\pi}$

The most common incorrect answer was $\int_0^4 f(t)dt + \int_4^6 f(t)dt = \frac{12}{\pi}$ or $\int_0^6 f(t)dt = \frac{12}{\pi}$.

Question 3e.

Marks	0	1	2	Average
%	72	18	10	**0.4**

Reflect *f* in the *y*-axis and translate 6 units left, $a = -1$ and $b = 1$, $c = 6 + 12n$, $n \in Z^+ \cup \{0\}$ and $d = 0$

Alternatively, reflect *f* in the *x*-axis and translate 6 units right, $a = 1$ and $b = -1$, $c = -6 + 12n$, $n \in Z^- \cup \{0\}$ and $d = 0$. There are other solutions.

This question was not answered well. Some students attempted to describe the transformations but gave incorrect or no values for *a*, *b*, *c* and *d*.

Question 3f.

Marks	0	1	2	Average
%	62	2	36	**0.8**

$\frac{15}{6\pi}$ OR $2 \times \frac{15}{\pi} = 12k$ OR $12k = 2\left(\int_0^4 f(t)dt - \int_4^6 f(t)dt\right)$, $k = \frac{5}{2\pi}$

Many students did not double their answer from Question 3d., giving $12k = \frac{15}{\pi}$, $k = \frac{5}{4\pi}$.

Other students had the correct method but wrote their final answer as $k = \frac{5\pi}{2}$.

Question 4a.

Marks	0	1	2	Average
%	17	4	78	**1.6**

$\int_0^5 \left(\frac{4x}{625}\left(5x^3 - x^4\right)\right)dx = \frac{10}{3}$

This question was done well. Some students worked out the median instead of the mean or evaluated $\int_0^5 \left(\frac{4}{625}\left(5x^3 - x^4\right)\right)dx$. Other students gave an approximate answer. Some students tried to treat f as a discrete random variable.

Question 4b.

Marks	0	1	2	Average
%	42	4	55	1.2

$$80\int_{2}^{5}\left(\frac{4}{625}\left(5x^3-x^4\right)\right)dx = 73 \text{ butterflies}$$

Some students found the probability but did not multiply by 80. Other students used a discrete random variable or the normal distribution. Some students evaluated $80\int_{0}^{2}\left(\frac{4}{625}\left(5x^3-x^4\right)\right)dx$ or

$80\int_{3}^{5}\left(\frac{4}{625}\left(5x^3-x^4\right)\right)dx$. Other students rounded to 74.

Question 4c.

Marks	0	1	2	Average
%	26	17	57	1.3

$\Pr\left(X \geq 4 \mid X \geq 2\right) = \dfrac{\int_{4}^{5}\left(\frac{4}{625}\left(5x^3-x^4\right)\right)dx}{\int_{2}^{5}\left(\frac{4}{625}\left(5x^3-x^4\right)\right)dx}$ = 0.2878 correct to four decimal places

Many students used conditional probability. Some students used $\Pr\left(X \leq 4 \mid X \leq 2\right)$. Other students rounded answers too early.

Question 4d.

Marks	0	1	Average
%	19	81	0.8

$X \sim N\left(14.1,\ 2.1^2\right),\ \Pr\left(16 \leq X \leq 18\right) = 0.1512$

This question was answered well. Some students rounded incorrectly, giving their answer as 0.1511 or 0.1516.

Question 4e.

Marks	0	1	Average
%	39	61	0.6

10.6 correct to one decimal place

This question was answered reasonably well. A common incorrect answer was 9.9.

Question 4fi.

Marks	0	1	Average
%	27	73	**0.8**

$X \sim \text{Bi}(36,\ 0.0527)$, $\Pr(X \geq 3) = 0.2947$

This question was answered well.

Question 4fii.

Marks	0	1	2	Average
%	58	19	23	**0.7**

$\Pr(6 \leq X \leq 36) = 0.010659....,\ \Pr(7 \leq X \leq 36) = 0.002436....,\ \text{smallest value of } n = 7$

A common incorrect answer was *n* = 6. Some students gave an answer without any working. Trial and error is an acceptable method.

Question 4fiii.

Marks	0	1	2	Average
%	45	13	42	**1.0**

$\text{E}(\hat{P}) = 0.0527$ correct to four decimal places, $\text{sd}(\hat{P}) = \sqrt{\dfrac{0.0527(1-0.0527)}{36}} = 0.0372$ correct to four decimal places

Some students found $\text{E}(X) = 36 \times 0.0527 = 1.8972$. Many students were able to find the standard deviation.

Question 4fiv.

Marks	0	1	2	Average
%	70	11	19	**0.5**

$\Pr(0.01546... < \hat{P} < 0.08993....) = \Pr(0.55659.... < X < 3.237....) = \Pr(1 \leq X \leq 3) = 0.7380$ correct to four decimal places

Many students were able to find the first interval. Some students used the normal distribution. Others rounded their answer to 0.738.

Question 4g.

Marks	0	1	2	Average
%	69	6	25	**0.6**

$\text{proportion} = 0.055,\ 0.055 + 1.96\sqrt{\dfrac{0.055 \times 0.945}{n}} = 0.0866 \text{ OR } 0.055 - 1.96\sqrt{\dfrac{0.055 \times 0.945}{n}} = 0.0234$

OR $p-1.96\sqrt{\frac{p(1-p)}{n}}=0.0234\ ....(1)$ and $p+1.96\sqrt{\frac{p(1-p)}{n}}=0.0866\ ....(2)$, $n=200$

Many students had the proportion as 0.0527 or 0.55 instead of 0.055. Others did not include the 1.96.

Question 5a.

Marks	0	1	Average
%	35	65	**0.7**

$y=-3a^2x+2a^3+1=1-a^3-3a^2(-a+x)$

This question was done reasonably well. An equation was required. Some students substituted $x=a$ into $y=-3a^2x+2a^3+1$, giving $y=1-a^3$ as the equation of the tangent. There appeared to be some transcription errors: $y=3a^2x+2a^3+1$ was sometimes seen.

Question 5b.

Marks	0	1	Average
%	37	63	**0.7**

$x=\frac{1+2a^3}{3a^2}$

This question was done reasonably well. There appeared to be some transcription errors: $x=\frac{1+2a^2}{3a^2}$ was sometimes seen.

Question 5c.

Marks	0	1	2	Average
%	28	15	57	**1.3**

$1+2a^3-3a^2x=f(x),\ x=-2a$

This question was answered well. Most students were able to equate their tangent line with $f(x)$. Some students gave the answer without showing any working. Other students unsuccessfully tried to solve $1+2a^3-3a^2x=f(x)$ by hand.

Question 5d.

Marks	0	1	2	3	Average
%	39	31	8	22	**1.2**

$A=\int_{-2a}^{1}\left(1+2a^3-3a^2x-f(x)\right)dx+\int_{1}^{\frac{1+2a^3}{3a^2}}\left(1+2a^3-3a^2x\right)dx=\frac{80a^6+8a^3-9a^2+2}{12a^2}$, or

$$A(a) = \text{Area of triangle} - \int_{-2a}^{1} f^{-1}(x)dx = \frac{1}{2}\left(\frac{2a}{3} + \frac{1}{3a^2} - -2a\right)(8a^3+1) - \int_{-2a}^{1}(1-x^3)dx$$

$$= \frac{80a^6 + 8a^3 - 9a^2 + 2}{12a^2}$$

A common incorrect definite integral was $\int_{-2a}^{\frac{1+2a^3}{3a^2}} \left(1+2a^3-3a^2x-f(x)\right)dx$ and

$\int_{-2a}^{a}\left(1+2a^3-3a^2x-f(x)\right)dx + \int_{a}^{\frac{1+2a^3}{3a^2}}\left(1+2a^3-3a^2x\right)dx$.

Question 5e.

Marks	0	1	2	Average
%	53	30	17	**0.7**

$A'(a) = 0$, minimum at $a = \frac{1}{10^{\frac{1}{3}}} = 10^{-\frac{1}{3}} = \frac{10^{\frac{2}{3}}}{10}$

Many students knew they needed to solve the derivative equal to zero but gave an incorrect minimum. Common incorrect answers were $a = -\frac{1}{2}$ and $a = \frac{1}{2}$.

Question 5f.

Marks	0	1	2	Average
%	95	1	4	**0.1**

$f\left(\frac{1}{10^{\frac{1}{3}}}\right) = f\left(\frac{10^{\frac{2}{3}}}{10}\right)$, $b = \frac{9}{10}$ or 0.9 or Area $= \int_0^{9-8b}\left(\text{tangent line} - f^{-1}(x)\right)dx$, $A'(b) = 0,\ b = 0.9$

This question was not answered well. Many students attempted the second method but were not successful.

Question 5g.

Marks	0	1	Average
%	95	5	**0.1**

$y = 3 - 3x$, $\frac{\pi}{2} - \tan^{-1}(3) = \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2} + \tan^{-1}(-3)$

This question was not answered well. Many students did not attempt this question. Some students rounded their answer to $18°$. An exact answer was required.