2020 VCAA Specialist Mathematics Exam

Written Examination 2

Wednesday 18 November 2020

Reading time: 3.00 - 3.15 am (15 minutes)
Writing time: 3.15 - 5.15 am (2 hours)

Structure of the Answer Book

Section	Number of Questions	Number of Questions to be Answered	Number of Marks
A	20	20	20
B	5	5	60
			Total 80

Section A: Answers

Question No.	1	2	3	4	5	6	7	8	9	10
Answer	D	E	C	D	E	B	C	B	E	B
Question No.	11	12	13	14	15	16	17	18	19	20
Answer	C	E	A	A	B	D	C	D	A	C

Definitions

Multiple Choices

Question 1

- $f(-1)=4, g(f(-1))=g(4))=6$
- Option D

Question 2

In[1]:=
```
MCQ2p[x_] := x^3 - 2 a x^2 + x - 1
```

In[2]:=
```
Solve[MCQ2p[-2] == 5, a]
```

Out[2]= $\{\{a \to -2\}\}$

- Option E

Question 3

In[4]:=
```
TraditionalForm[DSolve[f'[x] == 2/Sqrt[2 x - 3] && f[6] == 4, f[x], x]]
```

Out[4]//TraditionalForm=

$$\left\{\left\{f(x) \to 2\left(\sqrt{2x-3}-1\right)\right\}\right\}$$

- Option C

Question 4

In[6]:=
```
Expand[Solve[2 Cos[2 x - π/3] + 1 == 0, x]]
```

Out[6]= $\left\{\left\{x \to \text{ConditionalExpression}\left[-\frac{\pi}{6}+\pi c_1,\ c_1 \in \mathbb{Z}\right]\right\},\right.$
$\left.\left\{x \to \text{ConditionalExpression}\left[\frac{\pi}{2}+\pi c_1,\ c_1 \in \mathbb{Z}\right]\right\}\right\}$

- Option D

Question 5

In[9]:=
```
FullSimplify[(3 x + 2)/(5 - x)]
```

Out[9]= $-3-\frac{17}{-5+x}$

- x = 5, y = -3
- Option E

Question 6

- Two x-intercepts for $y = f'(x)$ where $x < 0$
- Hence two turning points for $y = f(x)$ where $x < 0$ for the x-coordinate of both turning points
- Hence, option B.

Question 7

- Recall that for $f(x) = e^{g(x)}$, $f'(x) = g'(x)\, e^{g(x)}$
- Recall that for $h(x) = g(k(x))$, $h'(x) = k'(x) \times g'(k(x))$
- Hence, for $f(x) = e^{g(x^2)}$, $f'(x) = 2\,x^2\, g'(x^2)\, e^{g(x^2)}$
- Hence, option C

Question 8

- Recall that $\mu = n \times p$, $n = 25$, $\mu = 1.4$
- $p = \frac{1.4}{25} = 0.056$

In[11]:=
```
1.4
---
25
```
Out[11]= 0.056

In[12]:=
```
Probability[x > 3, x ≈ BinomialDistribution[25, 0.056]]
```
Out[12]= 0.0484965

- Hence, option B

Question 9

- $\int_0^2 f(2\,(x+2))\,dx = \int_2^4 f(2\,x)\,dx$
- $\Leftrightarrow \int_0^2 f(2\,(x+2))\,dx = \frac{1}{2}\int_4^8 f(x)\,dx$
- $\Leftrightarrow \int_0^2 f(2\,(x+2))\,dx = \frac{5}{2}$
- Hence option E

Question 10

- Option B
- Since $\log_2\left(\left(2^k - 1\right) + 1\right) = \log_2\left(2^k\right) = k$

Question 11

In[13]:=
```
1 - Probability[x > 1.5, x ≈ NormalDistribution[0, 1]]
```
Out[13]= 0.933193

In[15]:=
```
Solve[
 Probability[x < 259, x ≈ NormalDistribution[250, σ]] == 0.9331927987311419`, σ]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[15]= {{σ → 6.}}

- Option C

Question 12

- Option E

Question 13

- Dilation by a factor of $\frac{1}{2}$ from the y-axis. AND Translation of 4 units left.
- Hence, due to matrices, option A

Question 14

In[16]:=
```
Solve[Probability[x > 5.2, x ≈ NormalDistribution[2 σ, σ]] == 0.9, σ]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[16]= {{σ → 7.23782}}

- Option A

Question 15

- Finding the x-intercepts:
  - Left graph: $y = -\frac{3}{2}x - a$, x-intercept: $\left(-\frac{2}{3}a, 0\right)$
  - Right graph: $y = 2x - a$, x-intercept: $\left(\frac{1}{2}a, 0\right)$
- Right graph can be cancelled as Area between $x = 0$ and $x = a$ is zero.
- Average value

In[22]:=
$$\frac{1}{3\,a} \times \left(\frac{1}{2} * 2\,a * \left(2 - \frac{2}{3}\right) a - \frac{1}{2} * a * \frac{2}{3}\,a\right)$$

Out[22]=
$$\frac{a}{3}$$

- Option B

Question 16

- Formulating the equation: $A(m) = \frac{1}{2}m(9 - m^2)$

In[23]:=
```
D[1/2 m (9 - m^2), m]
```

Out[23]= $-m^2+\frac{1}{2}\left(9-m^2\right)$

In[24]:=
```
Solve[-m^2 + 1/2 (9 - m^2) == 0, m]
```

Out[24]= $\left\{\left\{m\to-\sqrt{3}\right\},\left\{m\to\sqrt{3}\right\}\right\}$

In[27]:=
```
1/2 m (9 - m^2) /. m -> Sqrt[3]
```

Out[27]= $3\sqrt{3}$

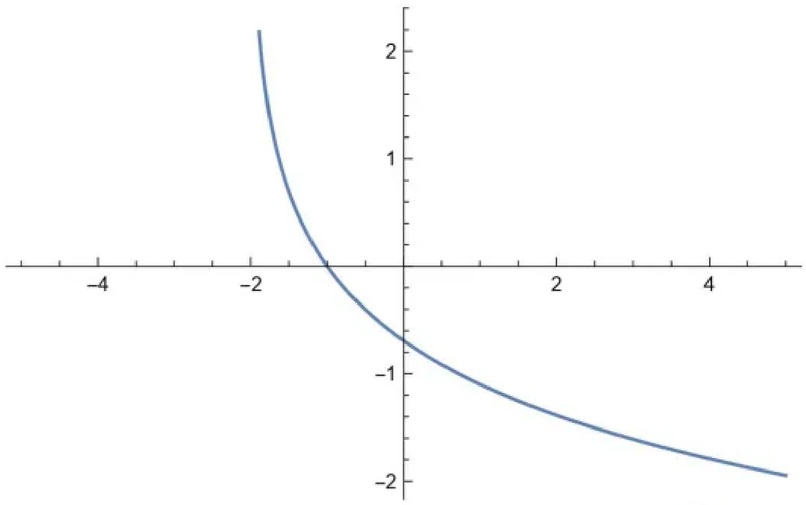
- Option D

Question 17

- Maximum occurs at $x=0$

In[29]:=
```
Plot[-Log[x + 2], {x, -5, 5}]
```

Out[29]=



In[28]:=
```
D[-Log[x + 2], x]
```

Out[28]= $-\frac{1}{2+x}$

In[34]:=
```
-1/(2 + x) /. x -> 0
```

Out[34]= $-\frac{1}{2}$

In[36]:=
```
Solve[-1/2 (0 - k) == -Log[2], k]
```

Out[36]= $\{\{k\to-2\,\mathrm{Log}[2]\}\}$

- Formulating the tangent : $y=\frac{-1}{2}\left(x+2\log_e(2)\right)$

Question 20

- Range of $f(x) \in \left[\frac{1}{2}, 1\right]$
- Given that $f(x) = \cos(a\,x)$, possible values of $a\,x \in \left[0 + 2\,k\,\pi, \frac{\pi}{3} + 2\,k\,\pi\right] \cup \left[\frac{5\,\pi}{3} + 2\,k\,\pi, 2\,\pi + 2\,k\,\pi\right]$
- Only option C satisfies, when $a = 1$.

Question 1

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

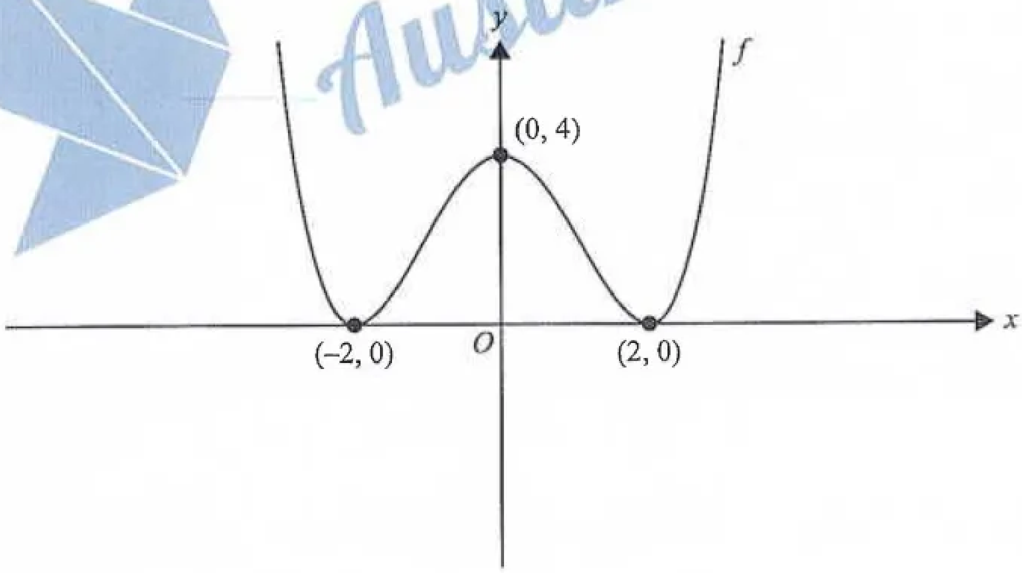
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

Let $f: R \to R, f(x) = a(x+2)^2(x-2)^2$, where $a \in R$. Part of the graph of f is shown below.



a. Show that $a = \frac{1}{4}$. 1 mark

As $(0,4)$ lies on $y=f(x)$

$f(0)=4 \Rightarrow a(2)^2(-2)^4 = 4$

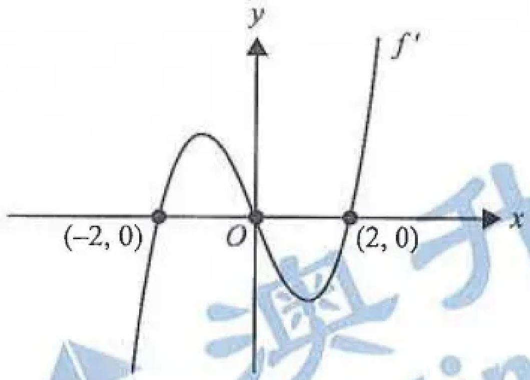
$a \times 16 = 4$

$a = \frac{4}{16} = \frac{1}{4}$

b. Express $f(x) = \frac{1}{4}(x+2)^2(x-2)^2$ in the form $f(x) = \frac{1}{4}x^4 + bx^2 + c$, where b and c are integers. 1 mark

$f(x) = \frac{1}{4}x^4 - 2x^2 + 4$

Part of the graph of the derivative function f' is shown below.



c. i. Write the rule for f' in terms of x. 1 mark

$f'(x) = x(x-2)(x+2)$ $\left(f'(x) = x^3 - 4x \right)$

ii. Find the minimum value of the graph of f' on the interval $x \in (0, 2)$. 2 marks

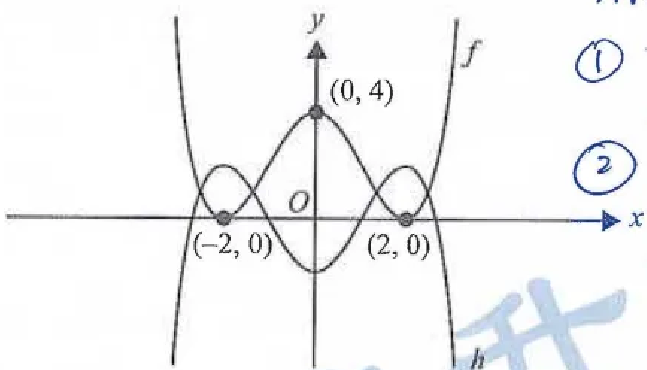
$f''(x) = 0$ (or let $g(x) = f'(x)$, $g'(x) = 0$)

$3x^2 - 4 = 0 \Rightarrow x = \pm\frac{2\sqrt{3}}{3}$

but $x \in (0, 2)$, $\therefore x = \frac{2\sqrt{3}}{3}$

$\therefore f'_{min}\left(\frac{2\sqrt{3}}{3}\right) = \frac{-16\sqrt{3}}{9}$

Let $h : R \to R$, $h(x) = -\frac{1}{4}(x+2)^2(x-2)^2 + 2$. Parts of the graphs of f and h are shown below.



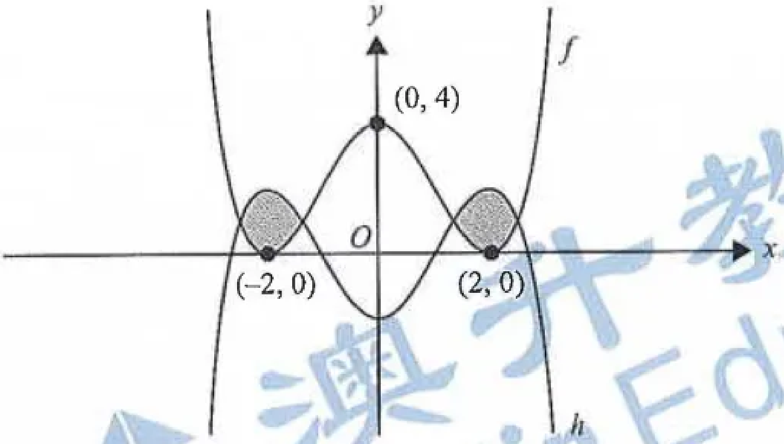
Altenatively (in that order)

① Translation by 2 units down (in negative y-direction)

② Reflection about x axis

d. Write a sequence of two transformations that map the graph of f onto the graph of h. 1 mark

- ① Reflection about x axis
- ② Translation by 2 units upwards (in the positive y-direction)



e. i. State the values of x for which the graphs of f and h intersect. 1 mark

$x = \pm\sqrt{6}$, $x = \pm\sqrt{2}$

ii. Write down a definite integral that will give the total area of the shaded regions in the graph above. 1 mark

$2\int_{-\sqrt{6}}^{-\sqrt{2}} (h(x) - f(x))\,dx$ OR $2\int_{\sqrt{2}}^{\sqrt{6}} (h(x) - f(x))\,dx$

iii. Find the total area of the shaded regions in the graph above. Give your answer correct to two decimal places. 1 mark

Area $= 2 \times 1.360547\ldots \approx 2.72$ units^2

f. Let D be the vertical distance between the graphs of f and h. $\longrightarrow$ Abs Value ?? Seriously?

Find all values of x for which D is at most 2 units. Give your answers correct to two decimal places. 2 marks

Let $D(x) = |f(x) - h(x)|$

$D(x) \leq 2$, solving for x gives

$-2.61 \leq x \leq -1.08$ or $1.08 \leq x \leq 2.61$

Question 2 (11 marks)

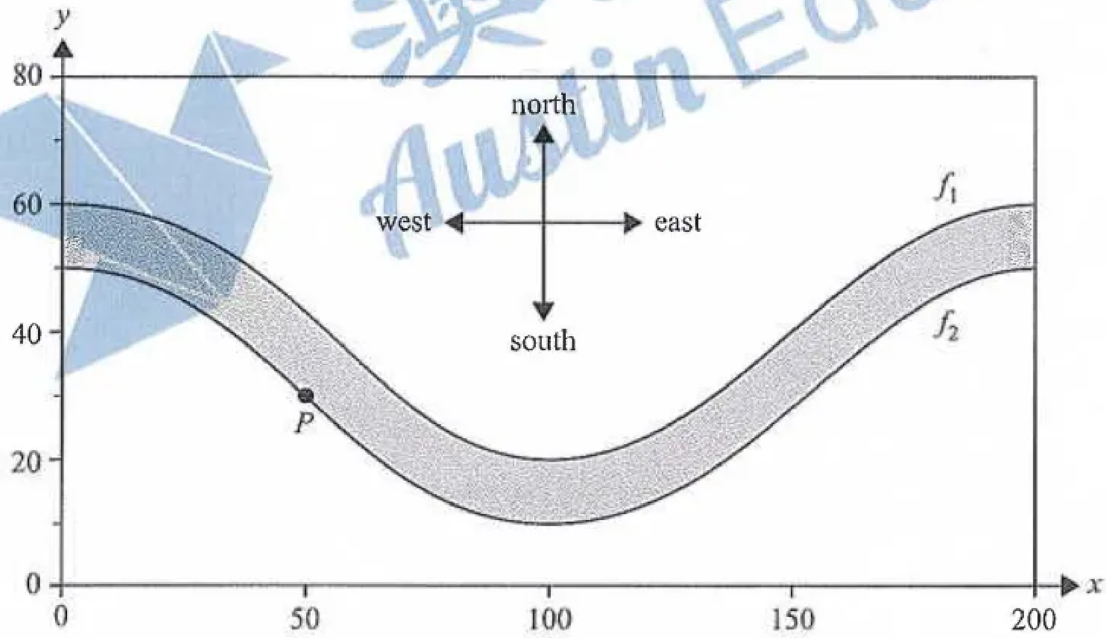
An area of parkland has a river running through it, as shown below. The river is shown shaded.

The north bank of the river is modelled by the function $f_1 : [0, 200] \rightarrow R,\ f_1(x) = 20\cos\left(\frac{\pi x}{100}\right) + 40$.

The south bank of the river is modelled by the function $f_2 : [0, 200] \rightarrow R,\ f_2(x) = 20\cos\left(\frac{\pi x}{100}\right) + 30$.

The horizontal axis points east and the vertical axis points north.

All distances are measured in metres.



A swimmer always starts at point P, which has coordinates (50, 30).

Assume that no movement of water in the river affects the motion or path of the swimmer, which is always a straight line.

a. The swimmer swims north from point P.

Find the distance, in metres, that the swimmer needs to swim to get to the north bank of the river. 1 mark

Distance $= f_1(50) - f_2(50)$

$= 10$ metres

b. The swimmer swims east from point P.

Find the distance, in metres, that the swimmer needs to swim to get to the north bank of the river. 2 marks

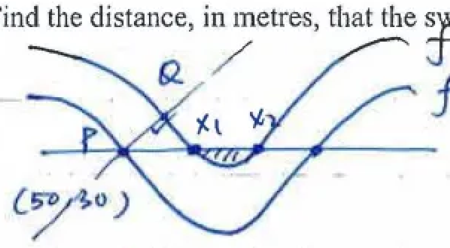


figure 2.1

Solve $f_1(x) = 30$ for x over $50 \le x \le 150$

$\Rightarrow x_1 = \frac{200}{3}$ $x_2 = \frac{400}{3}$

$\Rightarrow$ distance $= \frac{200}{3} - 50 = \frac{50}{3}$ metres

$\boxed{\therefore \frac{50}{3} \text{ metres}}$

c. On another occasion, the swimmer swims the minimum distance from point P to the north bank of the river.

Find this minimum distance. Give your answer in metres, correct to one decimal place. 2 marks

Find the line perpendicular to $y = f_2(x)$ at P: $x = 50$

$y = \frac{5x}{\pi} + \frac{10(3\pi - 25)}{\pi}$

Solve for x & y $\begin{cases} y = \frac{5x}{\pi} + \frac{10(3\pi-25)}{\pi} \\ y = f_1(x) \end{cases}$ $\Rightarrow$ $x_Q = 54.509036\ldots$ $y_Q = 37.17635\ldots$

(refer to part b. 2.1)

Min distance $= \sqrt{(54.5090\ldots - 50)^2 + (37.17635\ldots - 30)^2}$

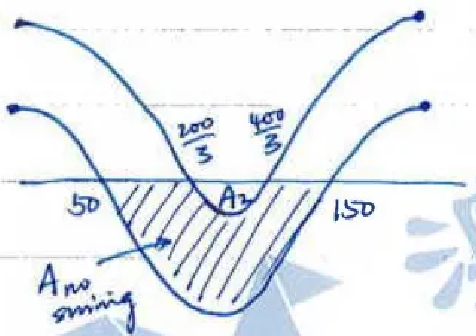
$= 8.5$ metres

d. Calculate the surface area of the section of the river shown on the graph on page 16, in square metres. 1 mark

Area $= \int_0^{200} (f_1(x) - f_2(x))\, dx = 2000 [m^2]$

e. A horizontal line is drawn through point P. The section of the river that is south of the line is declared a 'no swimming' zone.

Find the area of the 'no swimming' zone, correct to the nearest square metre. 3 marks



$A_{rea\ no\ swiming}$ =  − 

$= \int_{50}^{150} (30 - f_2(x))\, dx - \int_{\frac{200}{3}}^{\frac{400}{3}} (30 - f_1(x))\, dx$

$= 837.2484\ldots\ m^2$

$\approx 837\ m^2$

f. Scientists observe that the north bank of the river is changing over time. It is moving further north from its current position. They model its predicted new location using the function with rule $y = kf_1(x)$, where $k \geq 1$.

Find the values of k for which the distance **north** across the river, for all parts of the river, is strictly less than 20 m. 2 marks

let $D_k(x) = kf_1(x) - f_2(x)$

$D_k(x) = \underbrace{(20k-20)}_{A}\cos\left(\frac{\pi x}{100}\right) + \underbrace{(40k-30)}_{C}$

For $k \geq 1$, $A \geq 0$. $40k-30 \geq 10$

Now: Use Range properties:

$D_k \in [C-A, C+A]$

$= [20k-10, \underbrace{60k-50}_{Max}]$

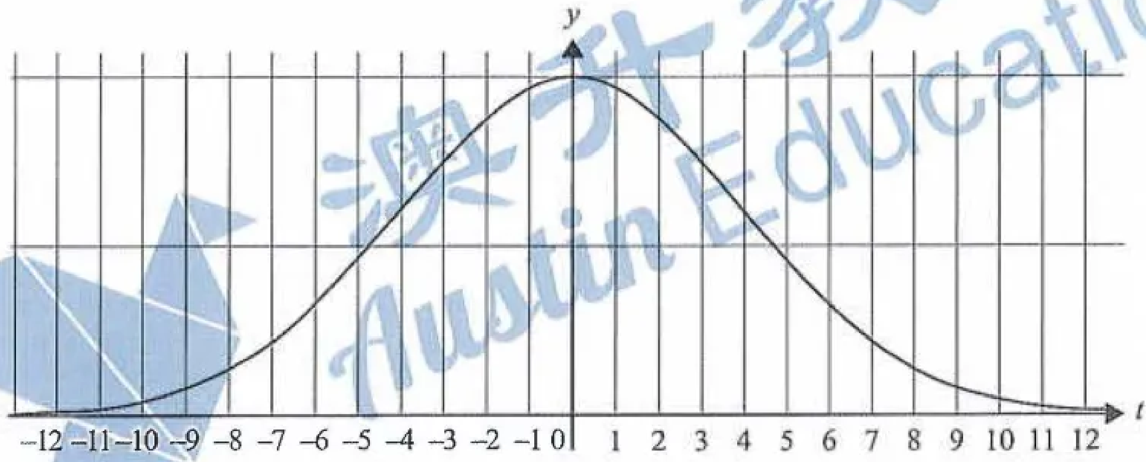
$60k - 50 < 20$

$k < \frac{70}{60}$

$\boxed{1 \leq k < \frac{7}{6}}$

Question 3 (12 marks)

A transport company has detailed records of all its deliveries. The number of minutes a delivery is made before or after its scheduled delivery time can be modelled as a normally distributed random variable, T, with a mean of zero and a standard deviation of four minutes. A graph of the probability distribution of T is shown below.



a. If $\Pr(T \le a) = 0.6$, find a to the nearest minute. 1 mark

$T \sim N(\mu = 0, \sigma^2 = 4^2)$

Using inv. norm on CAS

area $= 0.60$, $\mu = 0$, $\sigma = 4$

$a = 1.013388 \Rightarrow a \approx 1$ minute.

b. Find the probability, correct to three decimal places, of a delivery being no later than three minutes after its scheduled delivery time, given that it arrives after its scheduled delivery time. 2 marks

$\Pr(T < 3 \mid T > 0)$

$= \dfrac{\Pr(0 < T < 3)}{\Pr(T > 0)} \left(= \dfrac{0.27337272\ldots}{0.5} \right)$

$= 0.54674544\ldots \approx 0.547$

c. Using the model described on page 19, the transport company can make 46.48% of its deliveries over the interval $-3 \le t \le 2$.

It has an improved delivery model with a mean of k and a standard deviation of four minutes.

Find the values of k, correct to one decimal place, so that 46.48% of the transport company's deliveries can be made over the interval $-4.5 \le t \le 0.5$ 3 marks

Using integral eqn: $\int_{-4.5}^{0.5} \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-k}{4}\right)^2} dt = 0.46383518\ldots$

Note: $T \sim N(0, 4^2)$

$\Pr(-3 \le T \le 2)$

$= 0.46383518$

more accuracy

Solving for k numerically

$k = -2.5$ or -1.5

A rival transport company claims that there is a 0.85 probability that each delivery it makes will arrive on time or earlier.

Assume that whether each delivery is on time or earlier is independent of other deliveries.

d. Assuming that the rival company's claim is true, find the probability that on a day in which the rival company makes eight deliveries, fewer than half of them arrive on time or earlier. Give your answer correct to three decimal places. 2 marks

$X \sim Bi(n=8, p=0.85)$

$\Pr(X<4) = \Pr(0 \le X \le 3)$

$= 0.00285387\ldots \approx \underline{0.003}$

e. Assuming that the rival company's claim is true, consider a day in which it makes n deliveries.

i. Express, in terms of n, the probability that one or more deliveries will **not** arrive on time or earlier. 1 mark

$X_n \sim Bi(n, 0.15)$

$\Pr(X_n \ge 1) = 1 - \Pr(X_n = 0)$

$= 1 - {}^nC_0 (0.15)^0 (0.85)^n$

$= 1 - (0.85)^n$

$\left(1 - \left(\frac{17}{20}\right)^n \text{ ok}\right)$

Do we accept $\sum_{x=1}^{n} {}^nC_x (0.15)^x (0.85)^{n-x}$?

ii. Hence, or otherwise, find the minimum value of n such that there is at least a 0.95 probability that one or more deliveries will **not** arrive on time or earlier. 1 mark

$1 - (0.85)^n \ge 0.95 \Rightarrow n \ge 18.43\ldots$

$\boxed{\text{least no. of } n = 19}$

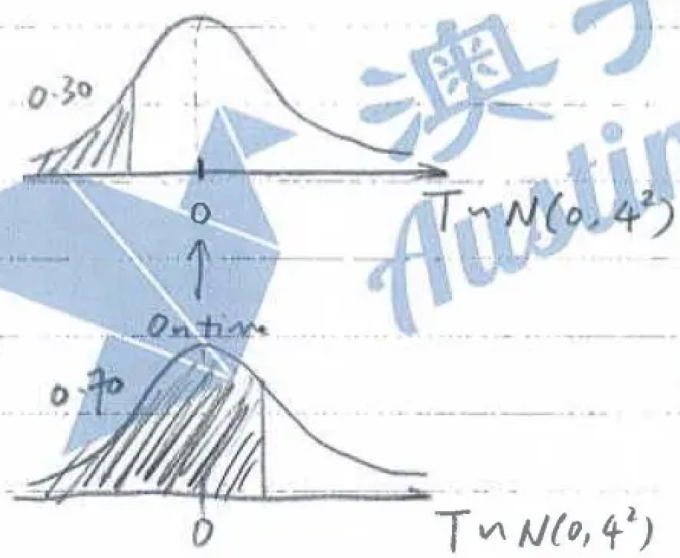
WTH is this? Too much wording!

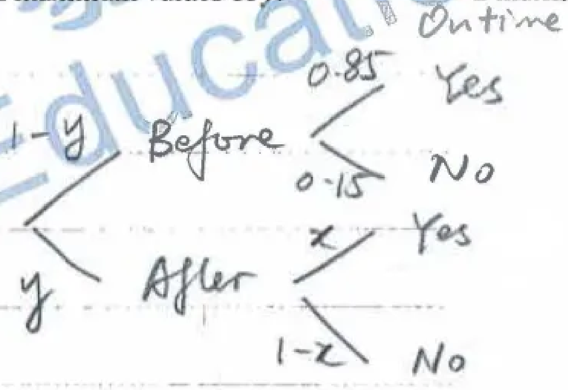
f. An analyst from a government department believes the rival transport company's claim is only true for deliveries made before 4 pm. For deliveries made after 4 pm, the analyst believes the probability of a delivery arriving on time or earlier is x, where $0.3 \le x \le 0.7$

After observing a large number of the rival transport company's deliveries, the analyst believes that the overall probability that a delivery arrives on time or earlier is actually 0.75

Let the probability that a delivery is made after 4 pm be y.

Assuming that the analyst's beliefs are true, find the minimum and maximum values of y. 2 marks





$(1-y) \cdot 0.85 + y \cdot x = 0.75$

$\Rightarrow y = \dfrac{2}{17-20x}$

Consider domain:

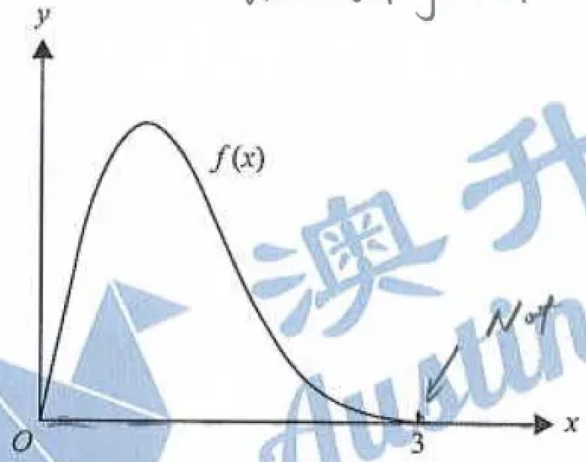
$0.3 \le x \le 0.7$

$\begin{cases} y_{min} = \dfrac{2}{11} \\ y_{max} = \dfrac{2}{3} \end{cases}$ i.e: $\dfrac{2}{11} \le y \le \dfrac{2}{3}$

Question 4 (13 marks)

The graph of the function $f(x) = 2xe^{(1-x^2)}$, where $0 \le x \le 3$, is shown below.

Wavelet function, ok..



a. Find the slope of the tangent to f at $x = 1$. 1 mark

$f'(1) = -2$

b. Find the obtuse angle that the tangent to f at $x = 1$ makes with the positive direction of the horizontal axis. Give your answer correct to the nearest degree. 1 mark

$\tan(\theta) = -2 \Rightarrow \theta = 116.5650..^\circ$

$\theta = 117^\circ$

c. Find the slope of the tangent to f at a point $x = p$. Give your answer in terms of p. 1 mark

$f'(p) = (2e - 4p^2 \cdot e)e^{-p^2}$

d. **i.** Find the value of p for which the tangent to f at $x = 1$ and the tangent to f at $x = p$ are perpendicular to each other. Give your answer correct to three decimal places. 2 marks

$$\left.\begin{array}{l} m_1 = -2 \\ m_2 = f'(p) \\ m_1 \times m_2 = -1 \end{array}\right\} \Rightarrow -2f'(p) = -1$$

$$f'(p) = \frac{1}{2}, 0 \le p \le 3$$

$$\Rightarrow p = 0.6552517... \approx \underline{0.655}$$

ii. Hence, find the coordinates of the point where the tangents to the graph of f at $x = 1$ and $x = p$ intersect when they are perpendicular. Give your answer correct to two decimal places. 3 marks

Tangent at $x = p \approx$ is

$y = \frac{1}{2}x + 1.991189...$ ... ①

Tangent at $x = 1$ is

$y = 4 - 2x$ ②

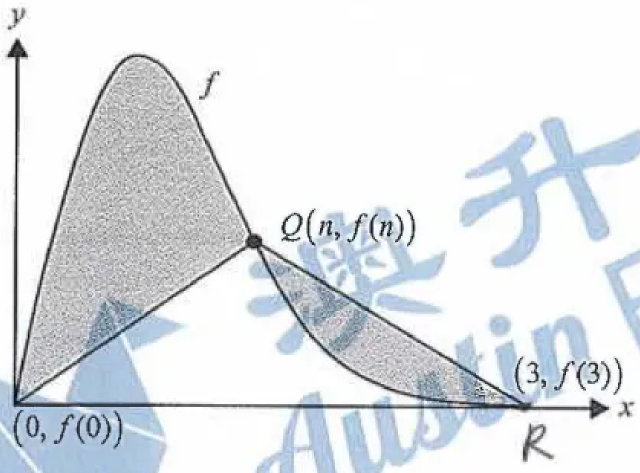
Solving ① & ② simultaneously

Coordinates of intersection is

$\Rightarrow (0.8035..., 2.3929...)$

$\therefore (0.80, 2.39)$

Two line segments connect the points $(0, f(0))$ and $(3, f(3))$ to a single point $Q(n, f(n))$, where $1 < n < 3$, as shown in the graph below.



e. **i.** The first line segment connects the point $(0, f(0))$ and the point $Q(n, f(n))$, where $1 < n < 3$.

Find the equation of this line segment in terms of n. 1 mark

$m_{OQ} = \dfrac{f(n)}{n} = 2e^{1-n^2}$

$\therefore$ Equation is $y_{OQ} = \left(2e^{1-n^2}\right) \cdot x$

ii. The second line segment connects the point $Q(n, f(n))$ and the point $(3, f(3))$, where $1 < n < 3$.

Find the equation of this line segment in terms of n. 1 mark

$$\left.\begin{array}{l} m_{QR} = \dfrac{f(3) - f(n)}{3 - n} \\ y - f(n) = m_{QR}(x - n) \end{array}\right\} \Rightarrow y_{QR} = \left(\frac{2ne}{(n-3)e^{n^2}} - \frac{6}{(n-3)e^8}\right) \cdot x + \left(\frac{6n}{(n-3)e^8} - \frac{6ne}{(n-3)e^{n^2}}\right)$$

iii. Find the value of n, where $1 < n < 3$, if there are equal areas between the function f and each line segment. Give your answer correct to three decimal places. 3 marks

$$\int_0^n \left(f(x) - y_{OQ}(x)\right) dx = \int_n^3 \left(y_{QR}(x) - f(x)\right) dx$$

solving for n where $1 < n < 3$: $\underline{\underline{n = 1.088}}$

Question 5 (13 marks)

Let $f : R \to R, f(x) = x^3 - x$.

Let $g_a : R \to R$ be the function representing the tangent to the graph of f at $x = a$, where $a \in R$.

Let $(b, 0)$ be the x-intercept of the graph of g_a.

a. Show that $b = \dfrac{2a^3}{3a^2 - 1}$. 3 marks

$f'(x) = 3x^2 - 1 \Rightarrow f'(a) = 3a^2 - 1$

$f(a) = a^3 - a$

$y - (f(a)) = f'(a)(x - a)$ for eqn of tangent at $x = a$

$y - (a^3 - a) = (3a^2 - 1)(x - a)$

sub $x = b, y = 0$: $0 - (a^3 - a) = (3a^2 - 1)(b - a)$

$\Rightarrow a - a^3 = (3a^2 - 1)(b - a)$

$\Rightarrow \dfrac{a - a^3}{3a^2 - 1} = b - a$, $b = \dfrac{a - a^3}{3a^2 - 1} + a$

$\Rightarrow b = \dfrac{a - a^3}{3a^2 - 1} + \dfrac{a(3a^2 - 1)}{3a^2 - 1} = \dfrac{a - a^3 + 3a^3 - a}{3a^2 - 1}$

$\Rightarrow \boxed{b = \dfrac{2a^3}{3a^2 - 1}}$

b. State the values of a for which b does not exist. 1 mark

b diminishes when $3a^2 - 1 = 0$

$a = \pm\dfrac{\sqrt{3}}{3}$

c. State the nature of the graph of g_a when b does not exist. 1 mark

Horizontal tangent lines (with only y-intercepts)

d. **i.** State all values of a for which $b = 1.1$. Give your answers correct to four decimal places. 1 mark

$a = -0.50517315\ldots$, $a = 0.8084077\ldots$

$a = 1.346765 3558\ldots \Rightarrow \begin{cases} a = -0.5052 \\ a = 0.8084 \\ a = 1.3468 \end{cases}$

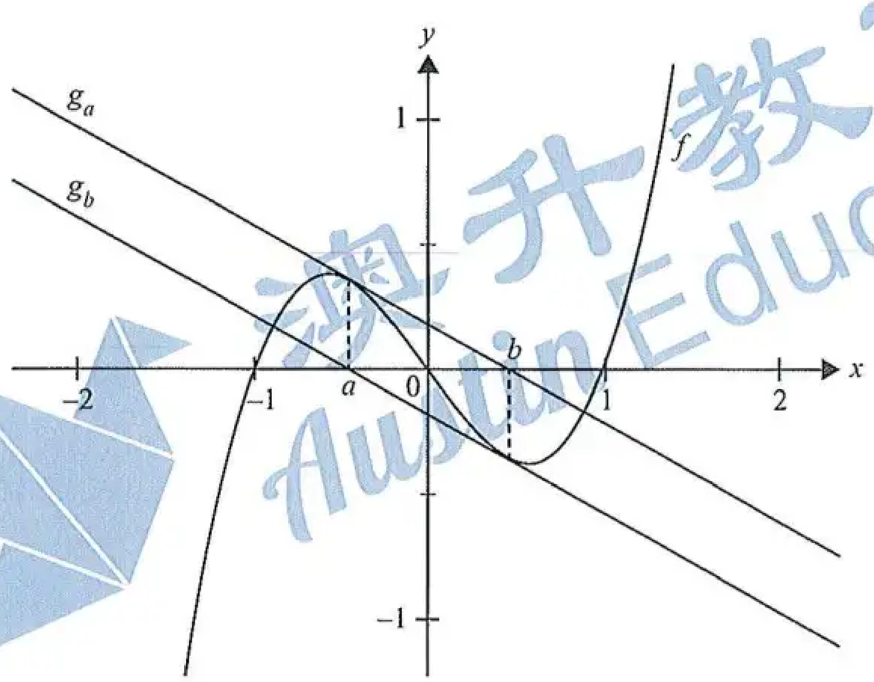
ii. The graph of f has an x-intercept at $(1, 0)$.

State the values of a for which $1 \le b < 1.1$. Give your answers correct to three decimal places. 1 mark

$-0.505 < a \le -0.5$ or $0.808 < a < 1.345$

The coordinate $(b, 0)$ is the horizontal axis intercept of g_a.

Let g_b be the function representing the tangent to the graph of f at $x = b$, as shown in the graph below.



e. Find the values of a for which the graphs of g_a and g_b, where b exists, are parallel and where $b \neq a$. 3 marks

Gradients at $x = a$	Gradients at $x = b$
$g'(a) = 3a^2 - 1$	$g'(b) = 3b^2 - 1$

$3a^2 - 1 = 3b^2 - 1$

$3a^2 - 1 = 3\left(\frac{2a^3}{3a^2-1}\right)^2 - 1 \Rightarrow$ $a = -1,\ a = 1$

$a = 0$

$a = -\frac{\sqrt{5}}{5},\ a = \frac{\sqrt{5}}{5}$

✗ But $a \neq b$

$\therefore a \neq \pm 1$ or 0 $\Rightarrow$ $\boxed{a = \pm\frac{\sqrt{5}}{5} \text{ only}}$

Let $p : R \to R,\ p(x) = x^3 + wx$, where $w \in R$.

f. Show that $p(-x) = -p(x)$ for all $w \in R$. 1 mark

$LHS = p(-x) = (-x)^3 + w(-x) = -x^3 - wx$

$RHS = -p(x) = -(x^3 + wx) = -x^3 - wx$

$\therefore LHS = RHS$, shown

A property of the graphs of p is that two distinct parallel tangents will always occur at $(t, p(t))$ and $(-t, p(-t))$ for all $t \neq 0$.

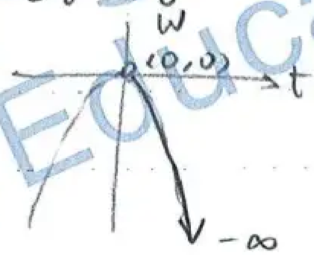
g. Find all values of w such that a tangent to the graph of p at $(t, p(t))$, for some $t > 0$, will have an x-intercept at $(-t, 0)$. 1 mark

The tangent to $y = p(x)$ at $x = t$ is $y = -2t^3 + 3t^2 \cdot x + w(x)$

With $x = -t$, $y = 0$, we get $-wt - 5t^3 = 0$

$\therefore\ w = -5t^2,\ t > 0$

$\therefore\ w < 0$



$\therefore\ w \in (-\infty, 0)$

h. Let $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$, where $m, n \in R\backslash\{0\}$ and $h, k \in R$.

State any restrictions on the values of m, n, h and k, given that the image of p under the transformation T always has the property that parallel tangents occur at $x = -t$ and $x = t$ for **all** $t \neq 0$. 1 mark

~~$m = 1,\ n = 1,\ h = 0,\ k = 0$.~~

Keep it unchanged

As long as the "odd function properties" is unchanged (i.e: rotational symmetry!)

$$\begin{cases} m \in \mathbb{R}\backslash\{0\} \\ n \in \mathbb{R}\backslash\{0\} \\ h = 0 \\ k \in \mathbb{R} \end{cases}$$

Ice on top of the cake!

END OF QUESTION AND ANSWER BOOK

DO NOT WRITE IN THIS AREA