NAME: ________________________________ **TEACHER:** ______________

MELBOURNE HIGH SCHOOL

MATHEMATICAL METHODS UNIT 3
Formative Assessment task 2

Dear Mathematical Methods students,

This is a formative-assessment task that forms part of your school-assessed coursework for Methods 3/4. It is designed to prompt you to investigate a section of the course in greater depth.

This is the **second** of the two tasks for SAC 1. We are anticipating the assessment data from these tasks may be validated after normal school operations. This validation could take the form of Assessment task run concurrently under exam conditions for all students.

We encourage you to examine the material in detail – investigating, exploring, and testing conjectures – rather than completing the tasks in a rush with little thought. Do not copy somebody else's responses, because you will then not only break MHS and VCAA rules for school-assessed coursework (thereby exposing yourself to the consequences of doing so) but also forfeit the opportunities the task provides for you to learn concepts and practice mathematical and other skills that will be useful later in the course and in your university studies.

Please submit your completed tasks via Canvas. Either print a hard copy, complete it by hand, and submit a high-quality scan in pdf format (*not* a photograph taken with a mobile-phone camera), or else write in this document electronically using your touchscreen laptop and stylus and submit your work as a pdf via Canvas.

We encourage you to use any suitable technology. Consider using Mathematica (which is free to all Victorian students under licence from DET) and Desmos in addition to, or instead of, your CAS calculator. You may also choose to use the TI CAS computer software instead of the handheld calculator.

Kind regards,
Ms Rawson and the Mathematical Methods Team

To assist with this task, the following is a definition of Odd and Even functions:
Odd functions satisfy the condition $f(-x) = -f(x)$ whereas even functions satisfy the condition $f(-x) = f(x)$.
You can identify odd and even functions based on their graphs. Odd functions are the same when rotated around the origin 180 degrees and even functions are the same when reflected in the y – axis.

NOTE: In this task, sketch all graphs over an appropriate domain, labelling all key features as exact values when reasonable. Otherwise, round to 2 decimal places.

Question1:

Determine if these functions are odd, even or neither by using both a sketch and an algebraic method

a. $f(x) = x(x-2)(x+2)$

b. $f(x) = 3\sin(2x)$

c. $f(x) = \dfrac{2}{1+2x^2} + 2$

d. $f(x)=\frac{1}{5}xe^{x^2}$

e. $f(x)=-4\cos(3x)+1$

Question 2:

Give an example of an odd, an even, and a function which is neither. Illustrate each with a sketch. (different functions to the ones in question 1. above)

Question 3:

Consider these two functions: $g(x) = x^2$ and $h(x) = 2\cos(2x) + 1$

a. Show algebraically, or with a sketch, that they are both even functions.

b. Write the product of $g(x) = x^2$ and $h(x) = 2\cos(2x) + 1$ as a new function $f(x)$

c. By using an algebraic technique, is this product function found above odd, even or neither?

d. Sketch this function over a suitable domain to confirm your answer to **c**. above.

Question 4:

a. Choose one odd function and one even function from question 1, or from your own choosing, and write the rule for the product of these two functions. Use algebra, or a sketch, to find out if this product function is odd, even, or neither.

b. For the general case, let $f(x)$ be an odd function and $g(x)$ be an even function. Use algebra to show whether the product function, $h(x) = f(x)g(x)$ is odd, even, or neither.

Question 5:

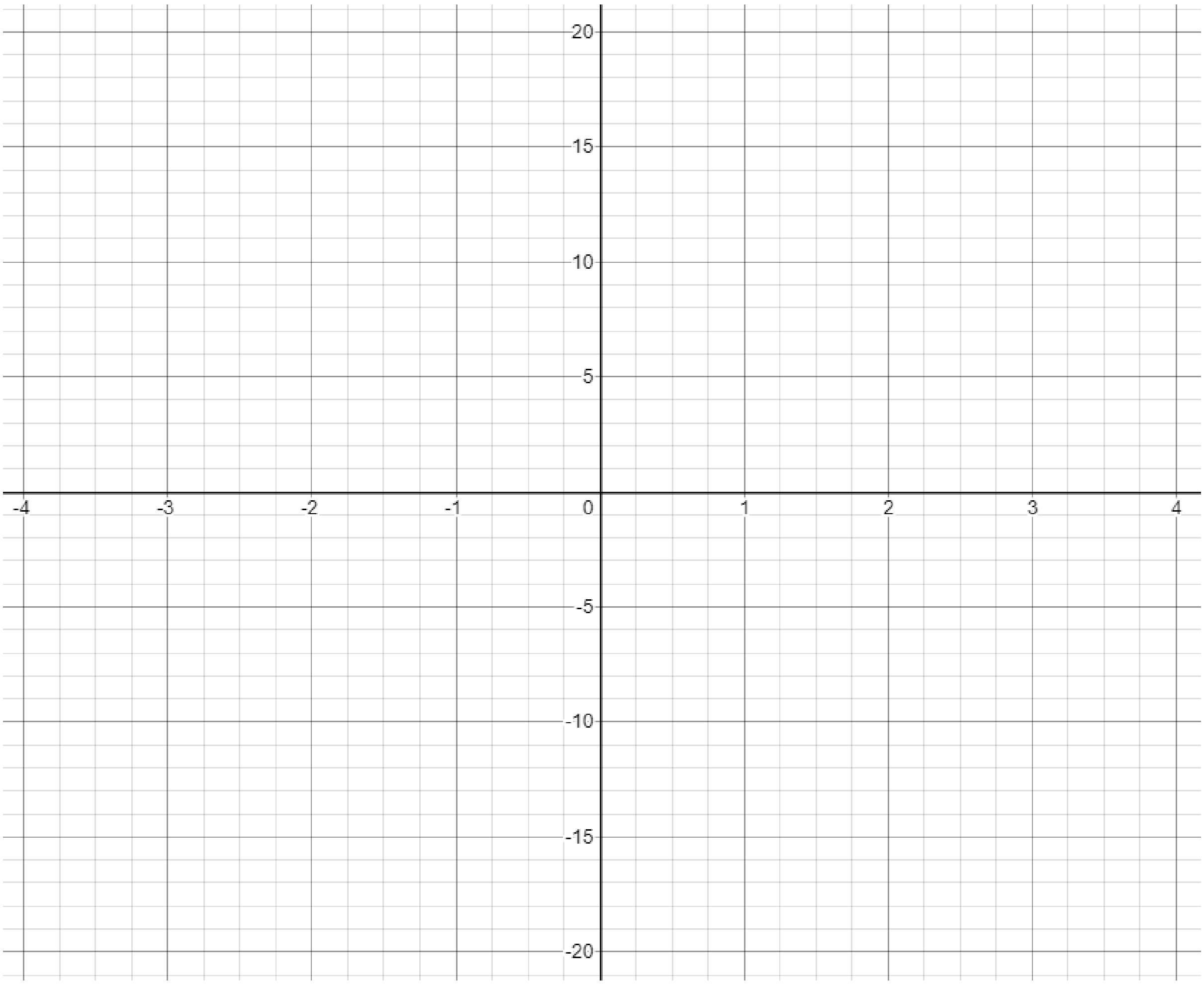
Consider the two even functions $g(x) = x^2$ and $h(x) = \cos(x)$

Find and confirm that $g(h(x))$ and $h(g(x))$ both exist and are both even functions.

Question 6:

Consider the odd function $f(x) = x^3 - 9x$

a. Sketch this function over the domain [-4,4].



b. Find the derivative $f'(x)$ and state if the derivative is an odd or even function.

c. Write down the equations of the tangent lines when $x = -2$ and $x = 2$

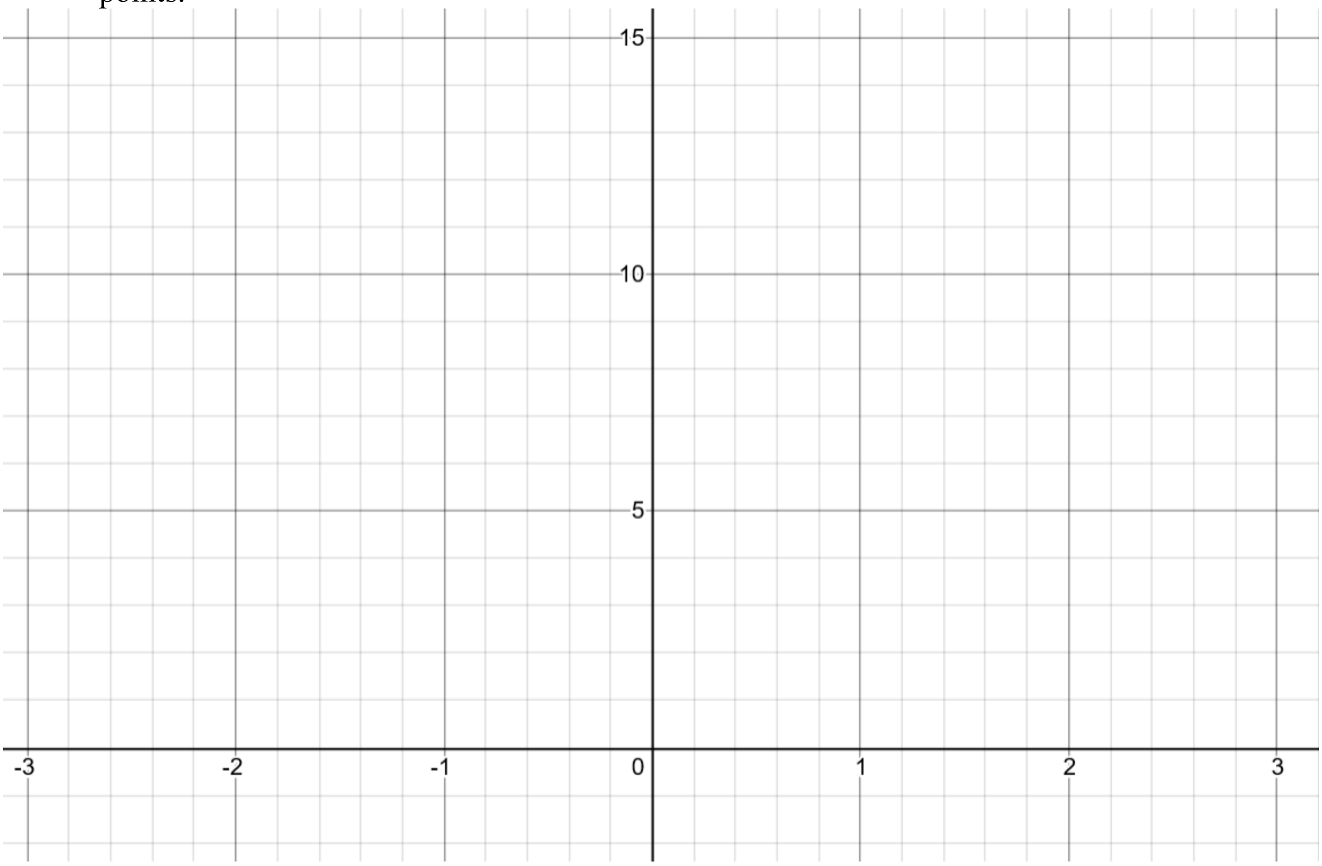
d. Include these 2 lines on your sketch.

e. What things do you observe about these 2 tangent lines?

Question 7:

Consider the even function $h:[-3,0)\cup(0,3]\to R,\ where\ h(x)=\frac{2}{x^2}+1$

a. Sketch include the equation(s) of any asymptote(s) if appropriate and the co-ordinates of all key points.



b. Find the derivative $h'(x)$ and state if the derivative function is odd, even or neither

c. Write down the equations of the tangent lines when $x=-2$ and $x=2$

d. Include these 2 lines on your sketch.

e. What did you observe, if anything, about these 2 tangent lines?

Question 8:

Choose **ONE** property of odd or even functions **NOT** covered in questions 1 to 7 above, to investigate and write a conclusion.

Use the investigative questions above as your guide.

Some ideas:

Product of 2 odd functions.

Composite functions of 2 odd functions or an odd and even function

The powers of odd or even functions

Is it possible for the derivative function to be odd/even if the original function is NOT odd/even?

Proofs of any of the properties stated in this task.

Antiderivatives of odd or even functions.

Proofs of properties about odd and even function derivatives from first principles