Neap

Trial Examination 2020

VCE Mathematical Methods Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

a. degree of the polynomial $P(x) = 3$ A1

b. $P(x) - Q(x) = 3x^3 - 2x^2 + 4x - 5 - (-4x^2 - 3)$

$= 3x^3 - 2x^2 + 4x - 5 + 4x^2 + 3$

$= 3x^3 + 2x^2 + 4x - 2$ A1

c. $P(-2) = 3(-2)^3 - 2(-2)^2 + 4(-2) - 5$

$= -24 - 8 - 8 - 5$

$= -45$ A1

Question 2 (3 marks)

Using re-expression:

$x^3 - 7x^2 + 7x + 15 = x^2(x-3) - 4x(x-3) - 5(x-3)$ A1

$= (x-3)(x^2 - 4x - 5)$ M1

Therefore, $b = -4$, $c = -5$ and $r = 0$. A1

Note: Students may use other acceptable methods.

Question 3 (1 mark)

No, the set of ordered pairs is not a function.

Any one of:

- There is an *x*-value with two different *y*-values: $(-3, 5)$ and $(-3, 10)$.
- The relation contains two sets of ordered pairs with the same first coordinate: $(-3, 5)$ and $(-3, 10)$.
- This set of ordered pairs does not pass the vertical line test as there is more than one value of *y* for the *x*-value of -3.

A1

Question 4 (7 marks)

a. **i.** x-intercept, $y = 0$

$$-2(x-5)^2 + 18 = 0$$
$$-2(x-5)^2 = -18$$
$$2(x-5)^2 = 18$$
$$(x-5)^2 = 9$$
$$x - 5 = \pm 3$$
$$x = 2 \text{ or } 8$$

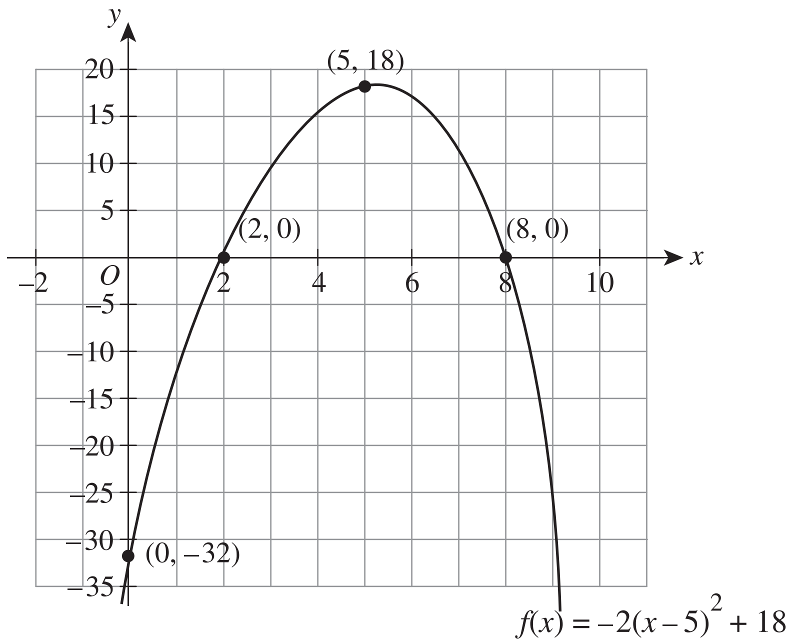
The x-intercepts are $(2, 0)$ and $(8, 0)$. A1

y-intercept, $x = 0$

$$y = -2(0-5)^2 + 18$$
$$= -2 \times 25 + 18$$
$$= -50 + 18$$
$$= -32$$

The y-intercept is $(0, -32)$. A1

ii.



correct shape and turning point (5, 18) A1

correct x-intercepts (2,0) *and* (8, 0) *and y-intercept* (0, –32) A1

b. domain: $x \in R$ A1

range: $y \in (-\infty, 18]$ A1

c. domain: $x \in (-\infty, 5]$ **OR** $x \in [5, \infty)$ A1

Question 5 (4 marks)

Let $y = f(x)$

Let $T = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$

$T^{-1} = \frac{1}{6-0}\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$

$T^{-1} = \frac{1}{6}\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$ M1

$\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$

$TX = X'$

$T^{-1}TX = T^{-1}X'$

$\frac{1}{6}\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$ M1

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$

$x = -\frac{1}{2}x' \qquad y = -\frac{1}{3}y'$ M1

$-\frac{1}{3}y' = -2\left(-\frac{1}{2}x'\right)^2 + 3\left(-\frac{1}{2}x'\right) + 5$

$-\frac{1}{3}y' = -2\left(\frac{1}{4}x'^2\right) - \frac{3}{2}x' + 5$

$-\frac{1}{3}y' = -\frac{1}{2}x'^2 - \frac{3}{2}x' + 5$

$y' = \frac{3}{2}x'^2 + \frac{9}{2}x' - 15$ A1

 MMU12EX1_SS_2020.FM

OR

$X' = TX$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$x' = -2x \Rightarrow x = \dfrac{-x'}{2}$ M1

$y' = -3y \Rightarrow y = \dfrac{-y'}{3}$ M1

$y = -2x^2 + 3x + 5$

$\dfrac{-y'}{3} = -2\left(\dfrac{x'}{2}\right)^2 + 3\left(\dfrac{-x'}{2}\right) + 5$ M1

$\dfrac{-y}{3} = \dfrac{x^2}{2} - \dfrac{3x}{2} + 5$

$y = -\dfrac{3}{2}x^2 + \dfrac{9x}{2} - 15$ A1

Question 6 (3 marks)

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{3(x+h)^2 - 4(x+h) + 5 - (3x^2 - 4x + 5)}{h}$ M1

$= \lim\limits_{h \to 0} \dfrac{3(x^2 + 2xh + h^2) - 4x - 4h + 5 - 3x^2 + 4x - 5}{h}$

$= \lim\limits_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 4h - 3x^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{3h^2 + 6xh - 4h}{h}$ M1

$= \lim\limits_{h \to 0} \dfrac{h(3h + 6x - 4)}{h}$

$= \lim\limits_{h \to 0} (3h + 6x - 4)$

$= 6x - 4$ A1

Question 7 (8 marks)

a. $f(x) = -2x^3 - 5x^2 + x - 2$

$f'(x) = -6x^2 - 10x + 1$ A1

b. i. gradient:

$f'(x) = -6x^2 - 10x + 1$

$f'(-2) = -6(-2)^2 - 10(-2) + 1$

$= -24 + 20 + 1$

$= -3$ A1

equation of the tangent at $(-2, -8)$:

$y - y_1 = m(x - x_1)$

$y + 8 = -3(x - (-2))$

$y + 8 = -3x - 6$

$y = -3x - 14$ A1

Consequential on answer to ***Question 7a.***

ii. gradient:

$f'(-2) = -3$

$m(\text{perpendicular}) = \frac{1}{3}$ A1

equation of the perpendicular line at $(-2, -8)$:

$y - y_1 = m(x - x_1)$

$y + 8 = \frac{1}{3}(x - (-2))$

$y + 8 = \frac{1}{3}x + \frac{2}{3}$

$y = \frac{1}{3}x + \frac{2}{3} - 8$

$y = \frac{1}{3}x + \frac{2}{3} - \frac{24}{3}$

$y = \frac{x}{3} - \frac{22}{3}$ A1

OR $y = \frac{1}{3}(x - 22)$

Consequential on answer to ***Question 7b.i.***

 MMU12EX1_SS_2020.FM

c. $\int_{-1}^{1} f(x)dx = \int_{-1}^{1} (-2x^3 - 5x^2 + x - \log_{10}100)dx$

$= \int_{-1}^{1} (-2x^3 - 5x^2 + x - 2)dx$

$= \left[-\frac{2x^4}{4} - \frac{5x^3}{3} + \frac{x^2}{2} - 2x\right]_{-1}^{1}$ M1

$= \left[-\frac{x^4}{2} - \frac{5x^3}{3} + \frac{x^2}{2} - 2x\right]_{-1}^{1}$

$= \left(-\frac{1}{2} - \frac{5}{3} + \frac{1}{2} - 2\right) - \left(-\frac{1}{2} + \frac{5}{3} + \frac{1}{2} + 2\right)$ M1

$= -\frac{5}{3} - 2 - \frac{5}{3} - 2$

$= -\frac{10}{3} - 4$

$= -\frac{22}{3}$ A1

OR $-7\frac{1}{3}$

Question 8 (2 marks)

$\frac{3^{4 \times 2x} \times 3^{2(x+3)}}{3^{-2x} \times 1} = 3 \times 3$

$\frac{3^{8x} \times 3^{2x+6}}{3^{-2x}} = 3^2$

$3^{8x + 2x + 6 + 2x} = 3^2$

$3^{12x+6} = 3^2$ M1

$12x + 6 = 2$

$12x = -4$

$x = -\frac{1}{3}$ A1

Question 9 (2 marks)

$$3^{\log_3(9)} - \log_2\sqrt{32} = 9 - \log_2(2^5)^{\frac{1}{2}}$$ M1

$$= 9 - \log_2\left(2^{\frac{5}{2}}\right)$$

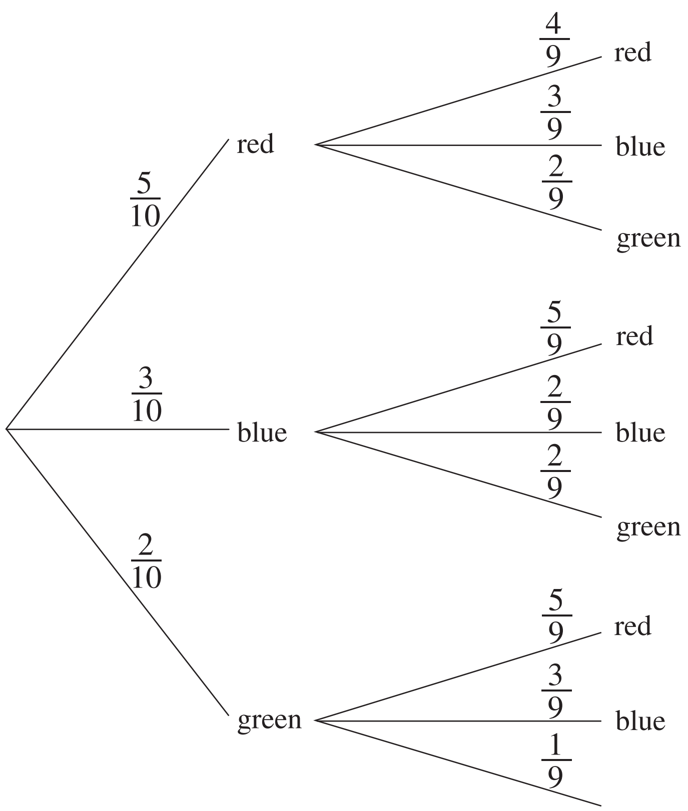
$$= 9 - \frac{5}{2}$$

$$= \frac{13}{2}$$ A1

OR $6\frac{1}{2}$

Question 10 (4 marks)

a.



Possible outcome	**Probability**
red, red	$\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$
red, blue	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
red, green	$\frac{1}{2} \times \frac{2}{9} = \frac{1}{9}$
blue, red	$\frac{3}{10} \times \frac{5}{9} = \frac{1}{6}$
blue, blue	$\frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$
blue, green	$\frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$
green, red	$\frac{1}{5} \times \frac{5}{9} = \frac{1}{9}$
green, blue	$\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$
green, green	$\frac{1}{5} \times \frac{1}{9} = \frac{1}{45}$

correct second branch of tree A1
correct possible outcomes A1
correct probabilities A1

b. $\Pr(\text{green given red first}) = \Pr(\text{green second})\Pr(\text{red first})$

$$= \frac{\Pr(\text{green second } \cap \text{ red first})}{\Pr(\text{red first})}$$

$$= \frac{\frac{1}{9}}{\frac{2}{9} + \frac{1}{6} + \frac{1}{9}}$$

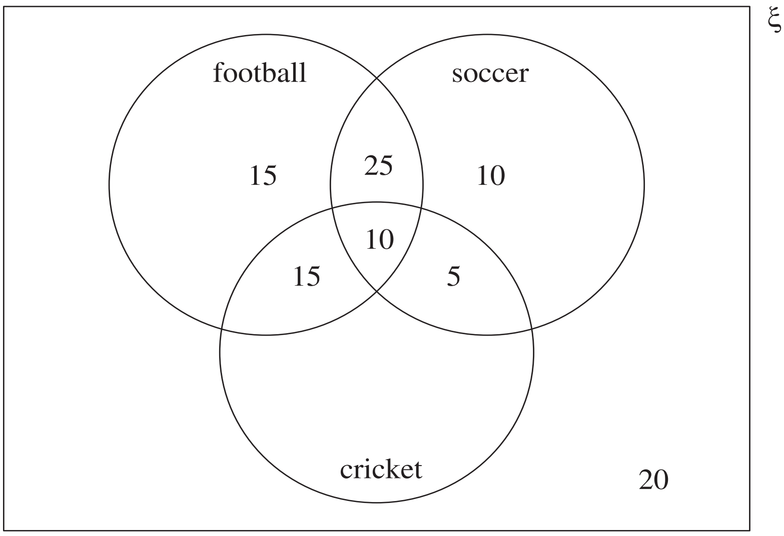
$$= \frac{\frac{2}{18}}{\frac{4}{18} + \frac{3}{18} + \frac{2}{18}}$$

$$= \frac{2}{18} \times \frac{18}{9}$$

$$= \frac{2}{9}$$

A1

Question 11 (3 marks)



universal symbol outside diagram and 20 students outside circles A1
correct 'football' circle A1
correct 'soccer' circle A1